

# Preface

The author is pleased to present *Finite Element Method in Mechanics*. This book will serve a wide range of readers, in particular, graduate students, Ph.D. candidates, professors, scientists, researchers in various industrial and government institutes, and engineers. Thus, the book should be considered not only as a graduate textbook, but also as a reference book to those working or interested in areas of finite element modelling of solid mechanics, heat conduction, and fluid mechanics.

The book is self-contained, so that the reader should not need to consult other sources while studying the topic. The necessary mathematical concepts and numerical methods are presented in the book and the reader may easily follow the subjects based on these basic tools. It is expected, however, that the reader should have some basic knowledge in the classical mechanics, theory of elasticity, and fluid mechanics.

The book contains 17 chapters, where the chapters cover the finite element modeling of all major areas of mechanics.

Chapter 1 presents the history of development of finite element method, where the key references are given and the progress of this science is discussed.

Chapter 2 is devoted to the basic mathematical concepts of finite element method. The method of calculus of variation is discussed and the distinction of boundary value problems versus the variational formulation is presented and several examples are given to make the reader familiar with the concepts of function and functional. The material then follows into the discussion of traditional Ritz and Galerkin methods. Numerical examples show the powerful nature of these numerical techniques.

Introduction to the finite element method is given in Chap. 3 with the discussion of elastic membrane. The subject of elastic membrane is selected because the height of membrane is approximated with finite element method. This gives a physical feeling for the finite element approximation to the reader.

Since a linear triangular element is employed to model the elastic membrane in Chap. 3, Chap. 4 discuss the one-, two-, and three-dimensional elements with linear and higher order approximation. The discussion gives a feeling to the reader that there is no limitation in the type of elements and the order of approximation, geometrically and mathematically. The subparametric, isoparametric, and superparametric elements are discussed and the natural coordinates are presented.

The finite element approximation of the field problems, harmonic and biharmonic, are given in Chap. 5.

Chapter 6 deals with the finite element approximation of the heat conduction equations. One-, two-, and three-dimensional conduction in solids are discussed and the transient heat conduction problems are presented. Both variational and Galerkin techniques are presented.

Up to this point, the reader learns how to obtain the element stiffness, capacitance, and force matrices for one element. He questions how a solution domain with many number of elements should be modeled and solved to obtain the required domain unknowns. A comprehensive treatment is given in Chap. 7 to give a proper tool to the reader to write his own computer program. Many numerical examples are solved to show the numerical scheme, and proper algorithms are given. The assembly of global matrices, bandwidth calculation, the method to apply the boundary conditions, and the Gauss elimination method are presented. The method of solution of the transient and dynamic finite element equations are then presented. The central difference method, the Houbolt Method, the Newmark Method, and the Wilson- $\theta$  method are presented. At this stage, the reader learns how to write his own computer problem. Now, he should learn how different problems of mechanics are formulated by the finite element approximation. These techniques are discussed in the following chapters.

Chapter 8 deals with the finite element approximation of beams. Static beam deflection equation, based on the Euler beam theory, is presented and the Galerkin and variational formulations are obtained. The axial, torsional, and lateral vibrations of beams are modeled. Finally, the vibrations of Timoshenko beam are presented.

Chapters 9 and 10 present the finite element formulations of elasticity problems based on Galerkin and variational formulations.

Torsion of prismatic bars and rods are given in Chap. 11 and quasistatic thermoelasticity theory is discussed in Chap. 12.

Chapter 13 is devoted to the finite element solution of viscous fluid mechanic problems. Derivation of the Navier-Stokes equations is presented and the finite element formulation of the two-dimensional fluid flow based on the velocity components and pressure are derived. In the following section, the vorticity transport model of the Navier-Stokes equations are obtained and the finite element formulations are derived. The method of solution of the resulting nonlinear finite element equation is presented.

Chapter 14 presents one-dimensional higher order elements. The local natural coordinate for the quadratic and cubic elements are derived and the Jacobian matrix is obtained. To describe the application, field problem for one-dimensional case is discussed and the element of the matrices are calculated. The chapter is completed with a discussion of layerwise theory for composite beams, where one-dimensional higher order element is used to discuss the problem.

The higher order element in two dimension is discussed in Chap. 15. The triangular element with quadratic and cubic shape functions are given in terms of the area element and the Jacobian matrix is calculated. The quadratic element is

employed to obtain the element matrices for a two-dimensional field problem. In the following, the quadrilateral element is discussed and its application to the field problem is presented.

Chapter 16 presents the linear coupled thermoelasticity problems, and their method of solution by finite element method. This chapter is unique in the literature of finite element analysis of solid elastic continuum. The most general form of the three-dimensional classical coupled thermoelasticity equations are considered and the finite element formulations are presented.

Computer programs for three different types of problems are given in Chap. 17. The first program is related to the elastic membrane problem, where Poisson's equation is solved. This program may be used for any other application of Poisson's equation, such as the steady-state heat conduction, torsion of prismatic bars, inviscid incompressible fluid flow problems, and the pressure in porous media. The second computer program handles two-dimensional elasticity problems, and the third computer program presents three-dimensional transient heat conduction problems. The programs are written in C++ environment.

At the end of all the chapters, except Chap. 1, there are a number of problems for students to solve. Also, at the end of each chapter, there is a list of relevant references.

The book was prepared over some 40 years of teaching the graduate finite element course. During this long period of time, the results of classwork assignments and student research are carefully gathered and put into this volume of work. The author takes this opportunity to thank all his students who made possible to provide this piece of work.

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