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★ **Variational methods in nonlinear field equations.**

Solitary waves, holomorphic solitons and vortices.

Springer Monographs in Mathematics.

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In the book under review, the authors take a bold and comprehensive approach to addressing the utility of variational techniques in the field of nonlinear equations and highlighting the underlying beauty of physical problems in the context of a variational framework. This book is clearly intended for professional research scientists and mathematicians who desire to understand the appropriate paradigmatic stage upon which one could build a variational framework for some nonlinear evolution equations. The exposition offered by Benci and Fortunato is highly explanatory, comprehensive, and in many ways novel.

The book opens with a chapter on general principles in which they offer a highlighted tour of the variational principle, the invariance principle, conservation laws and Hamilton-Jacobi theory. The authors do an outstanding job of clearly presenting these basic notions, though most researchers using this text will find the information in the first chapter less utilitarian and more emblematic of a beautiful development of an underlying theory already widely understood by specialists in this area of research. For instance, the authors present in the first chapter a rather non-traditional proof of Noether's theorem. The inclusion of this proof is less for the reader to gain an understanding of the utilization of Lie group symmetries in constructing conservation laws than for the reader to have a context of Noether's theorem in a suitable form for the applications presented later in the book.

The second chapter contains material upon which many results in this book are based. The heart of this chapter is a functional abstract framework through which solitons, solitary waves, and holomorphic solitons are defined. The chapter culminates with a series of abstract existence theorems contextualized in the aforementioned framework. These results are then used heavily throughout the remainder of the text.

Chapters 3 through 8 comprise the bulk of this text, offering applications of a variational framework in the context of the nonlinear Schrödinger equation, nonlinear Klein-Gordon equation, nonlinear Klein-Gordon-Maxwell equations, and nonlinear Schrödinger-Maxwell equations, as well as applications to nonlinear beam equations and vortices. The organization of this material is laid out in a thoughtful and meaningful manner. There are many "goodies" buried within the text. For instance, these reviewers found the inclusion of relativistic considerations in the chapter on the nonlinear Klein-Gordon equation to be a truly fun read. From space contraction and time dilation of solitary waves, considerations of the mass of solitary waves, the variational formulation of the Einstein equation, and a focus on understanding fields and particles in this interpretive context, it is clear that the authors had an eclectic audience in mind when writing this book.

In summary, this book is a useful, enlightening, and thought-provoking exposition of the research and findings over the last 35 years of using and implementing variational techniques to better understand the dynamics of nonlinear field equations. These reviewers would highly recommend this text be on the shelf of any applied mathemati-

cian/physicist who uses variational techniques to analyze nonlinear field equations.

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Variational Methods in Nonlinear Field Equations
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