

Preface

This volume contains contributions on the history, mathematical analysis, and numerical solution of constrained optimal control and optimization problems where a partial differential equation (PDE) or a system of PDEs appears as an essential part of the constraints. The appropriate treatment of such problems requires a fundamental understanding of the subtle interplay between optimization in function spaces and numerical discretization techniques and relies on advanced methodologies from the theory of PDEs and numerical analysis as well as scientific computing. The contributions reflect part of the work that has been done within the European Science Foundation (ESF) Networking Programme optimization with PDEs (OPTPDE). The OPTPDE programme has been launched in October 2008 for a 5-year period and has been supported by seventeen national science foundations and research institutions from 12 European countries:

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Austrian Science Research Fund, Austria
Fonds National de la Recherche Scientifique (FNRS)
National Fund for Scientific Research, Belgium
Akademie ved Ceske republiky (GACR)
Academy of Sciences of the Czech Republic, Czech Republic
Suomen Akatemia/Finlands Akademi
Academy of Finland, Finland
Deutsche Forschungsgemeinschaft (DFG)
German Research Foundation, Germany
Istituto Nazionale di Alta Matematica (INdAM)
National Institute for Advanced Mathematics, Italy,
Scuola Internazionale dei Studi Avanzati (SISSA)
Universita di Roma Tor Vergata (Dip. di Matematica),
Dip di Matematica F. Brioschi di Politecnico di Milano,
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A primary goal was to bring together experts from the optimization/optimal control and the numerical PDE communities and to use the emerging synergies to make significant progress in the mathematical treatment of challenging problems in PDE constrained optimization. To this end, a series of conferences on general PDE constrained optimization and workshops on specific topics have been organized that have both deepened existing cooperations and triggered new scientific relations between the participants. The contributions in this volume are original, peer-reviewed research articles by participants of the ESF OPTPDE Programme and their coworkers:

The contribution by Petr Beremlijski, Jaroslav Haslinger, Jiří L. Outrata, and Róbert Pathó deals with the numerical solution of shape optimization in frictional contact mechanics. The essential idea is to transform the discretized problem to a nonsmooth minimization problem which is then solved by a bundle trust method using the generalized differential calculus of Mordukhovich. Phase field methods for the recovery of a binary function from blurred and noisy data are a significant task in image processing and optimal control of PDEs. The article by Charles Brett, Charles M. Elliott, and Andreas S. Dedner presents an approach to the numerical solution of this inverse problem based on a combination of the Mumford-Shah model and a phase field approximation to the perimeter regularization.

Multigrid methods and adaptive sequential quadratic programming are two novel techniques for the numerical solution of shape optimization and optimal control problems associated with evolutionary partial differential equations that are applied in the contribution by Maurizio Falcone and Marco Verani. The approximation relies on the coupling between a proper orthogonal decomposition and the classical dynamic programming approach.

Adaptive finite elements for PDE constrained optimization represent a subject that emerged from the cooperation between the optimization and numerical analysis communities. The article by Alexandra Gaevskaya, Michael Hintermüller, Ronald H.W. Hoppe, and Caroline Löbhard is concerned with such techniques for the numerical solution of optimally controlled elliptic variational inequalities. Based on the equivalence with Mathematical Problems with Complementarity Constraints

(MPCCs), the convergence of discrete stationary points to a stationary point in function space is shown. Moreover, a residual-type a posteriori error estimator is developed which up to data oscillations provides both an upper and a lower bound for the global discretization error.

The history of constrained optimization is the subject of the contribution by Martin Gander, Felix Kwok, and Gerhard Wanner. Referring to a lot of original sources, the authors forge a bridge from the origins in the eighteenth century (Varignon, Johann Bernoulli, Lagrange) to Pontryagin's celebrated maximum principle and the modern theory of PDE constrained optimization.

Topology optimization using the concept of topological derivatives is a powerful tool for the optimal design in solid mechanics. The article by S.M. Giusti, Jan Sokolowski, and Jan Stebel deals with the application of this concept to frictionless contact problems by minimizing the structural compliance for a given amount of material. Several numerical examples demonstrate the robustness of the suggested approach.

The numerical solution of three-dimensional contact problems with orthotropic friction by using the Coulomb friction cone without any approximation is the main goal of the contribution by Jaroslav Haslinger, Radek Kučera, and Tomáš Kozubek. The suggested algorithm relies on an appropriate discretization of the dual variational formulation involving the Lagrange multipliers on the contact boundary. Its performance is illustrated by the documentation of numerical results for several model examples.

The article by Günter Leugering, Jan Sokolowski, and Antoni Żochowski is concerned with shape-topological differentiability of energy-type objective functionals for unilateral problems in domains with cracks. Using tools from nonsmooth analysis, the authors study the dependence of the Griffith shape functional on domain perturbations far from the cracks and obtain its directional shape and topological derivatives with respect to boundary variations of an inclusion.

The boundary stabilization for finite difference semidiscretizations of the one-dimensional wave equation with variable density and diffusion coefficients is the central theme of the article by Aurora Marica and Enrique Zuazua. Adding a suitable artificial viscosity to the finite difference approximation, by an application of the classical multiplier technique at the discrete level it is shown that the discrete decay rate is uniform as the mesh size tends to zero.

The theory of a posteriori error estimates of functional type is known to provide guaranteed upper and lower bounds for the global discretization error. In the contribution by Pekka Neittaanmäki and Sergey Repin, the theory is applied to finite element discretizations of distributed optimal control problems for second order elliptic boundary value problems. In this way, guaranteed bounds for the cost functional as well as computable error estimates for the state and the control functions are obtained.

A shape sensitivity analysis of the work functional for the compressible Navier-Stokes equations in a bounded domain with an obstacle is the subject of the contribution by Pavel I. Plotnikov and Jan Sokolowski. The main tool in the analysis is the Kuratowski-Mosco convergence of sequences of compact sets.

Moreover, establishing the continuity of typical cost functionals with respect to the Kuratowski-Mosco convergence, it is shown that the problem of minimizing the work of hydrodynamic forces in a class of obstacles with fixed volume admits a solution.

The contribution by Jean-Pierre Puel provides new results on local exact controllability and null controllability for the incompressible Navier-Stokes equations. The presented approach is based on new global Carleman estimates for the Stokes problem associated with the linearized equations and some fine interpolation results.

The editor would like to express his sincere thanks to those who have made it possible to produce this book. Particular thanks go to the European Science Foundation for including OPTPDE as one of the ESF PESC Networking Programmes and to the European national science foundations and research institutions listed above for their financial support. I am also indebted to the editors of the Lecture Notes in Computational Science and Engineering for considering this book as a volume within this series and to Claus Ascheron of Springer-Verlag for his continuous advice and support during the preparation and production of this volume.

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