Preface

The classical Pontryagin maximum principle (addressed to deterministic finite dimensional control systems) is one of the three milestones in modern control theory. The corresponding theory is by now well-developed in the deterministic infinite dimensional setting and for the stochastic differential equations. However, very little is known about the same problem but for controlled stochastic (infinite dimensional) evolution equations when the diffusion term contains the control variables, and the control domains are allowed to be non-convex. Indeed, it is one of the longstanding unsolved problems in stochastic control theory to establish the Pontryagin-type maximum principle for this kind of general control systems. This book aims to give a solution to this problem.

One of the main contributions (and also the most difficult part) in this work is to establish the well-posedness and "regularity" theory (see Chaps. 5–7) for the operator-valued backward stochastic evolution equation (BSEE for short) (1.10). Unlike the finite dimensional case, there exists essential difference between the vector-valued and operator-valued BSEEs. Indeed, in the infinite dimensional setting, there exists no such stochastic integration/evolution equation theory (in the previous literature) that can be employed to treat the well-posedness of such equation.

To overcome the above-mentioned difficulty, we need to introduce a new concept of solution, i.e., relaxed transposition solution, to the operator-valued BSEE (1.10), and study its well-posedness. This is motivated by the classical transposition method to solve the nonhomogeneous boundary value problems for (deterministic) partial differential equations [17], and especially the boundary controllability problem for (deterministic) hyperbolic equations [16]. We remark that, in the stochastic setting, the transposition method was firstly introduced in our previous paper [20], but one can find a rudiment of our method at [31, pp. 353–354].

Our new method has several advantages:

(1) The usual duality relationship is contained in our definition of solution, and therefore, we do NOT need to use the infinite dimensional Itô formula (valid only under some strong assumptions) to derive this sort of relation as usual to obtain the desired stochastic maximum principle.

(2) We do NOT need to use the martingale representation theorem and any other deep result in martingale theory, and therefore we can study the problem with general filtration.
(3) Thanks to its variational formulation, similarly to the classical finite element method of solving deterministic partial differential equations, our transposition method leads naturally numerical schemes to solve both vector-valued and operator-valued BSEEs (although the detailed analysis is beyond the scope of this book).

As a by-product of this work, we obtain (in Chap. 5) some weakly sequential Banach-Alaoglu-type theorems for uniformly bounded linear operators between Banach spaces. We believe that these sequential compactness results (say Theorem 5.4) have some independent interest and may be applied in other places.

We have tried our best to make this book to be as self-contained as possible. Also, for the readers' convenience, we provide considerably detailed proof for most of the results that appeared in the text. We expect that this book is useful for both beginners and experts who are interested in optimal control theory for stochastic evolution equations.

Finally, we mention that the first version of this book was posted at arXiv on April 15, 2012 (See http://arxiv.org/abs/1204.3275). Also, two recent papers "Du, K., Meng, Q.: A maximum principle for optimal control of stochastic evolution equations. SIAM J. Control Optim. **51**, 4343–4362 (2013)" and "Fuhrman, M., Hu, Y., Tessitore, G.: Stochastic maximum principle for optimal control of SPDEs. Appl. Math. Optim. **68**, 181–217 (2013)" generalized/improved part of the results in this book.

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