

Preface

Dear reader!

We are pleased to present the book *Nonlinear Deterministic Systems: A Short Course*. Nonlinear dynamics has gradually become an independent area of study over the last 30–35 years. However, this scientific field still continues to develop intensively. Nonlinear dynamics is based on input from a range of fundamental scientific disciplines, such as the nonlinear theory of oscillations and waves, the theory of dynamical systems, synergetics, statistical physics, and thermodynamics. When this field of research first began, the main problems to be tackled concerned the dynamical processes and effects realized in nonlinear deterministic systems. Research over the last 10–15 years has shown that complex and interesting phenomena caused by fluctuations can also be considered in the framework of nonlinear dynamics. Nowadays, a new direction has arisen as an integral part of nonlinear dynamics. It can be described as the nonlinear dynamics of stochastic systems. It is clear that stochastic processes cannot be analyzed in detail without a deep understanding of the dynamical properties of the relevant deterministic systems. Therefore, the nonlinear dynamics of deterministic systems is a fundamentally important part of modern nonlinear dynamics as a whole.¹

The book is based on selected lectures from the course entitled *Introduction to Nonlinear Dynamics*, delivered to students of the Physics Department of Saratov State University by one of the authors. The content of the lectures has been significantly revised and supplemented.

The book contains 15 chapters. Each can be considered as a lecture on the relevant problem and studied almost independently of the others. The first three chapters are devoted to the classical problems of the theory of dynamical systems. The definition and classification of dynamical systems are given in Chap. 1, the fundamentals of the linear theory of stability are discussed in Chap. 2 as applied to

¹Note that, in our book, only dissipative dynamical systems are analyzed and conservative (Hamiltonian) systems are not considered.

differential and discrete-time systems, and the elements of the theory of local and nonlocal bifurcations are described in Chap. 3.

Chapter 4 is devoted to the dynamics of self-sustained oscillatory systems with one degree of freedom. This is exemplified by the van der Pol oscillator with soft and hard excitation. The dynamics of systems with phase space dimension $N \geq 3$ is described qualitatively in Chap. 5. We focus on physical explanation of the onset of dynamical chaos and the description of typical properties of strange attractors. The next two chapters are devoted to the mechanisms of transition to chaotic dynamics through a cascade of period-doubling bifurcations and intermittency (Chap. 6) and through the destruction of quasiperiodic oscillations (Chap. 7). The problem of structural stability is discussed in Chap. 8. The notion of a nonhyperbolic attractor is introduced and chaotic attractors are classified.

Chapter 9 is devoted to recent results on the problem of Poincaré recurrence and describes a global approach to this problem, which is related to the Afraimovich–Pesin dimension of a Poincaré recurrence time sequence. The main theoretical features are confirmed by several simple examples of numerical calculations. Chapter 10 serves as a lecture on the fundamentals of the theory of fractals and its application to nonlinear dynamics. The concept of a fractal is introduced and the fractal nature of sets in the phase space of a dynamical system is discussed.

Chapters 11 and 12 are concerned with several basic models of oscillators with chaotic and quasiperiodic oscillations. The dynamics of the Anishchenko–Astakhov oscillator is analyzed in detail in Chap. 11. The theoretical and experimental results presented in the study of oscillator dynamics illustrate Shilnikov’s theorem for systems with a saddle-focus separatrix loop. An autonomous two-frequency oscillator is described in Chap. 12. It is shown that the oscillator can realize quasiperiodic two-frequency oscillations and transition to chaos through two-dimensional torus-doubling bifurcations.

The last three chapters examine synchronization effects in periodic (Chap. 13), two-frequency quasiperiodic (Chap. 14), and chaotic (Chap. 15) self-sustained oscillations.

The content of the book should satisfy the needs of students and researchers in the field of nonlinear dynamics. No special mathematical knowledge is assumed. The usual university courses on higher mathematics and the theory of oscillations would serve as a sufficient basis. The book is aimed mainly at masters and PhD students and young scientists working in this field. At the same time, some of the chapters could be used to give lectures to advanced natural science students in other fields.

The book has a list of recommended references that includes monographs and textbooks. These should be helpful to the reader who wishes to study the relevant chapters of the book in greater depth. We do not give references to the original scientific papers. If necessary, the reader can find the relevant information by using the detailed bibliographies provided in the recommended books.

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