

Contents

1	Introduction	1
1.1	Symmetry: Argument, Principle, and Leitmotif	1
1.2	Operations and Invariants	2
1.3	“Symmetries” in “Fundamental Physics”	4
1.3.1	What is Meant by “Fundamental Physics”?	4
1.3.2	“Physics” on Which Level of Description?	5
1.3.3	Which Kind of “Symmetry”?	9
1.4	The Scope of Symmetries	11
1.4.1	Ontology of Symmetries	11
1.4.2	Symmetry Groups in Fundamental Physics	13
1.4.3	The Use of Symmetries	15
1.5	Bibliographical Notes	17
2	Classical Mechanics	19
2.1	Newtonian and Analytical Mechanics	20
2.1.1	Newtonian Mechanics	20
2.1.2	Lagrange Form of Mechanics	22
2.1.3	Hamiltonian Formulation	24
2.1.4	Principle of Stationary Action	27
2.1.5	*Classical Mechanics in Geometrical Terms	32
2.2	Symmetries and Conservation Laws	37
2.2.1	Conservation Laws	37
2.2.2	Noether Theorem—A First Glimpse	43
2.2.3	Symmetry and Canonical Transformations	51
2.2.4	Conservation Laws and Symmetries	53
2.2.5	*Noether—Geometrically	63
2.3	Galilei Group	64
2.3.1	Transformations and Invariants of Classical Mechanics	64
2.3.2	Structure of the Galilei Group	65
2.3.3	Lie Algebra of the Galilei Group	66
2.4	Concluding Remarks and Bibliographical Notes	67

3	Electrodynamics and Special Relativity	69
3.1	Electrodynamics à la Maxwell	69
3.1.1	Maxwell Equations	69
3.1.2	Lorentz Boosts	70
3.2	Special Relativity	72
3.2.1	“Deriving” Special Relativity	72
3.2.2	Minkowski Geometry	76
3.2.3	Relativistic Mechanics	81
3.2.4	Relativistic Field Theory	84
3.3	Noether Theorems	90
3.3.1	Variational Symmetries in Field Theories	91
3.3.2	Global Symmetries and 1st Noether Theorem	94
3.3.3	Local Symmetries and 2nd Noether Theorem	101
3.3.4	Further Topics Relating to Variational Symmetries	106
3.4	Poincaré Transformations	113
3.4.1	Poincaré and Lorentz Groups	113
3.4.2	Poincaré Algebra	115
3.4.3	Galilei and Bargmann Algebra	117
3.4.4	Forms of Relativistic Dynamics	118
3.4.5	Kinematical Groups and Their Mutual Contractions	119
3.5	*Generalizations of Poincaré Symmetry	125
3.5.1	Conformal Symmetry	125
3.5.2	de Sitter Group	134
3.6	On the Validity of Special Relativity	138
3.7	Concluding Remarks and Bibliographical Notes	140
4	Quantum Mechanics	143
4.1	Principles of Quantum Mechanics	143
4.1.1	Hilbert Space	144
4.1.2	Operators	144
4.1.3	States, Observables, and Measurements	146
4.1.4	Time Evolution	148
4.2	Symmetry Transformations in Quantum Mechanics	150
4.2.1	Wigner Theorem	151
4.2.2	Symmetry Transformations and Observables	153
4.2.3	“Noether Theorem of Quantum Mechanics”	153
4.2.4	Symmetries and Superselection Rules	154
4.3	Quantum Physics and Group Representation	156
4.3.1	Why Group Representation?	156
4.3.2	Galilei Operators	156
4.3.3	Bargmann Group	160
4.3.4	Symmetries of the Schrödinger Equation	162
4.4	Concluding Remarks and Bibliographical Notes	168

5	Relativistic Field Theory	169
5.1	Representations of the Poincaré Group	171
5.1.1	Global Structure of ISO (3, 1)	172
5.1.2	Transformation of the Generators	172
5.1.3	The “Little Group”	173
5.1.4	Classification of Particles	176
5.2	Symmetry and Quantum Field Theory	181
5.2.1	Lorentz Symmetry Rules Field Variants	181
5.2.2	Representations of SL(2, \mathbb{C})	182
5.2.3	Field Variants	183
5.2.4	Quantum-Field Theoretical Symmetry Transformations	185
5.3	Actions	186
5.3.1	Requirements on a QFT Action	186
5.3.2	Scalar Fields	188
5.3.3	Spinor Actions	194
5.3.4	Gauge Vector Fields	203
5.3.5	Higher-Spin Fields	218
5.4	Spontaneous Symmetry Breaking	221
5.4.1	Goldstone Bosons	222
5.4.2	Nambu-Goldstone Model	224
5.4.3	Higgs Mechanism	227
5.5	Discrete Symmetries	230
5.5.1	General Preliminary Remarks and Definition of Terms	230
5.5.2	Space Inversion P	232
5.5.3	Time Reversal T	235
5.5.4	Charge Conjugation C	237
5.5.5	CPT Theorem	238
5.6	Effective Field Theories	240
5.6.1	EFT: The Very Idea	240
5.6.2	Historical Examples	241
5.6.3	Renormalization (Group)	243
5.6.4	Chain of Effective Theories	248
5.7	Concluding Remarks and Bibliographical Notes	251
6	Particle Physics	259
6.1	Particles and Interactions	259
6.1.1	Standard Model Constituents	259
6.1.2	Quarks as Building Blocks of Hadrons	262
6.1.3	Interaction Processes	270
6.1.4	Lagrangian of the Standard Model	273
6.2	Strong Interactions	274
6.2.1	Lagrangian of Quantum Chromo Dynamics	274

6.2.2	Symmetries of QCD	275
6.2.3	Theoretical Consistency and Experimental Support	277
6.3	Weak and Electromagnetic Interaction	278
6.3.1	Fermi-Type Model of Weak Interactions	280
6.3.2	Current Algebra	281
6.3.3	Glashow-Salam-Weinberg Model	285
6.3.4	Theoretical Consistency and Experimental Support	290
6.4	Paralipomena on the Standard Model	291
6.4.1	Limits of the Standard Model	291
6.4.2	Massive Neutrinos	293
6.4.3	Anomalies	297
6.4.4	Strong CP Problem	299
6.4.5	Standard Model and Effective Field Theories	301
6.5	Concluding Remarks and Bibliographical Notes	306
7	General Relativity and Gravitation	309
7.1	Introductory Remarks	309
7.2	Equivalence Principle	311
7.2.1	Different Versions of the Equivalence Principle	311
7.2.2	Reference Systems and Gravitation	314
7.2.3	Geodesics	315
7.2.4	The “Principle” of General Covariance	319
7.3	Riemann-Cartan Geometry	319
7.3.1	Tensors	320
7.3.2	Affine Connection and Covariant Derivative	321
7.3.3	Torsion and Curvature	324
7.3.4	Metric	325
7.3.5	Tetrads and Spin Connections	329
7.4	Physics in Curved Spacetime	334
7.4.1	Mechanics, Hydrodynamics, Electrodynamics	334
7.4.2	Coupling Relativistic Fields to Gravity	337
7.5	Geometrodynamics	339
7.5.1	Field Equations	339
7.5.2	Action Functionals for General Relativity	343
7.5.3	Covariance, Invariance, and Symmetries	356
7.5.4	Noether Identities and Conservation Laws	361
7.6	Modifications and Extensions of/to General Relativity	375
7.6.1	Interpreting GR as a Spin-2 Field Theory	375
7.6.2	Altering the Geometry	377
7.6.3	Gravitation as a Gauge Theory	381
7.6.4	Changing Structures and Modifying Principles	392
7.7	Concluding Remarks and Bibliographical Notes	399

8	*Unified Field Theories	401
8.1	Grand Unified Theories	401
8.1.1	Motivation and Basic Concepts.	401
8.1.2	SU(5) Grand Unification	404
8.1.3	SO(10) Grand Unification	408
8.1.4	Instead of a Conclusion	409
8.2	Kaluza-Klein Theory	409
8.2.1	Kaluza's and Klein's Contributions to the KK Theory.	409
8.2.2	The 5D Model	412
8.2.3	Beyond Five Dimensions: Einstein-Yang-Mills Theory.	419
8.2.4	Instead of a Conclusion	431
8.3	Supersymmetry	432
8.3.1	Why Supersymmetry?	432
8.3.2	Compelling Consequences of Fermi-Bose Symmetry.	434
8.3.3	Global Supersymmetry	436
8.3.4	Local Supersymmetry and Supergravity.	451
8.3.5	Instead of a Conclusion	458
8.4	Further Speculations	459
8.4.1	Compositeness and Technicolor	459
8.4.2	Strings and Branes	460
8.4.3	Gauge/Gravity Duality Conjecture.	467
9	Conclusion	469
9.1	Symmetries: The Road to Reality	469
9.1.1	Symmetry: The Golden Thread.	469
9.1.2	The " <i>Weltgesetze</i> " and Their Symmetries	473
9.1.3	History of Symmetry Considerations	476
9.2	Are Symmetries a Principle of Nature?	486
9.2.1	...and Other Philosophical Questions	486
9.2.2	Symmetries and the Unification of Physics	488
9.2.3	Laws of Nature and Principles of Physics	494
9.2.4	Origin of Symmetries	500
9.3	Physics Beyond Symmetries	501
9.3.1	Prominent Non-Symmetries	501
9.3.2	Other Notions of Fundamental Physics	502
9.3.3	Are we Biased, or Haughty, or Simply in a Specific World?	506
	Appendix A: Group Theory	509
A.1	Basics	509
A.1.1	Definitions: Algebraic Structures.	509
A.1.2	Mapping of Groups	513
A.1.3	Simple Groups	514

A.2	Lie Groups	517
A.2.1	Definitions and Examples.	517
A.2.2	Generators of a Lie Group	521
A.2.3	Lie Algebra Associated to a Lie Group	522
A.2.4	Inönü–Wigner Contraction of Lie Groups	526
A.2.5	Classification of Lie Groups.	528
A.2.6	Infinite-Dimensional Lie Groups.	530
A.3	Representation of Groups	532
A.3.1	Definitions and Examples.	532
A.3.2	Representations of Finite Groups	533
A.3.3	Representation of Continuous Groups	536
A.3.4	Examples: Representations of $SO(2)$, $SO(3)$, $SU(3)$	539
A.3.5	Projective Representations and Central Charges	546
	Appendix B: Spinors, \mathbb{Z}_2-gradings, and Supergeometry.	551
B.1	Spinors	551
B.1.1	Pauli and Dirac Matrices	552
B.1.2	Weyl Spinors	554
B.1.3	Spinors and Tensors	557
B.1.4	Dirac and Majorana Spinors	558
B.2	\mathbb{Z}_2 -Gradings	561
B.2.1	Definitions	561
B.2.2	Supertrace and Superdeterminant.	561
B.2.3	Differentiation and Integration	563
B.2.4	Pseudo-Classical Mechanics	565
B.3	*Supergeometry.	567
B.3.1	Superspace	567
B.3.2	Superfields	568
B.3.3	Superactions	574
B.4	*Supergroups.	580
B.4.1	$OSp(N/M)$ and the Super-Poincaré Algebra	580
B.4.2	$SU(N/M)$ and the Super-Conformal Algebra.	581
	Appendix C: Symmetries and Constrained Dynamics	583
C.1	Constrained Dynamics	583
C.1.1	Singular Lagrangians	583
C.1.2	Constraints as a Consequence of Local Symmetries.	585
C.1.3	Rosenfeld-Dirac-Bergmann Algorithm	586
C.1.4	First-Class Constraints and Symmetries	592
C.1.5	Second-Class Constraints and Gauge Conditions.	599
C.1.6	Constraints in Field Theories: Some Remarks.	601
C.1.7	Quantization of Constrained Systems.	602
C.2	Yang-Mills Type Theories	604
C.2.1	Electrodynamics	604

C.2.2	Maxwell-Dirac Theory	606
C.2.3	Non-Abelian Gauge Theories	608
C.3	Reparametrization-Invariant Theories	610
C.3.1	Immediate Consequences of Reparametrization Invariance.	610
C.3.2	Free Relativistic Particle	612
C.3.3	Metric Gravity	622
C.3.4	Tetrad Gravity	638
C.3.5	Einstein-Dirac-Yang-Mills-Higgs Theory	641
C.4	Alternative Approaches.	644
C.5	Constraints and Presymplectic Geometry	646
C.5.1	Legendre Projectability	646
C.5.2	Symmetry Transformations in the Tangent and Cotangent Bundle	648
C.5.3	Constraint Stabilization	651
 Appendix D: *Symmetries in Path-Integral and BRST		
	Quantization.	653
D.1	Basics	654
D.1.1	Path-integral Formulation of Quantum Mechanics.	654
D.1.2	Functional Integrals in Field Theory	659
D.1.3	Faddeev-Popov Ghost Fields in Theories with Local Symmetries	662
D.2	Noether and Functional Integrals	667
D.2.1	Noether Currents and Ward-Takahashi-Slavnov-Taylor Identities	667
D.2.2	Quantum Action and its Symmetries	668
D.2.3	BRST Symmetries.	670
D.2.4	Fujikawa: Fermionic Path Integrals and Anomalies	676
 Appendix E: *Differential Geometry		
E.1	Differentiable Manifolds	680
E.1.1	From Topological Spaces to Differentiable Manifolds.	680
E.1.2	Tensor Bundles	681
E.1.3	Flows and the Lie Derivative	684
E.1.4	Symplectic Manifolds	685
E.2	Cartan Calculus	686
E.2.1	Differential Forms	686
E.2.2	Differentiation with Respect to a Form	688
E.2.3	Hodge Duality*	688

E.2.4	Integration of Differential Forms and Stokes's Theorem.	690
E.2.5	Poincaré Lemma and de Rham Cohomology.	692
E.3	Manifolds with Connection	694
E.3.1	Linear Connection on Tensor Fields	694
E.3.2	Covariant Derivative	695
E.3.3	Torsion and Curvature	695
E.4	Lie Groups	698
E.4.1	Lie Algebra	698
E.4.2	Group Covariant Derivative	700
E.4.3	Group Curvature	701
E.4.4	Isometries and Coset Manifolds.	702
E.5	Fibre Bundles	706
E.5.1	Definition, Various Types, and Examples of Fibre Bundles	706
E.5.2	Connections in Fibre Bundles	709
E.5.3	Yang-Mills Gauge Field Theory in Fibre Bundle Language	712
E.5.4	Metric and Tetrad Gravity on a Bundle	714
Appendix F: *Symmetries in Terms of Differential Forms		719
F.1	Actions and Field Equations	719
F.1.1	The "World" Action	719
F.1.2	Field Equations	724
F.2	Symmetries	731
F.2.1	Generic Variational Symmetries	731
F.2.2	Lorentz Transformations.	735
F.2.3	Diffeomorphisms	737
F.3	Gravitational Theories.	744
F.3.1	Energy-Momentum Conservation.	744
F.3.2	Gravitational Theories Beyond Einstein	750
F.3.3	Topological Terms.	753
F.4	Gauge Theories	756
F.4.1	Global Symmetry.	756
F.4.2	Local Gauge Transformations	757
References		761
Index		783

Symmetries in Fundamental Physics

Sundermeyer, K.

2014, XXVIII, 788 p. 28 illus., Hardcover

ISBN: 978-3-319-06580-9