

Chapter 2

Theoretical Background

This chapter reviews the theoretical foundation of the work presented in this thesis. Section 2.1 outlines the main features of the Standard Model of particle physics. Section 2.2 gives an introduction to Supersymmetry. The content is taken from the referenced sources. Parts of this chapter are taken from the diploma thesis of the author [1] and have been adapted according to the latest developments.

2.1 The Standard Model of Particle Physics

Today's experimentally verified knowledge of the fundamental particles and their interactions is summarized in the Standard Model of particle physics (SM).

Within the Standard Model, the elementary constituents of matter are 12 spin-1/2 fermions and their respective antiparticles. They can be further classified according to their interactions into quarks and leptons, for each of which three generations of particle pairs exist. For both fermion species the second and third generations are heavier copies¹ of the first generation with identical quantum numbers. A summary of the Standard Model fermions is shown in Table 2.1 (left).

Quarks participate in the strong, weak and electromagnetic interactions and therefore carry color, electric charge and weak isospin. For each generation there is one up-type quark with electric charge $+2/3$ ("up", "charm", "top") and one down-type quark with $-1/3$ ("down", "strange", "bottom"). The lepton generations consist of one electron-type lepton ("electron", "muon", "tau") with electric charge -1 and a neutral almost massless lepton-neutrino. Whereas electron-type leptons are charged and participate in electromagnetic and weak interactions, neutrinos can only interact weakly due to their lack of electric charge.

The interactions between the matter particles are mediated by the spin-1 gauge bosons summarized in Table 2.1 (right). Their existence arises from invariance of the respective interaction Lagrangians under local symmetry transformations. The

¹ Not yet established for neutrinos.

Table 2.1 Summary of the experimentally measured spin- $\frac{1}{2}$ fermions (left) and the spin-1 gauge bosons (right) of the Standard Model of particle physics

Fermions	1. Generation	2. Generation	3. Generation	Bosons
Quarks	u $m=2.3^{+0.7}_{-0.5}$ MeV $Q=2/3$	c $m=1.28\pm0.03$ GeV $Q=2/3$	t $m=173.5\pm0.6\pm0.8$ GeV $Q=2/3$	γ $m=0$ $Q=0$
	d $m=4.8^{+0.7}_{-0.3}$ MeV $Q=-1/3$	s $m=95^{+5}_{-5}$ MeV $Q=-1/3$	b $m=4.19\pm0.03$ GeV $Q=-1/3$	g $m=0$ $Q=0$
Leptons	ν_e $m<2.05$ eV $Q=0$	ν_μ $m<0.19$ MeV $Q=0$	ν_τ $m<18.2$ MeV $Q=0$	Z^0 $m=91.19\pm0.002$ GeV $Q=0$
	e $m=0.511$ MeV $Q=-1$	μ $m=106$ MeV $Q=-1$	τ $m=1.78$ GeV $Q=-1$	W^\pm $m=80.39\pm0.02$ GeV $Q=\pm1$

The numbers in small font correspond to the electric charge Q in units of the elementary charge e and the mass m of the particles as given by [2]. The very small uncertainties on the masses of the charged leptons are not quoted. The existence of the spin-0 Standard Model Higgs boson has not yet been fully established at the time of writing and is therefore not listed here

underlying mathematical structure is the direct product of the internal symmetry groups $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$, where $SU(3)_C$ describes the strong force mediated by eight gluons and $SU(2)_L \otimes U(1)_Y$ the unified electroweak force mediated by W^\pm , Z bosons and the photon γ . Gravitational interactions are not incorporated into the Standard Model.

The unification of the weak and electromagnetic forces happens by virtue of the so-called Glashow-Salam-Weinberg (GSW) mechanism [3–5], in which a local gauge transformation of the left-handed weak isospin doublets $SU(2)_L$ and the $U(1)_Y$ multiplets and singlets with respect to hypercharge Y takes place. The invariance of the Lagrangian under these transformations necessitates the introduction of new massless vector fields $W^{1,2,3}$ and B , of which the physically observed W^\pm , Z and γ bosons are linear combinations. The physical eigenstates of the latter two bosons are additionally rotated by a so-called Weinberg-angle θ_W .

The Standard Model particles acquire their masses through interaction with a scalar Higgs background field, which spontaneously breaks the electroweak symmetry. The introduction of this breaking mechanism is necessary since gauge invariance requires massless gauge bosons, which contradicts experimental results. The neutral spin-0 Higgs boson associated to this mechanism has been the subject of many experimental searches. Recently the discovery of a new particle has been reported which could finally bring the long awaited experimental verification of this last missing piece of the Standard Model. Further details on this subject are given in Sect. 2.1.6.

The mathematical framework behind the Standard Model is a relativistic quantum field theory which can be derived using the Lagrangian formalism. The starting point is the Lorentz invariant scalar Lagrangian density \mathcal{L} which describes the dynamics of the system of interest. The application of Hamilton’s principle of least action, $\delta S = 0$, where the action S is defined as

$$S = \int d^4x \mathcal{L}, \quad (2.1)$$

then leads to the Euler–Lagrange equations from which the explicit equations of motion for the considered fields can be derived. While every global symmetry of the Lagrangian up to a total derivative leads to a conservation law according to Noether’s theorem [6], invariance with respect to local gauge transformations implies the introduction of new gauge fields which mediate the fundamental interactions of the Standard Model.

In the following sections the theory of the electromagnetic, weak, and strong interactions of the Standard Model as well as the Higgs mechanism is derived using the Lagrangian formalism.

2.1.1 Electromagnetic Interaction

The free propagation of the fermions of the Standard Model is described by the Dirac equation,

$$(i\gamma^\mu \partial_\mu - m)\psi = 0. \quad (2.2)$$

Here ψ is the four-component spinor representing the fermionic field, m the rest mass of the associated particle, and γ^μ are the Dirac matrices, defined as

$$\gamma^0 = \begin{bmatrix} \mathbb{1}_2 & 0 \\ 0 & \mathbb{1}_2 \end{bmatrix}, \quad \gamma^k = \begin{bmatrix} 0 & \sigma^k \\ -\sigma^k & 0 \end{bmatrix}, \quad k = 1, 2, 3, \quad (2.3)$$

where σ^k are the Pauli matrices,

$$\sigma^1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \sigma^2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \quad \sigma^3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}. \quad (2.4)$$

Every component of the spinor ψ is a function of the space-time coordinates, which in their contravariant form are written as

$$x^\mu = (t, x, y, z) = (x^0, x^1, x^2, x^3), \quad (2.5)$$

with the corresponding covariant derivative defined as

$$\partial_\mu = \frac{\partial}{\partial x^\mu} = \left(\frac{\partial}{\partial t}, \vec{\nabla} \right) = \left(\frac{\partial}{\partial x^0}, \frac{\partial}{\partial x^1}, \frac{\partial}{\partial x^2}, \frac{\partial}{\partial x^3} \right). \quad (2.6)$$

In the Lagrangian formalism the Dirac equation 2.2 follows from the Lagrangian

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi, \quad (2.7)$$

with the adjoint spinor $\bar{\psi}$ defined as

$$\bar{\psi} = \psi^\dagger \gamma^0, \quad (2.8)$$

where ψ^\dagger is the hermitian conjugate of the spinor. This Lagrangian is invariant under a global transformation of the type

$$\psi \rightarrow \psi' = e^{iq\alpha} \psi \quad (2.9)$$

which, following Noether's theorem, leads to the conservation of the electromagnetic current

$$j^\mu = q \bar{\psi} \gamma^\mu \psi \quad (2.10)$$

with the electric charge q . Here q is defined as $q = Qe$, where Q is the charge quantum number of the particle involved (see Table 2.1) and e is the elementary charge. The transformations of Eq. 2.9 with the real parameter α build the group of unitary transformations $U(1)$.

To introduce the electromagnetic interaction to the theory the symmetry of the Lagrangian has to be extended to local gauge transformations of the type

$$\psi \rightarrow \psi' = e^{iq\alpha(x)} \psi, \quad (2.11)$$

where the parameter α is now dependent on the position in space-time. To preserve the invariance of the Lagrangian density under such transformations, the derivative ∂_μ in Eq. 2.7 must be replaced by a new covariant derivative

$$D_\mu = \partial_\mu + iqA_\mu. \quad (2.12)$$

This expression contains a newly introduced vector field A_μ which is required to transform as

$$A_\mu \rightarrow A_\mu - \partial_\mu \alpha(x). \quad (2.13)$$

With these modifications the Lagrangian density of Eq. 2.7 reads

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi = \bar{\psi}i\gamma^\mu \partial_\mu \psi - m\bar{\psi}\psi - q\bar{\psi}\gamma^\mu \psi A_\mu. \quad (2.14)$$

Here the first two terms are equivalent to Eq. 2.7 and describe the free propagation of the fermion fields of the Standard Model. The last term accounts for the newly found interaction of the charged fermions with the vector field A_μ which can be identified with the photon γ . The coupling strength of the interaction is given by the electric charge q of the fermion. Since a mass term of the form $\mathcal{L}_\gamma = \frac{1}{2}m^2 A^\mu A_\mu$ is not gauge invariant, the theory requires the photon to be massless in accordance with experimental observation.

To complete the Lagrangian of the electromagnetic interaction a gauge invariant kinetic term for A_μ needs to be added to Eq. 2.14. The kinematics of the photon field

are governed by Maxwell's equations, which in their covariant form can be expressed in terms of the field strength tensor

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \quad (2.15)$$

The kinetic term of the photon in the Lagrangian, from which Maxwell's equations can be derived, is given by

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} \quad (2.16)$$

The final Lagrangian density of quantum electrodynamics is thus given by:

$$\begin{aligned} \mathcal{L}_{\text{QED}} = & \underbrace{\bar{\psi}i\gamma^\mu\partial_\mu\psi}_{\text{fermion kinetic term}} - \underbrace{m\bar{\psi}\psi}_{\text{fermion mass term}} - \underbrace{\frac{1}{4}F^{\mu\nu}F_{\mu\nu}}_{\text{photon kinetic term}} \\ & - \underbrace{q\bar{\psi}\gamma^\mu\psi A_\mu}_{\text{interaction term}}. \end{aligned} \quad (2.17)$$

2.1.2 Weak Interaction and Electroweak Unification

The first theory of the weak interaction was proposed by Fermi in 1933 [7]. The theory was motivated by the earlier postulation of the neutrino in 1927 by Pauli to explain the continuous energy spectrum of electrons from β decays. The Fermi theory describes the weak interaction with four-fermion vertices which is a valid approximation at energies much lower than the mass of the W-boson. The Lagrangian of this interaction is of the type current–current

$$\mathcal{L}_{\text{Fermi}} = \frac{G_F}{\sqrt{2}}J_\mu^\dagger J^\mu, \quad (2.18)$$

where the current is given by

$$J^\mu = \bar{\psi}\gamma^\mu\psi, \quad (2.19)$$

and $\psi, \bar{\psi}$ are spinors associated with the fermions of the interaction. The value of the constant G_F can be obtained for example from measurements of the muon lifetime [8, 9]. However, the form of the Lagrangian of Eq. 2.18 is not complete since it does not account for the parity violating nature of the weak interactions, which was first suspected by Lee and Yang [10] and then experimentally confirmed by Wu et al. in 1956 [11]. The Wu experiment observed that in β -decays of Cobalt 60 nuclei the emission of electrons occurs preferentially opposite to the direction of the spin of the nucleus. Since the electron momentum transforms like a vector and the spin like an axial vector this means that parity is not conserved in this process. This effect is

taken care of in the so-called V-A (vector minus axial vector) extension [12, 13] of Fermi's theory, which introduces chirality operators

$$\psi_L = P_L \psi = \frac{1}{2}(1 - \gamma^5)\psi \quad \psi_R = P_R \psi = \frac{1}{2}(1 + \gamma^5)\psi, \quad (2.20)$$

which project out the components of left- and right-handed chirality of the spinor ψ respectively. The matrix γ^5 is defined as

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3. \quad (2.21)$$

and the weak current is then of the type

$$J^\mu = \bar{\psi}_e \gamma^\mu \frac{(1 - \gamma^5)}{2} \psi_\nu, \quad (2.22)$$

where the example of the leptonic $e - \nu_e$ process has been chosen. Equation 2.22 has the form vector current minus axial-vector current which explains the name V-A theory. The simultaneous appearance of the two types of currents means that parity is not conserved in weak processes and only the left-chiral component of the fermions and the right-chiral component of anti-fermions participate in the interaction.

As mentioned above the Fermi theory as well as its V-A extension is only valid in a low energy regime. Above a certain threshold the calculated cross-section of a given process rises quickly with its energy which violates the unitarity of the theory.

The solution to this problem is the introduction of intermediate vector bosons, which, as in the electromagnetic interactions, arise from invariance of the Lagrangian under a given set of local gauge transformations. In fact, as will be shown in the following, a complete and renormalizable theory of the weak and electromagnetic interactions requires their unification into a common electroweak framework [3–5] based on the symmetry groups $SU(2)_L \otimes U(1)_Y$.

To describe the weak interactions with parity violation the symmetry $SU(2)_L$ is assumed. The left-handed fermions are ordered in doublets of weak isospin, e.g. for the first generation of leptons

$$\psi_L = \begin{pmatrix} \psi_{1,L} \\ \psi_{2,L} \end{pmatrix} = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}. \quad (2.23)$$

In analogy to Eq. 2.7 for the electromagnetic interactions, the Lagrangian density of two free, massless, left-handed fermions is given by

$$\mathcal{L}_L = \bar{\psi}_{1,L} (i\gamma^\mu \partial_\mu) \psi_{1,L} + \bar{\psi}_{2,L} (i\gamma^\mu \partial_\mu) \psi_{2,L} = \bar{\psi}_L (i\gamma^\mu \partial_\mu) \psi_L. \quad (2.24)$$

To obtain the mediators of the weak interaction, invariance of the Lagrangian under local gauge transformations of the form

$$\psi_L \rightarrow \psi'_L = e^{ig\alpha^a(x)T^a} \psi_L \quad (2.25)$$

is required. The 2×2 matrices T^a are the generators of $SU(2)_L$ and can be chosen as

$$T^a = \frac{\sigma^a}{2}, \quad a = 1, 2, 3, \quad (2.26)$$

where σ^a are the Pauli matrices given in Eq. 2.4. The $\alpha(x)$ are arbitrary space and time dependent functions, and g will be interpreted as the coupling constant of the interaction.

To preserve invariance of the Lagrangian under $SU(2)_L$ transformations, a new covariant derivative

$$D_\mu = \partial_\mu + igT^a W_\mu^a \quad (2.27)$$

is introduced along with three new vector boson fields W_μ^a , which are required to transform according to

$$W_\mu^a \rightarrow W_{\mu'}^a = W_\mu^a + g\epsilon^{abc}\alpha^b(x)W_\mu^c, \quad (2.28)$$

where ϵ is the Levi-Civita tensor. To account for the kinetic energy of the newly found gauge fields a tensor of the form

$$W_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a - g\epsilon^{abc}W_\mu^b W_\nu^c \quad (2.29)$$

is defined. The complete, gauge invariant Lagrangian of $SU(2)_L$ is then written as

$$\begin{aligned} \mathcal{L}_L &= \bar{\psi}_L(i\gamma^\mu D_\mu)\psi_L - \frac{1}{4}W^{a\mu\nu}W_{\mu\nu}^a \\ &= \underbrace{\bar{\psi}_L(i\gamma^\mu \partial_\mu)\psi_L}_{\text{fermion kinetic term}} - \underbrace{g\bar{\psi}_L(\gamma^\mu T^a W_\mu^a)\psi_L}_{\text{fermion—vector field interaction term}} - \underbrace{\frac{1}{4}W^{a\mu\nu}W_{\mu\nu}^a}_{\text{vector field kinetic and self-interaction term}} \end{aligned} \quad (2.30)$$

The first term describes the kinetic energy of the left-handed fermions, the second term their interaction with the vector fields, and the third term the kinetic energy of the vector fields and their self-interaction. The self-interaction of the vector fields originates from the fact that, contrary to $U(1)$ in the case of the electromagnetic interactions, the symmetry group $SU(2)_L$ is non-abelian i.e. its generators (Eq. 2.26) do not commute.

While the $SU(2)_L$ gauge theory alone describes many aspects of the weak interactions it is not a complete theory. For instance it is not able to explain the experimentally observed masses of the fermions and gauge bosons associated with the vector fields and it lacks a consistent description of the interactions mediated by the neutral Z boson.

To resolve these problems the electroweak formalism introduces a combination of $SU(2)_L$ and a new $U(1)_Y$ symmetry group, where Y is the so-called hypercharge.

The behaviour of the Lagrangian under $U(1)_Y$ gauge transformations is identical to that of the electromagnetic interactions and leads to a covariant derivative of the form

$$D_\mu = \partial_\mu + ig' \frac{Y}{2} B_\mu, \quad (2.31)$$

with a new vector field B_μ and a new coupling constant g' . In analogy to Eq. 2.15 the field strength tensor

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \quad (2.32)$$

is defined. In contrast to the $SU(2)_L$, the $U(1)_Y$ symmetry applies not only to particles of the left-handed doublets (Eq. 2.23) but also to right-handed particles which are ordered in isospin singlets. The $U(1)_Y$ invariant Lagrangian can thus be written as:

$$\mathcal{L}_Y = \bar{\psi} (i\gamma^\mu \partial_\mu) \psi + g' \frac{Y_R}{2} \bar{\psi}_R (\gamma^\mu B_\mu) \psi_R + g' \frac{Y_L}{2} \bar{\psi}_L (\gamma^\mu B_\mu) \psi_L - \frac{1}{4} B^{\mu\nu} B_{\mu\nu}, \quad (2.33)$$

where the couplings depend on the type and chirality of the particle considered.

The combined electroweak Lagrangian of the $SU(2)_L$ and $U(1)_Y$ gauge theories is then simply obtained from the sum of the Lagrangians of Eqs. 2.30 and 2.33. With the combined covariant derivative

$$D_\mu = \partial_\mu + ig T^a W_\mu^a + ig' \frac{Y}{2} B_\mu, \quad (2.34)$$

written out the complete $\mathcal{L}_{\text{electroweak}}$ is given by:

$$\begin{aligned} \mathcal{L}_{\text{electroweak}} = & \underbrace{-\frac{1}{4} W^{a\mu\nu} W_{\mu\nu}^a - \frac{1}{4} B^{\mu\nu} B_{\mu\nu}}_{\text{gauge field kinetic energy and self-interaction terms}} \\ & + \underbrace{\bar{\psi}_L \gamma^\mu (i\partial_\mu - \frac{1}{2} g \sigma^a W_\mu^a - \frac{1}{2} g' Y B_\mu) \psi_L}_{\text{left-chiral fermion kinetic energy and interaction term}} + \underbrace{\bar{\psi}_R \gamma^\mu (i\partial_\mu - \frac{1}{2} g' Y B_\mu) \psi_R}_{\text{right-chiral fermion kinetic energy and interaction term}} \end{aligned} \quad (2.35)$$

To relate this Lagrangian to the physically observed charged and neutral currents of the electroweak interactions the $SU(2)_L$ generators (Eq. 2.26) and thus the Pauli matrices are inserted into the covariant derivative 2.34 which then becomes

$$\begin{aligned} D_\mu = & \partial_\mu + \frac{ig}{2} \left(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} W_\mu^1 + \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} W_\mu^2 + \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} W_\mu^3 \right) + ig' \frac{Y}{2} B_\mu \\ = & \partial_\mu + \frac{ig}{2} \begin{pmatrix} W^3 & W^1 - iW^2 \\ W^1 + iW^2 & -W^3 \end{pmatrix}_\mu + ig' \frac{Y}{2} \begin{pmatrix} B & 0 \\ 0 & B \end{pmatrix}_\mu. \end{aligned} \quad (2.36)$$

The expression is simplified by introducing the ladder operators

$$T^\pm = \frac{1}{\sqrt{2}} (T^1 \pm iT^2) \quad \text{and} \quad W^\pm = \frac{1}{\sqrt{2}} (W^1 \mp iW^2), \quad (2.37)$$

where the W^\pm are identified with the charged gauge bosons. Equation 2.36 can then be written as

$$\begin{aligned} D_\mu &= \partial_\mu + \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & W^+ \\ W^- & 0 \end{pmatrix}_\mu + \frac{1}{2} \begin{pmatrix} gW^3 + g'YB & 0 \\ 0 & -gW_3 + g'YB \end{pmatrix}_\mu \\ &= \partial_\mu + \underbrace{(T^+W^+ + T^-W^-)}_{D_\mu^W} + \underbrace{\left(gT^3W^3 + g'\frac{Y}{2}B\right)}_{D_\mu^{\gamma Z}}, \end{aligned} \quad (2.38)$$

where the first term inserted into the Lagrangian 2.35 describes the interactions of the W^\pm gauge bosons with fermions

$$\begin{aligned} \mathcal{L}_{L, \text{charged}} &= ig\bar{\psi}_L i\gamma^\mu (T^+W^+ + T^-W^-)_\mu \psi_L \\ &= \frac{1}{\sqrt{2}} ig (\bar{\nu}_e, \bar{e})_L i\gamma^\mu \begin{pmatrix} 0 & W^+ \\ W^- & 0 \end{pmatrix}^\mu \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \\ &= -\frac{1}{\sqrt{2}} g (\bar{\nu}_{e,L} \gamma^\mu W_\mu^+ e_L + \bar{e}_L \gamma^\mu W_\mu^- \nu_{e,L}). \end{aligned} \quad (2.39)$$

The interaction of the fermions with the neutral gauge bosons originates from the term $D_\mu^{\gamma Z}$, which leads to the neutral interaction Lagrangian of the form

$$\begin{aligned} \mathcal{L}_{L, \text{neutral}} &= \bar{\psi}_L i\gamma^\mu i \left(gT^3W_\mu + g'\frac{Y}{2}B_\mu \right) \psi_L + \bar{\psi}_R i\gamma^\mu i \left(g'\frac{Y}{2}B_\mu \right) \psi_R \\ &= \sum_{\psi=e_L, e_R, \nu_L, \nu_R} \bar{\psi} i\gamma^\mu i \left(gT^3W_\mu + g'\frac{Y}{2}B_\mu \right) \psi. \end{aligned} \quad (2.40)$$

To obtain the physically observed photon and Z boson a change of base can be performed of the type

$$\begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} = \begin{pmatrix} \cos \theta_W & \sin \theta_W \\ -\sin \theta_W & \cos \theta_W \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}. \quad (2.41)$$

This corresponds to a rotation by the so-called Weinberg angle θ_W with which one obtains the interaction Lagrangian for the neutral current

$$\begin{aligned}
\mathcal{L}_{\gamma,Z} = & \sum_{e\nu} \bar{\psi} i \gamma^\mu i \underbrace{\left(g \sin \theta_W T^3 + g' \frac{Y}{2} \cos \theta_W \right)}_{\text{fermion-photon coupling}} A_\mu \psi \\
& + \sum_{e\nu} \bar{\psi} i \gamma^\mu i \underbrace{\left(g \cos \theta_W T^3 + g' \frac{Y}{2} \sin \theta_W \right)}_{\text{fermion-Z coupling}} Z_\mu \psi
\end{aligned} \tag{2.42}$$

Identifying the field A_μ with the photon with coupling constant equal to the Q times electric charge e leads to the expression

$$Qe = g \sin \theta_W T^3 + g' \frac{Y}{2} \cos \theta_W \tag{2.43}$$

Together with the Gell-Mann Nishijima equation [14, 15]

$$Q = T^3 + \frac{Y}{2}. \tag{2.44}$$

one obtains the relation between the Weinberg angle and the coupling constants:

$$e = g \sin \theta_W = g' \cos \theta_W \tag{2.45}$$

For the coupling of the Z boson to the fermions one obtains:

$$g_Z = g \cos \theta_W T^3 + g' \frac{Y}{2} \sin \theta_W = \frac{e}{\sin \theta_W \cos \theta_W} \left(T^3 - \sin^2 \theta_W Q \right) \tag{2.46}$$

With the above relations the interaction Lagrangian (Eq. 2.42) can be rewritten as

$$\mathcal{L}_{\gamma,Z} = \sum_{e\nu} i \bar{\psi} i \gamma^\mu \left(Qe A_\mu + g_Z Z_\mu \right) \psi \tag{2.47}$$

Since the field Z_μ is a combination of the field W_μ^3 and B_μ it couples to left- and right-handed fermions.

2.1.3 Spontaneous Symmetry Breaking and Higgs Mechanism

The formalism developed in the previous sections describes all phenomena of the electroweak interactions except for the masses of the gauge bosons and fermions. The particle masses can be incorporated into the Standard Model by introducing the Higgs mechanism [16–21] which spontaneously breaks the electroweak symmetry.

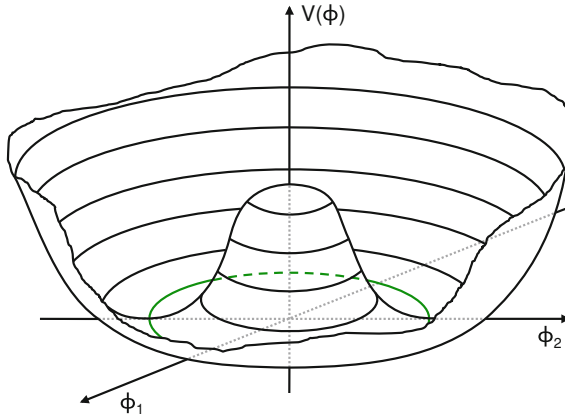


Fig. 2.1 Visualization of the Higgs potential in the complex (ϕ_1, ϕ_2) -plane. The *green circle* indicates the position of the degenerate minima of the potential

The Higgs-mechanism postulates a new complex scalar $SU(2)_L$ doublet field with hypercharge $Y = 1$. The electric charge Q of the two field components follows from the Gell-Mann Nishijima relation 2.44:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} \quad (2.48)$$

As can be seen above the complex nature of the fields leads to 4 degrees of freedom. The Lagrangian density of the spin-0 doublet follows the Klein-Gordon equation

$$\mathcal{L}_\phi = |D^\mu \phi|^2 - V(\phi) \equiv (D^\mu \phi)^\dagger (D_\mu \phi) - V(\phi), \quad (2.49)$$

where D_μ is the previously introduced covariant derivative of the electroweak interactions

$$D_\mu = \partial_\mu + igT^a W_\mu^a + ig' \frac{Y}{2} B_\mu, \quad (2.50)$$

and $V(\phi)$ is a newly postulated potential which is required to be invariant under local gauge transformations. This requires the potential to be symmetric in all four components which leads to the ansatz

$$V(\phi) = -\mu^2 |\phi|^2 + \lambda |\phi|^4, \quad (2.51)$$

where μ^2 and λ are real constants. For positive values of μ^2 the potential has the characteristic “Mexican Hat” type shape shown in Fig. 2.1. The potential has a circle of degenerate minima in the complex (ϕ_1, ϕ_2) -plane with a radius of

$$v = \sqrt{\frac{\mu^2}{\lambda}}, \quad (2.52)$$

which is referred to as the vacuum expectation value of the Higgs field.

The choice of a particular ground state of the vacuum on this circle breaks the symmetry spontaneously. Since the vacuum is electrically neutral the ground state

$$\langle \phi^+ \rangle = 0, \quad \langle \phi^0 \rangle = v, \quad (2.53)$$

can be chosen. An expansion around this ground state gives

$$\phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 + \eta_+(x) + i\xi_+(x) \\ v + \eta_0(x) + i\xi_0(x) \end{pmatrix}. \quad (2.54)$$

The fields $\eta_+(x)$, $\xi_+(x)$, $\xi_0(x)$ lead to massless Goldstone bosons [22, 23] which can be absorbed with a local gauge transformation of the type

$$\phi \rightarrow \phi' = e^{ig\alpha^a(x)T^a} \phi, \quad (2.55)$$

where the arbitrary functions $\alpha(x)$ are chosen accordingly. This gauge transformation results in the three spin degrees of freedom of the vector bosons of electroweak theory. The field ϕ is now given by

$$\phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + \eta^0(x) \end{pmatrix}, \quad (2.56)$$

where $\eta_0(x)$ can be associated with the field of the Higgs particle and will be denoted H in the following. Substituting Eq. 2.56 into the potential given in Eq. 2.51 and using Eq. 2.53 leads to

$$V(\phi) = \frac{1}{2} (2\mu^2) H^2 + \lambda v H^3 + \frac{1}{4} \lambda H^4, \quad (2.57)$$

where the first term describes the mass of the Higgs boson

$$m_H = \sqrt{2\mu^2}, \quad (2.58)$$

and the second and are third terms are associated with the self-interactions of the Higgs-field with 3 and 4 particle vertices.

The interaction of the Higgs boson with the vector boson of the electroweak theory is determined from the covariant derivative term of the Lagrangian

$$\begin{aligned}
\mathcal{L}_\phi &= |D_\mu \phi|^2 - V(\phi) \\
&= \left| \left(\partial_\mu + D_\mu^W + D_\mu^{\gamma Z} \right) \phi \right|^2 - V(\phi) \\
&= \left| \left(\partial_\mu + ig(T^+ W^+ + T^- W^-)_\mu + iQeA_\mu + ig_Z Z_\mu \right) \phi \right|^2 - V(\phi) \quad (2.59)
\end{aligned}$$

Due to the photon being massless all terms including the photon field A^μ are evaluated to zero. Also no mixing terms between W^\pm and Z fields occurs since $\phi^\pm = 0$ was chosen. Expression 2.59 thus becomes

$$\begin{aligned}
\mathcal{L}_\phi &= |\partial_\mu|^2 + |D_\mu^W|^2 + |D_\mu^{\gamma Z}|^2 - V(\phi) \\
&= \frac{1}{2} (\partial^\mu H) (\partial_\mu H) + \frac{1}{4} g^2 W_\mu^+ W_\mu^- (v + H)^2 + \frac{1}{2} g_Z^2 Z_\mu (v + H)^2 - V(\phi) \quad (2.60)
\end{aligned}$$

The first term represents the kinetic energy of the Higgs field H . Since g and v are constants, all terms with v^2 can be interpreted as mass terms, where the mass values are given by

$$m_W = \frac{1}{2} g v \quad \text{and} \quad m_Z = \frac{1}{2} \sqrt{g^2 + g'^2} v \quad (2.61)$$

for the W and Z bosons respectively. The terms

$$\frac{1}{2} g^2 W^+ W^- H \quad \text{and} \quad \frac{1}{4} g_Z^2 Z Z H \quad (2.62)$$

demonstrate that the Higgs—vector boson couplings are proportional to the masses of the vector bosons. Finally the terms

$$\frac{1}{4} g^2 v W^+ W^- H H \quad \text{and} \quad \frac{1}{2} g_Z^2 v Z Z H H \quad (2.63)$$

correspond to the interactions of two Higgs bosons and two vector bosons.

Since the value of μ in Eq. 2.58 is not known, the mass of the Higgs cannot be predicted from theory. The value of v can be derived e.g. from the experimentally measured values of M_Z and g_Z

$$v = \frac{2M_Z}{g_Z} = 246 \text{ GeV}. \quad (2.64)$$

While the formalism developed above incorporates the masses of the electroweak gauge bosons into the theory, the masses of the Standard Model fermions cannot be explained in the same way. In addition, standard fermion mass terms of the type

$$\mathcal{L}_m = -m\psi\psi = -m \left(\bar{\psi}_R \psi_L + \bar{\psi}_L \psi_R \right) \quad (2.65)$$

are not invariant under local $SU(2)_L$ or $U(1)_Y$ gauge transformations. This leads to the formulation of the so-called Yukawa interaction, which in the case of leptons it is given by

$$\mathcal{L}_{\text{Yukawa}} = -\lambda_l \left(\bar{L}_L \phi^0 L_R + \bar{L}_R \phi^+ L_L \right), \quad (2.66)$$

where L_L are the left-handed lepton doublets and L_R the lepton singlets. The Yukawa term is automatically invariant under $SU(2)_L$ since both ϕ and L belong to the same $SU(2)_L$ doublet. Looking at the first generation of leptons and using expression 2.56 one obtains

$$\mathcal{L}_{\text{Yukawa}} = -\lambda_e \frac{v}{\sqrt{2}} (\bar{e}_R e_L + \bar{e}_L e_R) - \lambda_e \frac{H}{\sqrt{2}} (\bar{e}_R e_L + \bar{e}_L e_R), \quad (2.67)$$

containing the electron mass term with mass

$$m_e = \lambda_e \frac{v}{\sqrt{2}} \quad (2.68)$$

and the coupling to the Higgs field proportional to that mass. Similar expressions can be derived for the remaining Standard Model fermions. In the case of the quarks the so-called Cabibbo-Kobayashi-Maskawa (CKM) matrix [24, 25] has to be introduced to take care of the fact that the mass eigenstates are rotated with respect to the flavour eigenstates.

In summary, the combined Lagrangian resulting from the Higgs mechanism and electroweak symmetry breaking together with the Yukawa terms in short-hand notation is given by

$$\begin{aligned} \mathcal{L}_{\text{Higgs}} = & \underbrace{\left| \left(\partial_\mu + \frac{1}{2} ig \sigma^a W_\mu^a + \frac{1}{2} ig' Y B_\mu \right) \phi \right|^2}_{\text{Higgs boson kinetic energy, } W^\pm/Z \text{ boson mass, and } W^\pm/Z\text{---Higgs boson interaction terms}} \\ & + \underbrace{\mu^2 |\psi|^2 - \lambda |\psi|^4}_{\text{Higgs boson mass and self-interaction terms}} \\ & + \underbrace{\left(M_1 \bar{\psi}_L \phi \psi_R + M_2 \bar{\psi}_L \phi^C \psi_R + h.c. \right)}_{\text{Yukawa fermion mass and fermion---Higgs interaction terms}}. \end{aligned} \quad (2.69)$$

2.1.4 Strong Interaction

The strong interaction between the quarks and gluons of the Standard Model is described by quantum chromodynamics (QCD). The QCD formalism is a local gauge theory based on the symmetry group $SU(3)_C$, where the index C stands for colour

charge and is a property of all strongly interacting particles. For quarks the colour charge can take the values “green”, “red”, “blue”, and gluons carry combinations of colour-anticolour. The quarks are assembled in colour triplets of the type

$$\psi = \begin{pmatrix} \psi_r \\ \psi_b \\ \psi_g \end{pmatrix}, \quad (2.70)$$

where the notation

$$\psi_r = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \psi_b = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad \psi_g = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad (2.71)$$

is adopted. The Lagrangian density for free quarks can then be written just as before

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi. \quad (2.72)$$

To derive the interaction, gauge invariance is required for local $SU(3)_C$ transformations of the type

$$\psi \rightarrow \psi' = e^{ig_s \alpha^a(x) T^a} \psi, \quad (2.73)$$

where g_s will be identified with the strong coupling constant and the $\alpha^a(x)$ are again arbitrary functions of space and time. The T^a , $a = 1..8$, are the generators of the non-abelian symmetry group $SU(3)$ which follow the commutation relation

$$[T^a, T^b] = if^{abc} T^c, \quad (2.74)$$

where f^{abc} is the so-called structure constant of the symmetry group, given by the tensor with values

$$\begin{aligned} f^{123} &= 1 \\ f^{458} &= f^{678} = \frac{\sqrt{3}}{2} \\ f^{147} &= f^{246} = f^{257} = f^{345} = f^{516} = f^{637} = \frac{1}{2} \end{aligned} \quad (2.75)$$

and antisymmetric permutations thereof, 0 otherwise. A possible representation of the generators $T^a = \lambda_a/2$ are the Gell-Mann matrices

$$\begin{aligned}
\lambda_1 &= \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, & \lambda_2 &= \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, & \lambda_3 &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\
\lambda_4 &= \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, & \lambda_5 &= \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, & \lambda_6 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \\
\lambda_7 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, & \lambda_8 &= \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.
\end{aligned} \tag{2.76}$$

For the Lagrangian density to stay invariant under $SU(3)_C$ symmetry transformations, the covariant derivative

$$D_\mu = \partial_\mu + ig_s T^a G_\mu^a \tag{2.77}$$

is introduced. It contains eight new vector fields G_μ^a which correspond to the massless gluons. These fields are required to transform as

$$G_\mu^a \rightarrow G'^a_\mu = G_\mu^a - \partial_\mu \alpha^a(x) - g_s f^{abc} \alpha^b(x) G_\mu^c. \tag{2.78}$$

The kinetic energy of the gluons is given by the field strength tensor

$$G_{\mu\nu}^a = \partial_\mu G_\nu^a - \partial_\nu G_\mu^a - g_s f^{abc} G_\mu^b G_\nu^c. \tag{2.79}$$

The complete, gauge invariant Lagrangian density of the strong interactions is thus given by

$$\begin{aligned}
\mathcal{L}_{\text{QCD}} &= \bar{\psi} (i\gamma^\mu D_\mu - m) \psi - \frac{1}{4} G^{a\mu\nu} G_{\mu\nu}^a \\
&= \underbrace{\bar{\psi} (i\gamma^\mu \partial_\mu - m) \psi}_{\text{quark kinetic and mass term}} - \underbrace{\frac{1}{2} g_s \bar{\psi} (\gamma^\mu \lambda^a G_\mu^a) \psi}_{\text{quark - gluon field interaction term}} - \underbrace{\frac{1}{4} G^{a\mu\nu} G_{\mu\nu}^a}_{\text{gauge field kinetic and self-interaction term}}.
\end{aligned} \tag{2.80}$$

The first term describes the kinetic energy and the mass of the quarks. The second term accounts for the interaction of the quarks with the gluon fields. The third term contains the kinetic energy of the gluon fields and their self interaction with 3 point and 4 point vertices. The self-interacting nature of the gluon fields originates from the fact that the symmetry group $SU(3)_C$ is non-abelian. The self-interaction is also responsible for the confinement of coloured particles, which means that only colourless particles, i.e. mesons consisting of a quark-antiquark pairs, or hadrons consisting of three quarks of different colour may exist freely. Not included in Eq. 2.80 is an additional CP-violating term inherent to QCD, for which no experimental evidence has been observed to date.

2.1.5 Total Lagrangian of the Standard Model

The combined Lagrangian density of the Standard Model as derived in the previous sections before electroweak symmetry breaking is given by

$$\begin{aligned}
\mathcal{L}_{\text{SM}} = & \underbrace{-\frac{1}{4}B^{a\mu\nu}B_{\mu\nu}^a - \frac{1}{4}W^{a\mu\nu}W_{\mu\nu}^a - \frac{1}{4}G^{a\mu\nu}G_{\mu\nu}^a}_{\text{gauge boson kinetic energy and self-interaction terms}} \\
& + \underbrace{\bar{\psi}_L\gamma^\mu\left(i\partial_\mu - \frac{1}{2}g\sigma^a W_\mu^a - \frac{1}{2}g'YB_\mu\right)\psi_L + \bar{\psi}_R\gamma^\mu\left(i\partial_\mu - \frac{1}{2}g'YB_\mu\right)\psi_R}_{\text{fermion kinetic energy and electroweak interaction terms}} \\
& + \underbrace{\left|\left(\partial_\mu + \frac{1}{2}ig\sigma^a W_\mu^a + \frac{1}{2}ig'YB_\mu\right)\phi\right|^2}_{\text{Higgs boson kinetic energy, } W^\pm/Z \text{ boson mass, and } W^\pm/Z\text{—Higgs boson interaction terms}} \\
& + \underbrace{\mu^2|\psi|^2 - \lambda|\psi|^4}_{\text{Higgs boson mass and self-interaction terms}} \\
& + \underbrace{\left(M_1\bar{\psi}_L\phi\psi_R + M_2\bar{\psi}_L\phi^C\psi_R + h.c.\right)}_{\text{Yukawa fermion mass and fermion—Higgs interaction terms}} \\
& - \underbrace{\frac{1}{2}g_s\bar{q}\gamma^\mu\lambda^a G_\mu^a q}_{\text{quark—gluon interaction term}}.
\end{aligned} \tag{2.81}$$

2.1.6 Experimental Verification of the Standard Model

The Standard Model of particle physics in its simplest form with the assumption of massless neutrinos is defined by 19 parameters: 9 fermion masses, 3 mixing angles and 1 phase from the CKM matrix, 1 strong CP parameter, 3 coupling constants for the electromagnetic, weak, and strong interactions, and 2 boson masses such as e.g. that of the Z and Higgs particles. With the exception of the mass of the Standard Model Higgs boson, 18 out of these 19 parameters have been determined experimentally at some precision and are summarized in Ref. [2]. With the recent observation of a new particle in the search for the Standard Model Higgs boson the experimental determination of the last missing parameter of the Standard Model seems within close reach. After a review of the theoretical and indirect experimental bounds on the mass of the Higgs boson, the status of the direct searches at the LHC is summarized in this section.

While the mass of the Higgs boson cannot be derived from the known parameters of the Standard Model, some constraints on its upper and lower bounds arise from theoretical arguments. Firstly, the requirement of unitarity of the theory in longi-

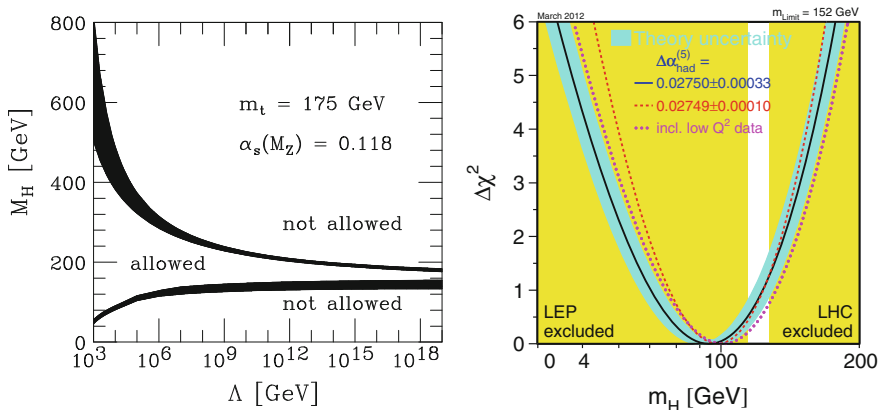


Fig. 2.2 *Left:* Lower and upper bounds on the mass of the Higgs boson M_H from vacuum stability and triviality considerations respectively as a function of the cut-off scale Λ up to which the Standard Model is assumed to be valid. The mass of the top quark and the strong coupling constant are assumed to be at the values given in the figure. The *black bands* indicate the impact of various uncertainties. Taken from Ref. [26]. *Right:* Distribution of $\Delta\chi^2 = \chi^2 - \chi^2_{\min}$ from a global fit to electroweak precision data as a function of different Higgs masses m_H . The preferred value of m_H corresponds to the minimum of the curve at 94^{+29}_{-24} GeV. Values $m_H > 152$ GeV are excluded at 95 % confidence level. The *blue band* corresponds to the theoretical uncertainty from unknown higher order corrections. Results are shown for two values of $\Delta\alpha_{\text{had}}^{(5)}$, which corresponds to different contributions of the 5 lighter quarks to the QED fine structure constant. The *yellow shaded areas* show the mass range excluded by the LEP (*left*) and LHC (*right*) experiments. Taken from Ref. [27]

nal W^+W^- scattering necessitates a Higgs particle with a mass $m_H \lesssim 870$ GeV [28]. This bound can be reduced further with so-called triviality considerations, where the evolution of the quartic Higgs coupling λ as it occurs in Eq. 2.57 is examined as a function of the energy scale Q . The behaviour of $\lambda(Q)$ is governed by the renormalization group equations which are described in Sect. 2.2. For low values of $Q^2 \ll v^2$, where v^2 is identified with the electroweak breaking scale, λ converges $\rightarrow 0$. The resulting Higgs potential has no longer the characteristic “Mexican hat” shape shown in Fig. 2.1 and the theory becomes “trivial” since no Higgs self-interactions occur. In the opposite case, $Q^2 \gg v^2$, the quartic coupling eventually becomes infinite and develops a so-called Landau pole at a cut-off energy Λ , where the validity of the Standard Model ends. For a given value of Λ one can then determine an upper limit on the mass of the Higgs boson, e.g. in the case of the Planck scale $\Lambda \sim 10^{16}$ GeV, $m_H \lesssim 200$ GeV is required [29].

Lower theoretical bounds on the Higgs mass are derived from vacuum stability considerations, which imply that at low values of λ and m_H contributions from fermions and gauge bosons become significant and can result in negative overall values for the quartic coupling constant. This would imply a Higgs potential without minimum and thus lead to an instability of the vacuum. The lower and upper theoretical bounds on the mass of the Higgs boson from vacuum stability and triviality arguments as a function of the cut off scale Λ are shown in Fig. 2.2 (left).

While theoretical considerations provide a rough window of possible values of the mass of the predicted Higgs particle, experimental measurements are necessary to prove its existence and determine its precise properties. Indirect experimental constraints on m_H arise from global fits to the observables of electroweak precision measurements [27, 30], to which radiative corrections from the Standard Model Higgs boson are expected. The current fit results for m_H as a function of $\Delta\chi^2 = \chi^2 - \chi_{\min}^2$ are shown in Fig. 2.2 (right). The preferred value for the mass of the Higgs boson corresponds to the minimum of the curve at a value of 94^{+29}_{-24} GeV. At the upper bound, values $m_H > 152$ GeV can be excluded at 95 % confidence level.

Direct searches for the Higgs boson at the LEP [34], Tevatron [35–37], and LHC [38, 39] experiments have further narrowed the window of possible Higgs masses down to a region between 116 and 127 GeV. In that mass range the ATLAS and CMS experiments have recently reported the observation of a new Higgs-like boson [31, 32, 40, 41]. The statistical significance of the observation corresponds to approximately seven standard deviations as shown on Fig. 2.3 (top). The measured masses are 125.2 ± 0.3 (stat.) ± 0.6 (syst.) GeV and 125.8 ± 0.4 (stat.) ± 0.4 (syst.) GeV for the ATLAS and CMS experiments respectively. According to the measurements performed to date the new particle seems to couple to W and Z bosons as expected, whereas the signal strength in the $H \rightarrow \gamma\gamma$ channel is observed to be somewhat higher as shown in Fig. 2.3 (bottom left). The evidence for couplings to quarks and leptons is weaker at present and will require more data for precise measurements. The measurements of the spin of the new particle are ongoing. Spin-1 can be ruled out since the new particle is observed in di-boson states. As shown in Fig. 2.3 (bottom right) a spin-0 positive parity state (0^+) as expected for the Standard Model Higgs boson seems preferred over 2^- , but the measurements are not fully conclusive yet. A similar tendency is observed with respect to the 0^- and 2^+ hypotheses.

The fact that the mass of the observed Higgs-like particle falls within the narrow window predicted by the fits to electroweak precision data as well as the previously discussed theoretical bounds is a remarkable success of particle physics.

2.2 Supersymmetry

As shown in the previous part of this chapter, the Standard Model of particle physics has been very successful in describing the fundamental particles and their interactions at the currently accessible energy scales in high energy physics. It is expected, however, that more comprehensive theories are necessary to explain the physical phenomena in higher energy regimes. Such theories originate from the Standard Model's inability to clarify important theoretical questions, such as the arbitrariness of gauge couplings, mixing angles and particle masses, the lack of an explanation for gauge symmetry, quantum numbers and generations, the Higgs-mass fine-tuning problem and eventually the incorporation of gravity into a unified theory. In addition significant cosmological observations, such as cold dark matter, dark energy, the

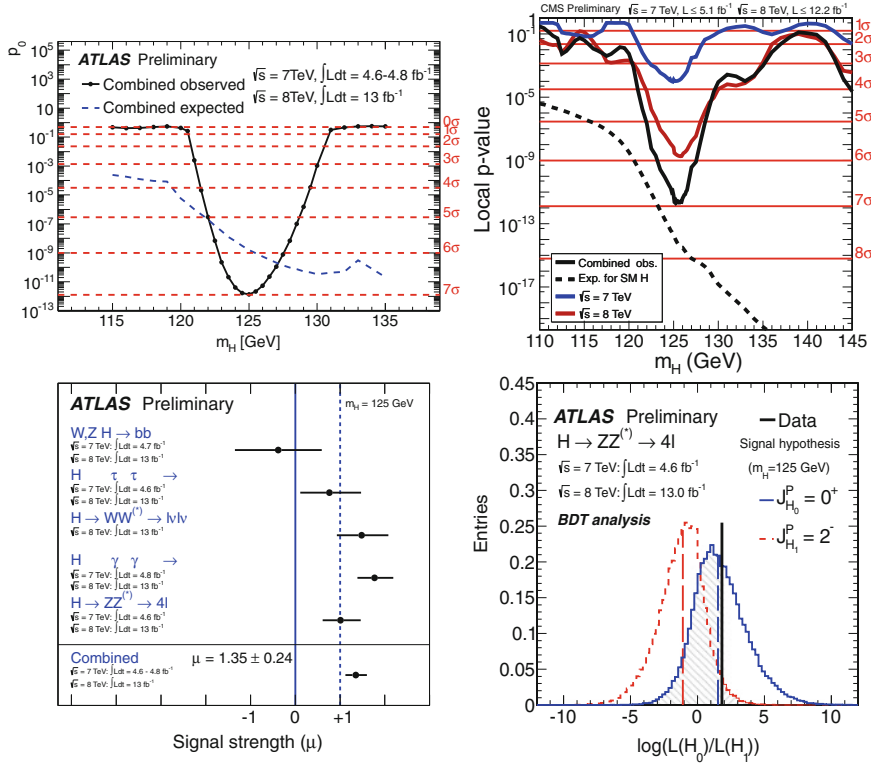


Fig. 2.3 *Top*: The observed local p-value is shown for a combination of the considered search channels as a function of possible Higgs boson masses m_H using data from the combined 2011 and 2012 LHC data-taking campaigns at 7 and 8 TeV in proton–proton collision. The *dashed line* corresponds to the expected local p-value under the hypothesis of a Standard Model Higgs boson. The results of the ATLAS searches are shown on the *left* and those of the CMS experiment on the *right*. *Bottom left*: Measurements of the signal strength parameter μ under the assumption of a Standard Model Higgs boson with $m_H = 125$ GeV for the individual search channels and their combination. *Bottom right*: Distributions of the log-likelihood ratio generated with pseudo-experiments assuming the spin 0^+ hypothesis and testing the spin 2^- hypothesis in $H \rightarrow ZZ^* \rightarrow 4\ell$ events. The data corresponds to the *solid vertical line*. The *shaded areas* are equivalent to the observed p-values for compatibility with the tested 2^- hypothesis (*right shaded area*) and the assumed 0^+ hypothesis (*left shaded area*). Figures taken from Refs. [31–33]

observed matter-antimatter asymmetry as well as results from neutrino physics are not addressed within the Standard Model.

The ultimate goal of particle physics is to consistently explain and integrate all these phenomena into a single Theory of Everything (ToE) valid up and beyond the Planck energy scale² $M_P \approx 10^{19}$ GeV, including quantum gravitational effects.

² The Planck scale is defined as the energy scale at which the effects of gravity become comparable to the other forces and quantum gravity can no longer be ignored.

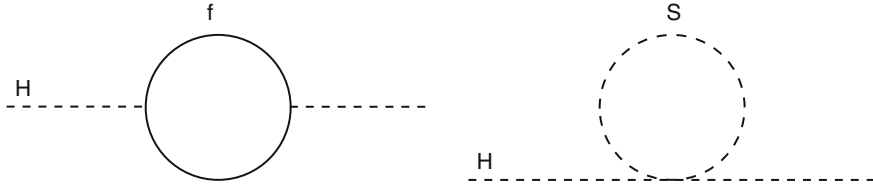


Fig. 2.4 One-loop quantum correction to the physical Higgs mass from fermion loop (*left*) and scalar loop (*right*) diagrams

Superstring models [42] are promising candidates on the way to such a ToE, but they are not yet entirely understood and experimental validation is currently beyond technical possibilities.

An essential requirement for most String models is Supersymmetry (SUSY). SUSY is a fundamental symmetry between fermions and bosons introducing a set of new partner particles with opposite spin statistics for each Standard Model particle. The possible implications of SUSY on the electroweak scale make it one of the best-motivated theories beyond the Standard Model. As will be shown in the following, Supersymmetry suggests very elegant solutions to many open questions of the Standard Model.

The Hierarchy Problem. A strong argument for Supersymmetry at the electroweak scale is the so-called Hierarchy Problem [43]. It describes the unnatural discrepancy between the energy scale of the renormalized Higgs-boson mass and that of its bare mass at the lowest order of perturbation theory. This tremendous difference is caused by large quantum corrections to Higgs-boson processes, such as fermion loops as illustrated in Fig. 2.4 (left). These contributions result in quadratically divergent correction terms to the physical Higgs mass:

$$m_H^2 = m_0^2 - \frac{|\lambda_f|^2}{8\pi^2} \Lambda^2 + \mathcal{O}\left(\ln \frac{\Lambda^2}{m_f^2}\right) \quad (2.82)$$

Here m_0 is the bare mass of the Higgs-boson at Born-level, λ_f is the Yukawa coupling of the process of interest, m_f the mass of the involved fermion, and Λ a cut-off parameter that can be interpreted as the upper validity limit of the theory and is usually identified with the GUT scale at $\Lambda_{GUT} \approx 10^{16}$ GeV.

In contrast, the mass of the Higgs Boson m_H , as shown in the previous sections, is expected to lie within 116–127 GeV. It follows then from Eq. 2.82 that m_0^2 must be known up to a precision of ~ 24 significant digits to yield the correct value of m_H . From a theoretical point of view it is unlikely that such a fine-tuning of parameters in every order of perturbation theory is realized in nature.

A more elegant solution to this problem is provided by Supersymmetry, where the supersymmetric partner particles, due to their half-spin difference, contribute with opposite sign loop corrections to those of the Standard Model. In this way fermion loop processes cancel with loop diagrams from bosonic SUSY particles as shown in

Fig. 2.4 (right). Likewise Standard Model boson contributions are absorbed by their respective fermionic superpartners.

For the case of a scalar particle S with mass m_S the corrections can be written as

$$\Delta m_H^2 = \frac{\lambda_S}{16\pi^2} \left(\Lambda^2 - 2m_S^2 \ln \frac{\Lambda}{m_S} + \dots \right). \quad (2.83)$$

A comparison between Eqs. 2.82 and 2.83 shows that the quadratic divergences can be absorbed, if there are two scalar contributions and $\lambda_S = |\lambda_f|^2$ holds between the couplings. In fact this relation is an intrinsic property of Supersymmetry as will be shown in the following sections. Now the remaining one loop correction can be approximated as

$$\Delta m_H^2 \approx \mathcal{O} \left(\frac{\alpha}{\pi} \right) (m_S^2 - m_f^2), \quad (2.84)$$

where α can be identified with a typical coupling constant. Hence, if Supersymmetry is broken, the masses of the fermions and their supersymmetric partners must lie close together to allow for a natural value of m_H without artificial fine tuning:

$$|m_S^2 - m_f^2| \lesssim 1 \text{ TeV}^2 \quad (2.85)$$

This is one of the strongest motivations to expect Supersymmetry at the electroweak scale.

Grand Unification. Motivated by the evolution of the Standard Model coupling constants with energy, Grand Unified Theories (GUTs) aim to provide a unified description of the electroweak and strong interactions at high energies. The underlying idea is to embed the Standard Model's gauge groups $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ into a universal symmetry group G_{GUT} , representing a single interaction with one coupling constant at a unification scale M_{GUT} .

This requires the three previously defined coupling constants g , g' , and g_s of the SM, if rewritten according to [44]

$$\begin{aligned} \alpha_1 &= (5/3)g'^2/(4\pi) = 5\alpha/(3\cos^2\theta_W), \\ \alpha_2 &= g^2/(4\pi) = \alpha/\sin^2\theta_W, \\ \alpha_3 &= g_s^2/(4\pi), \end{aligned} \quad (2.86)$$

to intersect at M_{GUT} .

The energy dependence of the parameters α_i is provided by the renormalization formalism. Using a specific renormalization scheme,³ the contributions of vacuum polarization processes to the boson propagators are taken into account. The corresponding renormalization group equations at one-loop level are given by [47]

³ Here the so-called modified minimal subtraction scheme \overline{MS} is used [45, 46].

$$\frac{d\alpha_i}{dt} = \frac{b_i}{2\pi}\alpha_i^2, \quad i = 1, 2, 3, \quad (2.87)$$

with $t = \ln(Q^2/\mu_R^2)$, where Q^2 stands for the energy of the interaction and μ_R^2 is the renormalization scale. The analytical solution of Eq. (2.87),

$$\frac{1}{\alpha_i(Q^2)} = \frac{1}{\alpha_i(\mu_R^2)} - \frac{b_i}{2\pi} \ln\left(\frac{Q^2}{\mu_R^2}\right), \quad (2.88)$$

describes the so-called ‘running’ of the coupling constants as a function of energy. The coefficients b_i carry intrinsic information about the particle content of the underlying model. In the case of the Standard Model they have been determined to [48]

$$b_i = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -22/3 \\ -11 \end{pmatrix} + N_{Fam} \begin{pmatrix} 4/3 \\ 4/3 \\ 4/3 \end{pmatrix} + N_{Higgs} \begin{pmatrix} 1/10 \\ 1/6 \\ 0 \end{pmatrix}, \quad (2.89)$$

where $N_{Fam} = 3$ stands for the number of generations and $N_{Higgs} = 1$ is the number of Higgs doublets.

Using Eqs. (2.88), (2.89) and the experimentally measured values for the couplings, one can extrapolate to high energies to examine a possible unification [44]. The result of this extrapolation is shown in Fig. 2.5 (left), which clearly indicates that a unification in a single point is not natural. In fact it is ruled out by more than 7 standard deviations [44] and so is a minimal GUT based on the Standard Model.

To maintain the idea of a Grand Unified Theory, new physics must exist between the electroweak and Planck scale to alter the behaviour of the α_i . This is where Supersymmetry enters the picture. Assuming a minimal supersymmetric model as described in Sect. 2.2.3, the extended particle content changes the coefficients b_i to [48]

$$b_i = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -6 \\ -9 \end{pmatrix} + N_{Fam} \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} + N_{Higgs} \begin{pmatrix} 3/10 \\ 1/2 \\ 0 \end{pmatrix}, \quad (2.90)$$

where $N_{Fam} = 3$ remains and an additional Higgs doublet is introduced $N_{Higgs} = 2$. It turns out, with these parameters a unification becomes possible at $\sim 10^{16}$ GeV as illustrated in Fig. 2.5 (right). Furthermore the most perfect intersection of the coupling constants can be obtained if the masses of the SUSY particles are of the order of 1 TeV [44]. This provides yet another very strong motivation for low energy Supersymmetry.

Dark Matter. A third argument in favour of Supersymmetry is its possible explanation for dark matter. Dark matter is a hypothetical form of matter that cannot be observed directly, but whose existence can be inferred from gravitational effects on visible matter, such as the rotation of galaxies or the structure formation in the universe. Latest cosmological results, e.g. from WMAP-data [50], suggest that the cold dark matter content makes up about 23 % of the energy density of the universe.

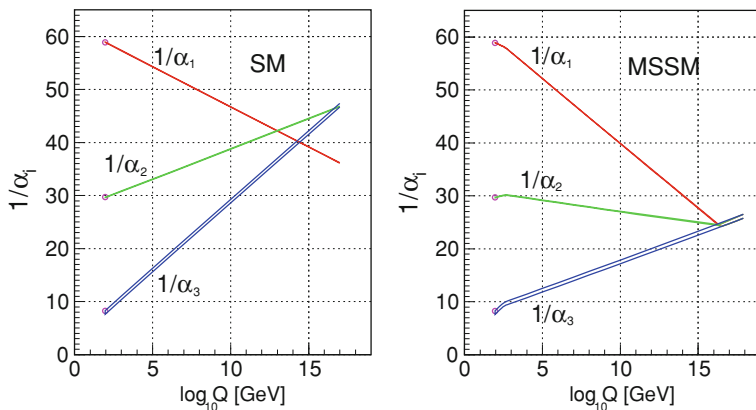


Fig. 2.5 Evolution of the inverse coupling constants with energy. Within the Standard Model (*left*) a unification is not natural, whereas for minimal supersymmetric models (*right*) the coupling constants intersect at $M_{GUT} \approx 10^{16}$ GeV [44]. The thickness of the lines indicates the error on the coupling constants. The plots are taken from [49]

Candidate particles for dark matter include the so-called weakly-interacting massive particles (WIMPs). These hypothetical neutral particles only participate in the gravitational and weak interactions and thus are extremely difficult to detect. Neutrinos are the only WIMP-like particles within the Standard Model. They are, however, not massive and abundant enough to provide an explanation for dark matter.

In R-Parity conserving SUSY models (see Sect. 2.2.3) the lightest supersymmetric particle (LSP) is stable and can be neutral and only weakly interacting. It therefore exhibits all features of a WIMP and constitutes a possible dark matter candidate.

Gravity. As a last important point the connection between Supersymmetry and Gravity is briefly mentioned. As will be further explained in Sect. 2.2.5, by making Supersymmetry a local symmetry the principles of both theories can be unified into a single concept, called Supergravity.

Supergravity is non-renormalizable and therefore not a candidate for a Theory of Everything, but it can be understood as an effective description of physical phenomena including gravity at energies below the Planck scale [51].

In conclusion, a variety of reasons point to Supersymmetry as a possible theory for physics beyond the Standard Model. In the following sections of this chapter the mathematical formalism of Supersymmetry is introduced step-by-step.

2.2.1 Supersymmetry Algebra

In this section the group-theoretical foundations of Supersymmetry are reviewed. As a starting point the so-called Poincaré group is considered, one of the most fundamental symmetry groups in physics. It contains the full symmetry of special relativity, including the four translations in Minkowski space, plus the three rotations

and boosts of the Lorentz group. The known elementary particles are irreducible representations of the Poincaré group.

To obtain a unified theory of all interactions, it is desirable to combine the internal gauge symmetries of the Standard Model, represented by the Lie groups $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$, with the structure of space-time provided by the Poincaré group. Unfortunately, the existence of such a group is ruled out for any but the trivial case, as was shown by Coleman and Mandula [52] in 1967. However, after Wess and Zumino found the first supersymmetric model in 1974 [53], this theorem had to be significantly generalized. In fact it was shown by Haag, Lopuszanski, and Sohnius [54] a year later that under weaker assumptions a non-trivial extension to the Poincaré symmetry is possible, namely *Supersymmetry*.

Thus Supersymmetry is a space-time symmetry. Its generators Q transform bosonic states into fermionic states and vice-versa:

$$Q|Boson\rangle = |Fermion\rangle, \quad Q|Fermion\rangle = |Boson\rangle \quad (2.91)$$

It follows that Q and its hermitian conjugate Q^\dagger must have fermionic character and carry spin-1/2. Within the easiest supersymmetric extension of the Poincaré group, the above mentioned Wess-Zumino model, the generators are required to satisfy the following algebra,

$$\{Q_\alpha, Q_\alpha^\dagger\} = 2\sigma_{\alpha\dot{\alpha}}^\mu P_\mu \quad (2.92)$$

$$\{Q_\alpha, Q_\beta\} = \{Q_\alpha^\dagger, Q_\beta^\dagger\} = 0 \quad (2.93)$$

$$[Q_\alpha, P^\mu] = [Q_\alpha^\dagger, P^\mu] = 0, \quad (2.94)$$

where P^μ denotes the four-momentum in space-time, σ^μ are the Pauli matrices and $\alpha, \dot{\alpha}, \beta, \dot{\beta}$ are the indices of two-component Weyl spinors.

Direct implications of this supersymmetric algebra are the following:

- The bosonic or fermionic states and their respective superpartners with opposite spin statistics are ordered in supersymmetric multiplets, so-called *supermultiplets*. The supermultiplets are the irreducible representations of the supersymmetric algebra.
- Superpartners must have equal mass, since $-P^2$ commutes with both Q and Q^\dagger as well as all generators of the Poincaré group.
- Superpartners must have equal gauge quantum numbers, such as charge, isospin and color, since Q and Q^\dagger commute with all generators of the internal gauge symmetry groups.
- A concatenation of two supersymmetric transformations leads to a translation in space-time, since the square of the generator Q is equal to P_μ . For local supersymmetric transformations this implies a connection between Supersymmetry and General Relativity as will be shown later on.
- The bosonic and fermionic degrees of freedom of each supermultiplet are related as $n_B = n_F$.

The latter statement needs a short explanation: As stated earlier the fermionic generators of Supersymmetry Q and Q^\dagger map the bosonic subspace B onto a fermionic subspace F and vice-versa. It is also known that for a linear mapping $f : X \rightarrow Y$ the relation $\dim(Y) \leq \dim(X)$ holds. Thus, for a two-fold supersymmetric transformation $B \rightarrow F \rightarrow B$, the equation $\dim(F) = \dim(B)$ must apply, since the concatenation of two SUSY transformations maps the bosonic subspace onto itself. The same argument holds for the fermionic subspace and thus the bosonic and fermionic degrees of freedom in each supermultiplet must be equal.

Following this rule the possible constellations of supermultiplets are examined. The easiest example is the so-called *chiral supermultiplet*. Each chiral supermultiplet consists of one fermion with two spin degrees of freedom $n_F = 2$ and two real scalar fields with $n_B = 1$ each. The two real components are equivalent to one complex scalar field. Per naming convention, the supersymmetric scalar particle states receive an “s”-prefix to their name (“s-fermion”), to distinguish them from the original particle.

Next are the gauge supermultiplets. They contain massless spin-1 vector bosons with two helicity states and again a spin-1/2 fermion with $n_F = 2$ as superpartner. Here the superparticles are indicated by an “ino”-suffix to the name of the corresponding gauge boson (“gaugino”).

Depending on the underlying supersymmetric model there are also other possible constellations. An explicit discussion of the supermultiplets and their particle content will be given in Sect. 2.2.3 for the case of the Minimal Supersymmetric Standard Model.

2.2.2 Supersymmetric Lagrangians

In this section the Lagrangian formalism of Supersymmetry is derived. The goal is to obtain a general formulation of the supersymmetric field theory with its particle masses and interactions. Following closely the approach in [43], the Lagrangians of the chiral and gauge supermultiplets and their respective field interactions are introduced step-by-step.

2.2.2.1 The Chiral Supermultiplet

To begin with, the simplest possible SUSY model, containing a single left-handed two component Weyl fermion ψ and a complex scalar field ϕ as its superpartner is considered. This is equivalent to a theory with only one chiral supermultiplet, also referred to as the massless non-interacting Wess-Zumino model [53]. The Lagrangian density of this model is of the form

$$\mathcal{L}_{\text{chiral,free}} = \mathcal{L}_{\text{scalar}} + \mathcal{L}_{\text{fermion}} = -\partial^\mu \phi^* \partial_\mu \phi - i\psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi, \quad (2.95)$$

and contains two kinetic terms for the scalar and fermionic states.

Now minimal supersymmetric transformations of the scalar fields of the type

$$\delta\phi = \epsilon\psi \quad \text{and} \quad \delta\phi^* = \epsilon^\dagger\psi^\dagger \quad (2.96)$$

are introduced, where ϵ is an infinitesimal fermion-like parameter. The scalar part of the Lagrangian transforms under these equations as

$$\delta\mathcal{L}_{\text{scalar}} = -\epsilon\partial^\mu\psi\partial_\mu\phi^* - \epsilon^\dagger\partial^\mu\psi^\dagger\partial_\mu\phi. \quad (2.97)$$

According to Hamilton's principle the action (Eq. 2.1) must stay invariant under any symmetry transformations. Therefore the Lagrangians before and after the transformation must be equivalent up to a total derivative $\mathcal{L}' = \mathcal{L} + \partial_\mu\Lambda^\mu$. This implies the following transformations of the fermion fields:

$$\delta\psi_\alpha = i(\sigma^\mu\epsilon^\dagger)_\alpha\partial_\mu\phi \quad \text{and} \quad \delta\psi^\dagger_{\dot{\alpha}} = -i(\epsilon\sigma^\mu)_{\dot{\alpha}}\partial_\mu\phi^* \quad (2.98)$$

Using Pauli matrix identities and commutation relations for partial derivatives one obtains

$$\delta\mathcal{L}_{\text{fermion}} = \epsilon\partial^\mu\psi\partial_\mu\phi^* + \epsilon^\dagger\partial^\mu\psi^\dagger\partial_\mu\phi - \partial_\mu\left(\epsilon\sigma^\nu\bar{\sigma}^\mu\psi\partial_\nu\phi^* + \epsilon\psi\partial^\mu\phi^* + \epsilon^\dagger\psi^\dagger\partial^\mu\phi\right). \quad (2.99)$$

Comparing 2.97 and 2.99 one sees that the two contributions cancel up to a total derivative and thus leave the action invariant as required.

However, for the SUSY algebra to be valid off-shell, this formalism needs to be generalized by the introduction of an auxiliary field F . F has no kinematic term and its Lagrangian density is given by

$$\mathcal{L}_{\text{auxiliary}} = F^*F. \quad (2.100)$$

The auxiliary fields transform as

$$\delta F = i\epsilon^\dagger\bar{\sigma}^\mu\partial_\mu\psi \quad \text{and} \quad \delta F^* = -i\partial_\mu\psi^\dagger\bar{\sigma}^\mu\epsilon, \quad (2.101)$$

and to keep the action invariant Eq. 2.98 have to be modified with additional F -terms

$$\delta\psi_\alpha = i\left(\sigma^\mu\epsilon^\dagger\right)_\alpha\partial_\mu\phi + \epsilon_\alpha F \quad \text{and} \quad \delta\psi^\dagger_{\dot{\alpha}} = -i\left(\epsilon\sigma^\mu\right)_{\dot{\alpha}}\partial_\mu\phi^* + \epsilon^\dagger_{\dot{\alpha}}F^*. \quad (2.102)$$

The need for the auxiliary field F becomes apparent, if one compares the scalar and fermionic degrees of freedom on- and off-shell. In the on-shell case, the complex scalar field ϕ has two real components corresponding to the two helicity states of the fermion field ψ . Going off-shell, however, ψ becomes a two-dimensional complex object with four real degrees of freedom. To balance out this inequality the complex field F with two additional degrees of freedom must be introduced.

The final Lagrangian of the free chiral supermultiplet can now be written as

$$\mathcal{L}_{\text{chiral,free}} = -\partial^\mu \phi^{*i} \partial_\mu \phi_i - i\psi^{\dagger i} \bar{\sigma}^\mu \partial_\mu \psi_i + F^{*i} F_i, \quad (2.103)$$

where i is the index over all flavour and gauge degrees of freedom.

Interactions. The interactions between the scalar and fermion fields inside the supermultiplets are now considered. It can be shown [43] that the most general renormalizable form of the interaction Lagrangian is given by

$$\mathcal{L}_{\text{chiral,int}} = \left(-\frac{1}{2} W^{ij} \psi_i \psi_j + W^i F_i \right) + c.c., \quad (2.104)$$

where W^{ij} and W^i are functions of the scalar fields participating in the interaction. The invariance of $\mathcal{L}_{\text{chiral,int}}$ under SUSY transformations implies the following form of the W^{ij} and W^i [43]

$$W^{ij} = \frac{\partial^2}{\partial \phi_i \partial \phi_j} W = \frac{1}{2} M^{ij} + \frac{1}{6} y^{ijk} \phi_k, \quad (2.105)$$

$$W^i = \frac{\partial W}{\partial \phi_i} = \frac{1}{2} M^{ij} \phi_j + \frac{1}{6} y^{ijk} \phi_j \phi_k, \quad (2.106)$$

where W is the so-called superpotential

$$W = \frac{1}{2} M^{ij} \phi_i \phi_j + \frac{1}{6} y^{ijk} \phi_i \phi_j \phi_k. \quad (2.107)$$

Here M^{ij} is the symmetric fermion mass matrix and y^{ijk} the Yukawa coupling of two fermion fields with one scalar. It can be seen that the superpotential contains only bilinear and trilinear scalar coupling terms and no fermionic contributions.

The auxiliary field terms of Lagrangians 2.103 and 2.104 are $F_i F^{i*} + W^i F_i + W_i^* F^{i*}$. They can be reformulated in terms of the superpotential by applying the Euler–Lagrange equations of motion for F^i leading to

$$F_i = -W_i^* \quad \text{and} \quad F^{i*} = -W^i. \quad (2.108)$$

With this the auxiliary-free representation of the total Lagrangian of the interacting chiral supermultiplet is obtained:

$$\begin{aligned} \mathcal{L}_{\text{chiral}} = & -\partial^\mu \phi^{i*} \partial_\mu \phi_i - V(\phi, \phi^*) - i\psi^{i\dagger} \bar{\sigma}^\mu \partial_\mu \psi_i - \frac{1}{2} M^{ij} \psi_i \psi_j - \frac{1}{2} M_{ij}^* \psi^{i\dagger} \psi^{j\dagger} \\ & - \frac{1}{2} y^{ijk} \phi_i \psi_j \psi_k - \frac{1}{2} y_{ijk}^* \phi^{i*} \psi^{j\dagger} \psi^{k\dagger} \end{aligned} \quad (2.109)$$

Here V is referred to as the scalar potential

$$V(\phi, \phi^*) = W^k W_k^* = F^{*k} F_k = M_{ik}^* M^{kj} \phi_i^* \phi_j + \frac{1}{2} M^{in} y_{jkn}^* \phi_i \phi_j^* \phi^{k*} \\ + \frac{1}{2} M_{in}^* y^{jkn} \phi_i^* \phi_j \phi_k + \frac{1}{4} y^{ijn} y_{klm}^* \phi_i \phi_j \phi^{k*} \phi^{l*}. \quad (2.110)$$

It can be seen that the scalar potential contains cubic and quartic scalar coupling terms as well as a mass term with the same mass matrix as the fermionic part in Eq. 2.109. This leads to the expected mass-degenerate partner states inside each supermultiplet.

Equation 2.109 also demonstrates that the coupling strength of a scalar particle with two fermion fields is of the order y^{ijk} , whereas Eq. 2.110 implies $(y^{ijk})^2$ for a quartic scalar process. This is equivalent to the relation $\lambda_S = \lambda_f^2$ that was postulated in the introduction of Sect. 2.2 for the solution of the hierarchy problem.

2.2.2.2 The Gauge Supermultiplet

The gauge supermultiplets consist of massless gauge bosons A_μ^a and their gaugino superpartners λ_a

$$\begin{pmatrix} A_\mu^a \\ \lambda_a \end{pmatrix}, \quad (2.111)$$

where a is the index over the group representations, e.g. $a = 1, \dots, 8$ for $SU(3)_C$. The corresponding supersymmetric transformations are given by

$$\delta_{gauge} A_\mu^a = \partial_\mu \Lambda^a + g f^{abc} A_\mu^b \Lambda^c \quad (2.112)$$

$$\delta_{gauge} \lambda^a = g f^{abc} \lambda^b \Lambda^c. \quad (2.113)$$

Here Λ is an infinitesimal gauge transformation parameter, g represents the coupling strength of the interaction and f^{abc} the structure constant in the case of a non-abelian theory. The total Lagrangian density of the gauge supermultiplets must again leave the action invariant under Eqs. 2.112 and 2.113. It is given by:

$$\mathcal{L}_{gauge} = -\frac{1}{4} F^{a\mu\nu} F_{\mu\nu}^a - i \lambda^{\dagger a} \bar{\sigma}^\mu D_\mu \lambda^a + \frac{1}{2} D^a D^a \quad (2.114)$$

The first term describes the kinetic energy with the gauge field tensor

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c, \quad (2.115)$$

where the A_μ are the respective gauge field components of the interaction. The second term in Eq. 2.114 expresses the kinetic energy of the gaugino fields and includes a covariant derivative

$$D_\mu \lambda^a = \partial_\mu \lambda^a + g f^{abc} A_\mu^b \lambda^c, \quad (2.116)$$

which contains the interactions between the gauge and gaugino fields. With the third term in Eq. 2.114 again an auxiliary field D is introduced to account for the inequality of degrees of freedom within the supermultiplets on- and off-shell. In this case only one additional degree of freedom is necessary and thus D has one real component. As in the case of the chiral Lagrangian this field vanishes when going on-shell.

2.2.2.3 The Combined Supersymmetric Lagrangian

Now the results for the Lagrangian of the chiral and gauge supermultiplets can be combined. First, the derivatives of the scalar and fermion fields need to be replaced by the respective covariant derivative to preserve gauge invariance:

$$\partial_\mu \psi \rightarrow D_\mu \psi = \partial_\mu \psi + ig A_\mu^a T^a \psi \quad (2.117)$$

$$\partial_\mu \phi \rightarrow D_\mu \phi = \partial_\mu \phi + ig A_\mu^a T^a \phi \quad (2.118)$$

Here T^a stands for the generators of the gauge groups, and one can see that Eqs. 2.117 and 2.118 yield the couplings between the gauge bosons and the scalar and fermionic fields of the chiral supermultiplet.

There are also possible couplings between the gaugino fields λ^a and the D auxiliary fields. It can be shown [43] that possible renormalizable couplings of this sort contribute with terms:

$$-\sqrt{2}g (\phi^* T^a \psi) \lambda^a - \sqrt{2}g \lambda^{\dagger a} (\phi^\dagger T^a \phi) + g (\psi^* T^a \phi) D^a \quad (2.119)$$

The D -term in Eq. 2.114 and the last term in 2.119 combine to the equation of motion

$$D^a = -g (\phi^* T^a \phi). \quad (2.120)$$

Since 2.120 contains only scalar fields it is then usually written with the scalar potential

$$V(\phi, \phi^*) = W_i^* W^i + \frac{1}{2} g^2 (\phi^* T \phi)^2. \quad (2.121)$$

The results of this chapter can now be summarized in the total Lagrangian density for Supersymmetry:

$$\begin{aligned}
\mathcal{L}_{SUSY} = & \underbrace{-D^\mu \phi^* D_\mu \phi}_{\text{scalars}} \underbrace{-i\psi^\dagger \bar{\sigma}^\mu D_\mu \psi}_{\text{fermions}} \underbrace{-\frac{1}{2} \left(W^{ij} \psi_i \psi_j + W^{ij*} \psi^{i\dagger} \psi^{j\dagger} \right)}_{\text{fermion mass term and Yukawa coupling}} \\
& \underbrace{-W^i W_i^* - \frac{1}{2} g^2 (\phi^* T \phi)^2}_{\text{scalar potential}} \underbrace{-\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu}}_{\text{gauge bosons}} \underbrace{-i\lambda^{\dagger a} \bar{\sigma}^\mu D_\mu \lambda^a}_{\text{gauginos}} \\
& \underbrace{-\sqrt{2}g (\phi^* T^a \psi) \lambda^a - \sqrt{2}g \lambda^{\dagger a} (\psi^\dagger T^a \phi)}_{\text{additional couplings}} \underbrace{+ \mathcal{L}_{\text{soft}}}_{\text{soft breaking term}} \quad (2.122)
\end{aligned}$$

The first two terms are the kinetic terms of the scalar and fermionic fields in the chiral supermultiplet. They are followed by the fermion mass terms and the Yukawa coupling of scalar and fermionic fields, written in terms of the superpotential 2.107. The mass terms of the scalar fields and their associated interactions are contained in the scalar potential. The next three terms account for the kinetic energy and interactions of the gauge bosons and their gaugino superpartners plus additional couplings as discussed above. For completeness the SUSY breaking term $\mathcal{L}_{\text{soft}}$ is also included and will be discussed in Sect. 2.2.4.

2.2.3 The Minimal Supersymmetric Standard Model

In the previous section of this chapter the general theoretical features of Supersymmetry have been discussed. Now the Minimal Supersymmetric Standard Model (MSSM) is discussed specifically. It represents the simplest possible supersymmetric extension of the Standard Model and has long been the main focus of experimental searches. Implications of a possible Higgs boson in the mass range of 126 GeV and recent experimental bounds on the MSSM will be discussed in Sect. 2.2.7.

2.2.3.1 Particle content of the MSSM

The MSSM contains the minimal number of couplings and fields. The field content is described in terms of:

- Chiral supermultiplets with SM leptons and quarks and their associated scalar superpartners (“squarks” and “sleptons”).
- Gauge supermultiplets with SM gauge bosons and associated superpartners (“gauginos”).

A summary of all MSSM multiplets can be found in Tables 2.2 and 2.3. For the chiral supermultiplets, each SM fermion has two helicity states, which transform differently under gauge symmetry. Therefore they must both have their own scalar superpartner. This is illustrated in Table 2.2, e.g. for the left- and right-handed electrons e_L and e_R and their corresponding scalar particles \tilde{e}_L and \tilde{e}_R .⁴ The latter two scalar fields are

⁴ The tilde symbol is used to denote supersymmetric partners of SM particles.

Table 2.2 Content of the chiral supermultiplets in the Minimal Supersymmetric Standard Model

Name		Scalar ϕ ($S = 0$)	Fermion ψ ($S = 1/2$)
Squarks, Quarks (3 Generations)	Q_1	$(\tilde{u}_L, \tilde{d}_L)$	(u_L, d_L)
	\bar{U}_1	\tilde{u}_R^*	u_R^\dagger
	\bar{D}_1	\tilde{d}_R^*	d_R^\dagger
Sleptons, Leptons (3 Generations)	L_1	$(\tilde{\nu}_e, \tilde{e}_L)$	(ν_e, e_L)
	\bar{E}_1	\tilde{e}_R^*	e_R^\dagger
Higgs, Higgsino	H_u	(H_u^+, H_u^0)	$(\tilde{H}_u^+, \tilde{H}_u^0)$
	H_d	(H_d^0, H_d^-)	$(\tilde{H}_d^0, \tilde{H}_d^-)$

The symbols Q_i and L_i stand for the supermultiplets containing $SU(2)_L$ doublets, while \bar{U}_i , \bar{D}_i , and \bar{E}_i contain the corresponding conjugate right-handed singlet states. Table adapted from [43]

completely independent and the index refers only to the handedness of the associated SM particles. All fermions in the chiral supermultiplets are defined in terms of left-handed Weyl-spinors. Conjugations are therefore applied to the right-handed fields of Table 2.2.

In the supersymmetric Higgs-sector, the scalar Higgs fields are accommodated in the chiral supermultiplets along with their “Higgsino” superpartners (see Table 2.2). However, within the MSSM one Higgs-doublet is not sufficient. In particular, gauge anomalies from triangle diagrams, as they are known from the SM, do not cancel within the MSSM unless a second doublet is introduced. Two Higgs-doublets are also required to give mass to all matter fermions by means of electroweak symmetry breaking. The doublets are of the form

$$\begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}, \quad (2.123)$$

with weak Isospin $Y = \pm 1/2$ to induce the necessary Yukawa couplings to all up- and down-like quarks. As can be seen from Eq. 2.123 there are 4 complex or 8 real degrees of freedom in the Higgs doublets. As in the Standard Model three phases are absorbed by Goldstone bosons leaving 5 physical Higgs eigenstates:

- h , a light neutral scalar Higgs particle.
- H , a heavy neutral scalar Higgs Particle.
- A , a neutral CP-odd pseudoscalar Higgs particle.
- H^\pm , two charged scalar Higgs particles.

The gauge supermultiplets are shown in Table 2.3. They contain the mediators of the SM interactions and their spin-1/2 superpartners. These are 8 gluons and gluinos in the case of $SU(3)_C$ for QCD and W^\pm, W^0, B^0 with superpartners for $SU(2)_L \otimes U(1)_Y$. After electroweak symmetry breaking the latter four mix to the mass eigenstates Z^0 and γ and the respective gaugino combinations “Zino” \tilde{Z}^0 and “Photino” $\tilde{\gamma}$.

Electroweak gauginos also form new mass eigenstates with the Higgsinos of same charge. In the neutral sector $\tilde{W}^0, \tilde{B}^0, \tilde{H}_u^0, \tilde{H}_d^0$ mix to four “Neutralinos” $\tilde{\chi}_{1,2,3,4}^0$, whereas the charged $\tilde{W}^\pm, \tilde{H}_u^\pm$ and \tilde{H}_d^\pm form the “Charginos” $\tilde{\chi}_{1,2}^\pm$. The

Table 2.3 Content of the gauge supermultiplets in the Minimal Supersymmetric Standard Model

Name	Boson A^μ ($S = 1$)	Fermion λ ($S = 1/2$)
Gluon, Gluino	g	\tilde{g}
W-Bosons, Winos	W^\pm, W^0	$\tilde{W}^\pm, \tilde{W}^0$
B-Boson, Bino	B^0	\tilde{B}^0

Table adapted from [43]

mixing is possible because the participating states have identical quantum numbers as can be inferred from Table 2.3.

It should be noted here that if Supergravity (see Sect. 2.2.5) is to be included into the theory, one must also introduce an additional type of supermultiplet containing the spin-2 graviton and its spin-3/2 superpartner (“gravitino”).

2.2.3.2 R-parity

An important quantum number in the MSSM is R-parity. Whereas in the Standard Model baryon- and lepton-numbers are automatically conserved, the MSSM theoretically allows interaction terms that violate this symmetry. To avoid this undesired effect, the conservation of R-parity

$$R = (-1)^{3(B-L)+2S} \quad (2.124)$$

is imposed. B and L are lepton and baryon numbers and S the spin of the participating particles in the process. All SM fields carry R-parity $R = +1$, all superfields $R = -1$. In addition R-parity is a multiplicative quantum number. It follows that in this case supersymmetric particles can only be produced in pairs by SM particles due to

$$R_{total} = R_1^{SM} \cdot R_2^{SM} = 1^2 = R_1^{SUSY} R_2^{SUSY} = (-1)^2 = 1. \quad (2.125)$$

Another consequence of R-parity conservation is the stability of the lightest supersymmetric particle. Due to $R = -1$ it cannot decay into SM matter and due to its mass not into any other supersymmetric particle. It is therefore considered a candidate for dark matter.

In some supersymmetric models, which are not the subject of this thesis, the violation of R-parity is allowed to the extent that it is compatible with the observed proton lifetime.

2.2.3.3 Interactions of the MSSM

In the MSSM the generic superpotential of Eq. 2.107 is replaced by [43]

$$W_{MSSM} = y_u \bar{U} Q H_U - y_d \bar{D} Q H_d - y_e \bar{E} L H_d + \mu H_u H_d, \quad (2.126)$$

where Q , L , \bar{U} , \bar{D} , \bar{E} , H_u , and H_d stand for the superfields of the respective multiplets shown in Table 2.2. The indices for each generation of quarks and leptons are suppressed in this vector notation. The 3×3 matrices y_u , y_d and y_e are the corresponding Yukawa couplings and determine the CKM mixing angles and masses after electroweak symmetry breaking. The last term is the supersymmetric Higgs term with mass parameter μ .

The supersymmetric Yukawa interactions can be derived from Eq. 2.126. They describe cubic and quartic couplings of fermions, sfermions, Higgs, and Higgsino fields. The coupling of SM gauge bosons to supersymmetric particles is governed by the kinetic terms of the SUSY Lagrangian. The MSSM gauge interactions can be obtained by interchanging any two of the three participating particles in a SM gauge interaction by their respective superpartners. In this way one can also obtain gaugino-sfermion-fermion interactions, which are possible through the additional renormalizable couplings, corresponding to the first two terms in Eq. 2.119.

2.2.4 Breaking of Supersymmetry

If Supersymmetry exists it must be broken, since no mass-degenerate superpartners of the SM particles have been found. The breaking mechanism should preserve the renormalizability of the theory as well as the cancellation of quadratic divergences to maintain the hierarchy of the energy scales as discussed in the introduction. A symmetry breaking with these basic SUSY properties can be introduced into the theory by adding a so-called “soft-breaking” term in the Lagrangian density. The general form of this term is [43]

$$\mathcal{L}_{\text{soft}} = - \left(\frac{1}{2} M_a \lambda^a \lambda^a + \frac{1}{6} a^{ijk} \phi_i \phi_j \phi_k + \frac{1}{2} b^{ij} \phi_i \phi_j + c.c. \right) - (m^2)_j^i \phi^j \phi_i^*, \quad (2.127)$$

with squared scalar mass terms $(m^2)_j^i$ and b^{ij} , cubic scalar couplings a^{ijk} , and gaugino mass terms M_a for each gauge group. It can be seen that 2.127 contains only scalar and gaugino terms and thus breaks the symmetry by giving masses to the associated particles. A phenomenological explanation of this breaking mechanism will follow in the next section. In the case of the MSSM the soft breaking term specializes to [43]

$$\begin{aligned} \mathcal{L}_{\text{soft}}^{\text{MSSM}} = & - \frac{1}{2} \left(M_3 \tilde{g} \tilde{g} + M_2 \tilde{W} \tilde{W} + M_1 \tilde{B} \tilde{B} + c.c. \right) \\ & - (\bar{U} a_U Q H_u - \bar{D} a_D Q H_d - \bar{E} a_E L H_d + c.c.) \\ & - Q^\dagger m_Q^2 Q - L^\dagger m_L^2 L - \bar{U} m_U^2 \bar{U}^\dagger - \bar{D} m_D^2 \bar{D}^\dagger - \bar{E} m_E^2 \bar{E}^\dagger \\ & - m_{H_u}^2 H_u^* H_u - m_{H_d}^2 H_d^* H_d - (b H_u H_d + c.c.). \end{aligned} \quad (2.128)$$

The first line represents the mass terms of winos, gluinos and binos, and the third line those of the squarks and sleptons with hermitian 3×3 mass matrices. Here the

tildes on the scalar superfields $Q, \bar{U}, \bar{D}, L, \bar{E}$ are again suppressed for readability. The second line contains the cubic scalar couplings of Eq. 2.127 with the matrices a_U, a_D, a_E , where again all three generations contribute. The last line corresponds to the soft breaking contributions from the squared Higgs-mass terms $m_{H_u}^2$ and $m_{H_d}^2$ plus one b^{ij} -type term.

Equation 2.128 demonstrates the complexity of the spontaneously broken MSSM. In total 105 new parameters are introduced: 21 masses, 36 mixing angles, 40 CP-violating phases in the squark and slepton sector and 5 real and 3 CP-violating parameters in the Higgs-sector. However, not all of these parameters are independent. In particular flavour and CP-conserving relations reduce the number of degrees of freedom significantly.

The most common framework for SUSY breaking models is based on a so-called “hidden sector”, in which the symmetry is spontaneously broken [55, 56]. The introduction of this hidden sector is necessary, since none of the MSSM fields, which are referred to as the “visible sector”, can have a non-zero vacuum expectation value in order to not violate gauge invariance.⁵ Therefore the underlying idea of the hidden sector is, that the spontaneous SUSY breaking is communicated down to the observable MSSM sector via hypothetical flavour-blind messenger fields. This mediation mechanism, however, is highly dependent on the model framework assumed.

Some of the most common scenarios are:

- Gauge Mediated Supersymmetry Breaking (GMSB)
- Anomaly Mediated Supersymmetry Breaking (AMSB)
- Gravity Mediated Supersymmetry Breaking (MSUGRA)

In GMSB the SUSY breaking is invoked by the minimal gauge group interactions $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$, where the associated gauge bosons couple to the messenger fields to make the breaking observable. AMSB is a special case of gravity-mediated SUSY breaking, where there are no direct tree-level couplings between the sectors and the masses of the sparticles are generated with higher order loop corrections. MSUGRA is one of the most commonly assumed scenarios in experimental searches. It will be discussed in more detail in the following section.

2.2.5 Minimal Supergravity

The principles of Supergravity were already briefly mentioned in the introduction of Sect. 2.2. The underlying idea is that supersymmetry, when made a local symmetry, yields both, an effective field theory for energies below the Planck scale and an elegant mechanism for SUSY breaking. In this way Supergravity, as any local gauge theory, necessitate the introduction of new gauge fields, the spin-2 graviton and its spin-3/2

⁵ Spontaneous breaking of global Supersymmetry would require non-zero vacuum expectation values of either the F or D auxiliary fields (see Sect. 2.2.2).

gravitino superpartner. In an unbroken theory the masses of both particles are zero. The couplings, however, still scale with the dimensionful Newton's constant and are thus proportional to $\sim 1/M_{\text{Planck}}$. Therefore the associated terms in the Lagrangian are non-renormalizable and so is the concept of Supergravity, which hence does not represent a full theory of quantum gravity.

The breaking mechanism of Supergravity, also referred to as super-Higgs-mechanism, takes place in two steps: First the spontaneous breaking of *global* Supersymmetry yields a massless Weyl fermion called Goldstino. This Goldstino has two degrees of freedom which are subsequently absorbed through the spontaneous breaking of *local* Supersymmetry to give mass to the gravitino. The graviton, however, is still massless and thus the degeneracy in the gravitational supermultiplet is broken.

The breaking happens in the hidden sector with a vacuum expectation value $\langle F \rangle$ and a non-renormalizable coupling to the visible sector of strength $\sim 1/M_{\text{Planck}}$ as mentioned above. In the case of vanishing gravitational interactions $M_{\text{Planck}} \rightarrow \infty$ and vacuum expectation value $\langle F \rangle \rightarrow 0$, the soft breaking mass terms m_{soft} must also vanish. This leads to the following approximative formula

$$m_{\text{soft}} \sim \frac{\langle F \rangle}{M_{\text{Planck}}}, \quad (2.129)$$

which implies for $m_{\text{soft}} \lesssim 1 \text{ TeV}$ a scale of $\sqrt{\langle F \rangle} \approx 10^{11} \text{ GeV}$ for the hidden sector. The F-Field here refers to the auxiliary field of Sect. 2.2.2, which is related to the superpotential as in Eq. 2.106.

An attractive, but quite constrained scenario, is that of minimal Supergravity (MSUGRA) which is often also referred to as constrained MSSM (CMSSM). It implies universal soft breaking terms and thereby reduces greatly the number MSSM parameters at the unification scale to:

- m_0 , the universal scalar mass,
- $m_{1/2}$, the universal gaugino mass,
- A_0 , the universal trilinear Higgs-sfermion-sfermion couplings,
- $\tan \beta$, the ratio of the vacuum expectation values of the two Higgs-doublets,
- $\text{sign} \mu$, the sign of the Higgsino mass parameter.

All parameters of the MSSM at the electroweak scale can be obtained from these five GUT scale parameters by application of the renormalization group equations as is illustrated in Fig. 2.6 for one particular MSUGRA/CMSSM scenario. The mass spectrum of a given model at the electroweak scale depends on the chosen GUT-scale parameters. Figure 2.7 shows the spectrum for a typical MSUGRA/CMSSM scenario.

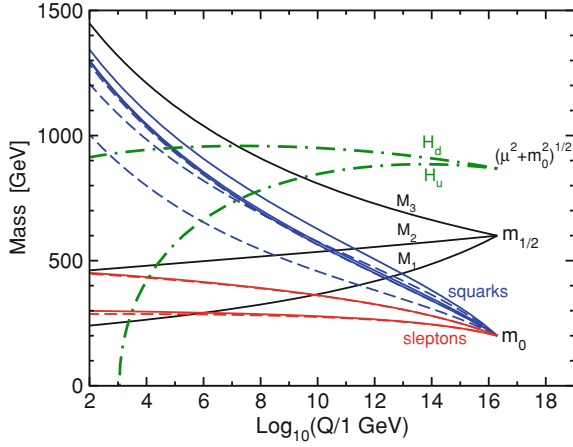


Fig. 2.6 Evolution of the scalar and gaugino mass parameters from the GUT scale to the electroweak scale for a MSUGRA/CMSSM scenario ($m_0 = 200 \text{ GeV}$, $m_{1/2} = 600 \text{ GeV}$, $A_0 = -600 \text{ GeV}$, $\tan \beta = 10$, $\text{sign } \mu > 0$) based on the renormalization group equations. Taken from Ref. [43]

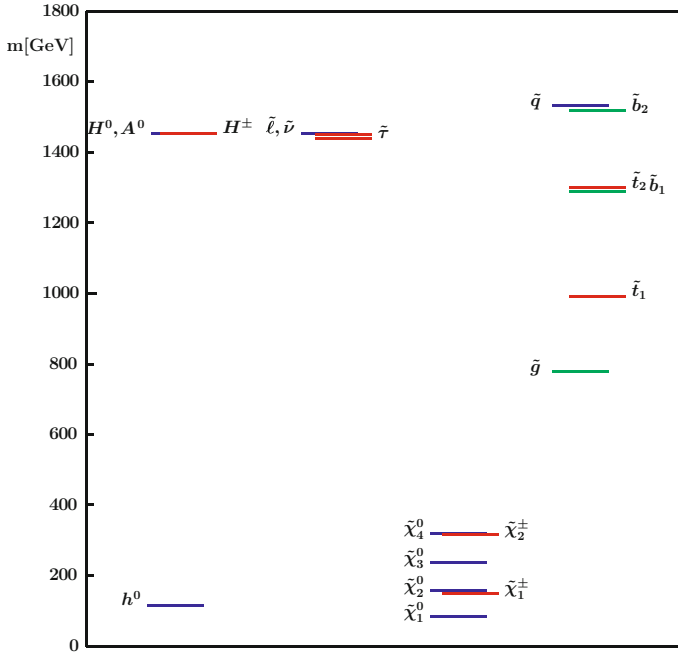


Fig. 2.7 Mass spectrum of a typical MSUGRA scenario with $m_0 = 1450 \text{ GeV}$, $m_{1/2} = 300 \text{ GeV}$, $A_0 = 0$, $\tan \beta = 10$, $\mu > 0$ generated with ISAJET 7.58 [57]

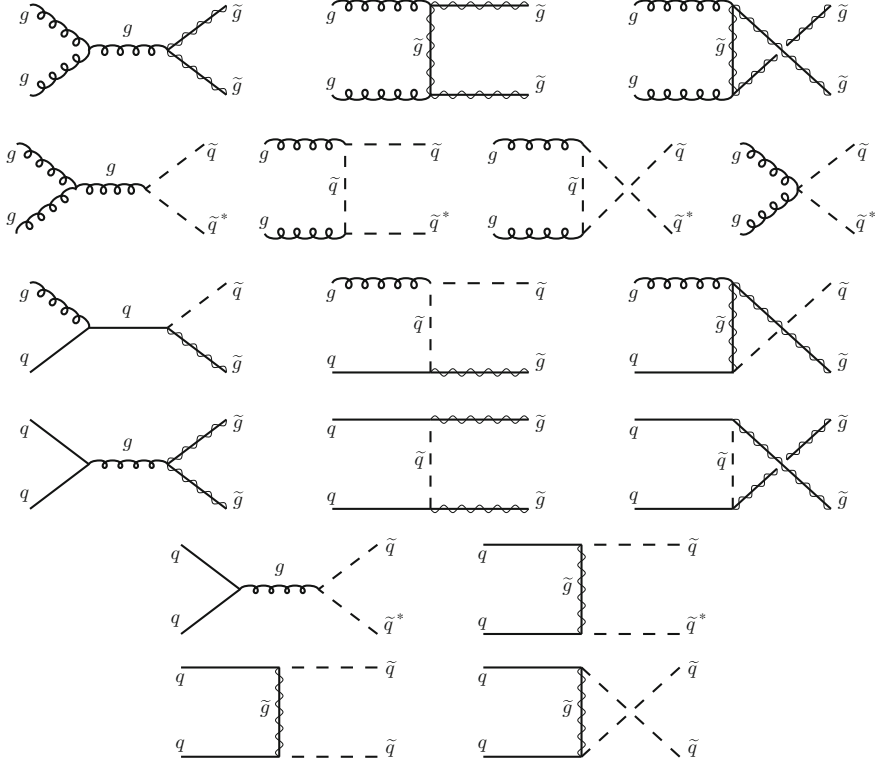


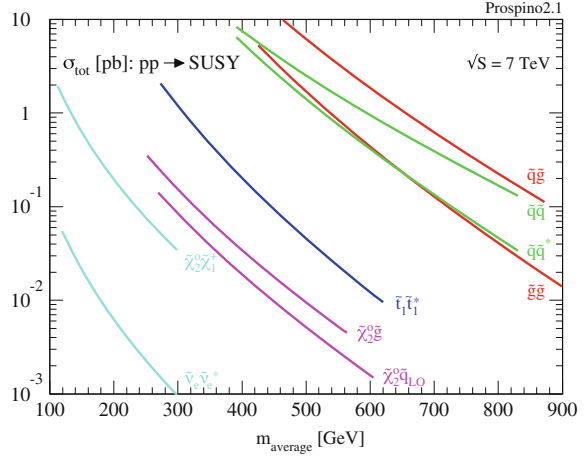
Fig. 2.8 Leading order Feynman diagrams for the strong production of SUSY particles at the LHC, with gluon–gluon, gluon–quark, and quark–quark initial states from *top to bottom*. Taken from [43]

2.2.6 Production and Decay Processes at the LHC

Supersymmetric particles, if they exist at energies accessible by the Large Hadron Collider, can be produced via the electroweak and strong interactions. Within R-parity conserving scenarios, such as the MSSM, sparticles are produced in pairs. The dominant production processes at the LHC are those of the strong interactions via gluon–gluon and gluon–quark processes and to a smaller extent via quark–quark interactions. The possible Feynman graphs of such processes at leading order are shown in Fig. 2.8. The electroweak production processes lead to charginos, neutralinos, and sleptons in the final state via electroweak vector bosons in the s-channel, and t-channel squark exchange. The corresponding diagrams can be found e.g. in Ref. [43]. For most supersymmetric mass spectra, the strong production cross-sections dominate over those of the weak processes as can be seen from Fig. 2.9.

Squarks typically decay, if kinematically allowed, through the process $\tilde{q} \rightarrow q\tilde{g}$ or otherwise to charginos or neutralinos according to $\tilde{q} \rightarrow q^{(\prime)}\tilde{\chi}^{0/(\pm)}$. The direct decay to the lightest neutralino $\tilde{\chi}_1^0$ is kinematically preferred and can dominate for

Fig. 2.9 Next-to-leading order MSSM SUSY production cross-sections in proton-proton collisions as a function of the average final state mass [58]



right handed squarks since the $\tilde{\chi}_1^0$ has a large admixture of the Bino. The left-handed squarks, in turn, may preferentially decay into heavier charginos and neutralinos due to the stronger couplings to the Wino. Couplings of squarks to gauginos with a large Higgsino admixture are usually suppressed except for third generation squarks where the Yukawa couplings are large due to the heavy quark masses. Decays are also possible through virtual quarks if not kinematically allowed on-shell.

Gluinos decay through the process $\tilde{g} \rightarrow q\tilde{q}$ or if the latter is kinematically forbidden via a virtual squark directly to charginos or neutralinos according to $\tilde{g} \rightarrow qq^{(\prime)}\chi^{0/(\pm)}$. If squarks are not kinematically accessible the decay $\tilde{g} \rightarrow g\chi^0$ is possible.

As discussed previously the charginos and neutralinos are mixtures of the electroweak gauginos and Higgsinos. Depending on the mass spectrum of the SUSY scenario considered, the following two-body decays are in principle possible [43]:

$$\begin{aligned}\tilde{\chi}_i^0 &\rightarrow Z\tilde{\chi}_j^0, W\tilde{\chi}_j^\pm, h^0\tilde{\chi}_j^0, \tilde{l}l, \nu\tilde{\nu}, A^0\tilde{\chi}_j^0, H^0\tilde{\chi}_j^0, H^\pm\tilde{\chi}_j^\pm, q\tilde{q} \\ \tilde{\chi}_i^\pm &\rightarrow W\tilde{\chi}_j^0, Z\tilde{\chi}_1^\pm, h^0\tilde{\chi}_1^\pm, \tilde{l}l, \nu\tilde{l}, A^0\tilde{\chi}_1^\pm, H^0\tilde{\chi}_1^\pm, H^\pm\tilde{\chi}_j^0, q\tilde{q}'\end{aligned}\quad (2.130)$$

If these decays are kinematically excluded, they are replaced again by 3-body decays with off-shell gauge bosons, which decay on to fermions.

Sleptons typically decay to leptons and charginos or neutralinos according to $\tilde{l} \rightarrow l\tilde{\chi}^0, \nu\tilde{\chi}^\pm$ and $\tilde{\nu} \rightarrow \nu\tilde{\chi}^0, l\tilde{\chi}^\pm$. As for the quarks one has to distinguish between left- and right-handed sleptons, which preferentially decay to Wino or Bino like gauginos respectively.

Due to the cascade-like nature of supersymmetric decays described above, the expected experimental signature at the LHC for most MSSM-like models consists of several jets, missing transverse energy from the undetected LSP, and possible other objects, such as for example leptons.

2.2.7 Status and Implications of Experimental Bounds

While no evidence for supersymmetric particles has been found to date, experimental efforts have helped to significantly constrain the allowed regions of supersymmetric parameter space, in particular in the context of the well-studied MSUGRA/CMSSM scenarios. Some of the most important experimental bounds and their implications are described below. Figure 2.10 visualizes these bounds in the plane of universal scalar and gaugino masses, for scenarios with $\mu > 0$, $A_0 = 0$, and $\tan \beta = 10$ (left) as well as $\tan \beta = 50$ (right).

Cosmological constraints. It has previously been mentioned that in R-parity conserving SUSY models, the lightest supersymmetric particle (LSP) exhibits all features of a weakly interacting massive particle (WIMP), which in turn constitutes a candidate for cold dark matter. For MSUGRA/CMSSM scenarios this LSP with WIMP-features is the lightest neutralino $\tilde{\chi}_1^0$.

According to cosmological models, WIMPs were created in the early universe, when the energy density was high enough for the production processes. Thereafter they remained in thermal equilibrium until temperatures dropped below $m_{\tilde{\chi}_1^0}$ and production was suppressed. At the same time increasing distances between the particles reduced annihilation rates until today's "freeze-out" relic density $\Omega_{\tilde{\chi}_1^0}^0$ at a temperature of ~ 2.7 K was reached. Precise measurements of this relic density have been performed by the WMAP collaboration [50].⁶ The results can be translated into constraints of supersymmetric parameter space by the following mechanism:

The relic density is proportional to the inverse expectation value of the thermally averaged inclusive annihilation and co-annihilation cross section of the LSP pair times its relative velocity $\Omega_{\tilde{\chi}_1^0}^0 \propto 1/\langle \sigma v \rangle$ (e.g. [66]). The cross-section σ depends highly on the particular supersymmetric model and its associated masses and couplings. To find the allowed regions of parameter space for a model with universal parameters at the GUT scale like MSUGRA/CMSSM, one first needs to evaluate the renormalization group equations to obtain the MSSM parameters in the weak regime. Then a SUSY-spectrum generator can be used to determine higher-order contributions to masses and couplings. These are then fed into calculations of the annihilation inclusive cross section σ to yield the corresponding neutralino density. The bounds resulting from the latest WMAP measurements correspond to the green shaded areas in Fig. 2.10.

Constraints from $b \rightarrow s\gamma$. Another restriction on the parameter space results from measurements of the flavour-changing process $b \rightarrow s\gamma$. In the SM this decay involves loops containing W-bosons and up-type quarks as shown in Fig. 2.11 (top left). In the MSSM additional diagrams with SUSY contributions are possible as shown in Fig. 2.11 (top right and bottom). Measurements of the corresponding inclusive branching ratio, i.e. $b \rightarrow X_s \gamma$ (see [67]) put constraints on the kinematically allowed regions of parameter space, when compared with precise SM calculations.

⁶ Measurements are also expected from the Planck collaboration (e.g. [65]).

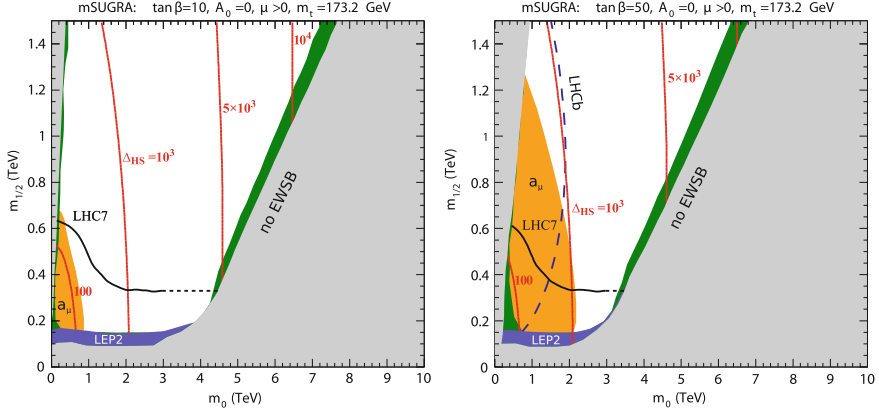


Fig. 2.10 Overview of experimental and theoretical constraints on MSUGRA/CMSSM parameter space in the plane of universal scalar and gaugino masses, for scenarios with $\mu > 0$, $A_0 = 0$, and $\tan \beta = 10$ (left) as well as $\tan \beta = 50$ (right). The green shaded area corresponds to WMAP cosmological constraints [50], the blue shaded area to limits from LEP2 chargino searches [59], the blue dashed line to LHCb $B_s \rightarrow \mu^+ \mu^-$ constraints [60], the orange shaded area to the region favoured by a_μ measurements [61], and the black solid line to the bounds from direct searches at the LHC [62, 63]. The red lines indicate the amount of fine-tuning according to the definition in Ref. [64]. Taken from Ref. [64]

Constraints from $B_s \rightarrow \mu^+ \mu^-$. Within the Standard Model the flavour-violating process $B_s \rightarrow \mu^+ \mu^-$ can occur at one-loop level with a highly suppressed branching ratio of the order of 10^{-9} . In MSUGRA/CMSSM scenarios this branching ratio is enhanced at large values of $\tan \beta$ due to additional flavour violating processes at one-loop level emerging from the supersymmetric Higgs sector. However, the upper limit on the $B_s \rightarrow \mu^+ \mu^-$ branching ratio is constrained by a recent measurement of the LHCb collaboration [60] to approximately less than two times the Standard Model expectation, which leaves little room for SUSY contributions. The region where the branching ratio calculated in MSUGRA/CMSSM falls below the experimentally allowed range is indicated with a dashed blue line in Fig. 2.10 (right).

Muon anomalous magnetic moment. The magnetic moment of the muon

$$\mu = \frac{eg}{2mc} \mathbf{S}, \quad (2.131)$$

with spin operator \mathbf{S} , contains a gyromagnetic factor g , which is expected to have a value of two, plus small higher-order corrections. Deviations of 3σ with respect to state-of-the-art theoretical calculations of the quantity $a_\mu = (g-2)/2$ [68] have been observed by the Muon $g-2$ Collaboration [61]. This discrepancy can be interpreted as a SUSY contribution, mainly through additional neutralino-smuon and chargino-sneutrino loops. The measurement can be accommodated in the MSSM preferentially for positive values of the Higgsino mass parameter μ and higher values of $\tan \beta$. The

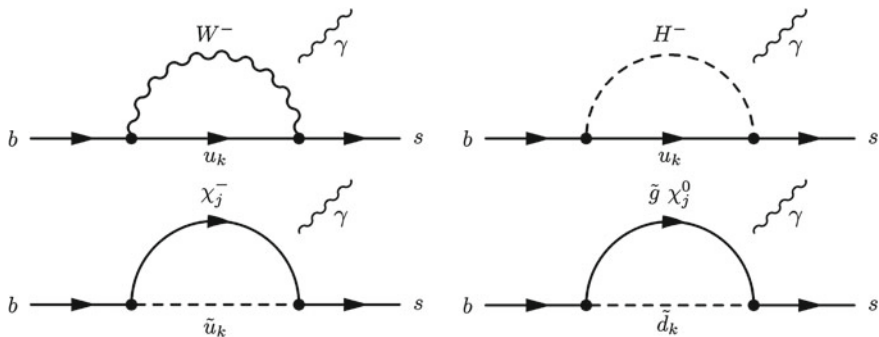


Fig. 2.11 Possible $b \rightarrow s\gamma$ processes in the Standard Model (*top left*) and supersymmetric models (*top right and bottom*). The photon line may be attached in all possible ways. Taken from Ref. [51]

region favoured by this measurement corresponds to the shaded area labeled a_μ in Fig. 2.10.

Collider constraints. Direct searches at high energy physics collider experiments provide the most stringent bounds on the masses of supersymmetric particles. The current exclusion limits on gluino and squark production within MSUGRA/CMSSM based on LHC data taken in 2011 at 7 TeV center-of-mass energy are denoted by the solid black line labelled LHC7 in Fig. 2.10. The original results by the ATLAS and CMS collaborations are shown in Fig. 2.12.

In addition the LHC experiments have published numerous analyses with interpretations outside the MSUGRA/CMSSM framework. One example is the SUSY search presented in this thesis, which is interpreted in terms of so-called simplified models, a concept introduced in Chap. 6.

Apart from the limits on sparticle masses from direct searches the most far-reaching implications on supersymmetric theories to date come from the recent observation of a new Higgs-boson like particle at the LHC. While in the Standard Model the mass of the Higgs boson is a free parameter of the theory as described in Sect. 2.1.3, in the MSSM it is bound to values below the Z-boson mass in addition to radiative corrections, which lead to a total upper limit of $M_H \lesssim 135$ GeV [70–72]. The fact that the newly found particle lies within that rather narrow mass range may be interpreted as a hint in favour of weak scale Supersymmetry. However, the measured mass range of this particle around 126 GeV requires a considerable amount of radiative corrections which are expected to originate mostly from the supersymmetric top quark. These corrections, in turn, require a careful tuning of the mass parameters of the MSSM to cancel contributions to the Higgs potential and to arrive at a vacuum expectation value within the electroweak regime. This problem is also known as the “little hierarchy problem”. The amount of fine-tuning⁷ that is necessary for a given MSUGRA/CMSSM scenario is indicated by the red lines in Fig. 2.10.

⁷ The high-scale fine-tuning definition described in Ref. [64] is used.

A number of theoretical proposals exist to alleviate the effects of a heavy Higgs boson on naturalness considerations. These are subject to ongoing studies and discussions inside the theoretical particle physics community. Two examples are given below.

Amongst one of the most well-known approaches is the so-called Next-to-minimal Supersymmetric Standard Model (NMSSM), which introduces a new gauge-singlet chiral supermultiplet S . In the simplest version of the NMSSM, the term $\mu H_u H_d$ in the MSSM superpotential (Eq. 2.126) is replaced by $\lambda H_1 H_2 S + \frac{\kappa}{3} S^3$ [73], where λ is the coupling of S to H_u and H_d , and κ is the self-coupling of the singlet field. In the NMSSM the μ -term is then generated dynamically via electroweak symmetry breaking where S takes the vacuum expectation value v_S resulting into an effective μ -term, $\mu_{\text{eff}} = \lambda v_S$. This mechanism allows to circumvent the so-called μ -problem of the MSSM [74], which emerges from the explicit appearance of the μ -term in the MSSM superpotential. While the value of μ is expected to be at the electroweak scale to allow for spontaneous symmetry breaking in the supersymmetric Higgs sector, there is no natural explanation why this value should be so small compared to e.g. the Planck scale, and why it should be of the same order of magnitude as the soft Supersymmetry breaking mass terms given that their physical origins are essentially unrelated. In the NMSSM the additional coupling λ of S to H_u and H_d can lead to larger masses of the lightest neutral CP-even Higgs boson than in the MSSM and thus makes the NMSSM a more natural candidate for Supersymmetry in the light of the recent experimental results.

Another path that is being followed by the theoretical community is that of “Natural” Supersymmetry (NSUSY). An overview can be found in [75] and references therein. NSUSY is a collective term for supersymmetric models, in which the squarks of the third generation are expected to be of the order of 1 TeV whereas the remaining supersymmetric quarks and leptons as well as the electroweak gauginos are essentially decoupled at energies of ~ 10 –50 TeV. The gluino mass can be of the order of several TeV. These scenarios still allow for Supersymmetry with small fine-tuning and are to date consistent with the experimentally excluded limits on squark and gluino masses at the LHC.

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