

# Preface

Nonsmooth optimization refers to the general problem of minimizing (or maximizing) functions that are typically not differentiable at their minimizers (maximizers). These kinds of functions can be found in many applied fields, for example in image denoising, optimal control, neural network training, data mining, economics, and computational chemistry and physics. Since classical theory of optimization presumes certain differentiability and strong regularity assumptions for the functions to be optimized, it cannot be directly utilized. The aim of this book is to provide an easy-to-read introduction to the theory of nonsmooth optimization and also to present the current state of numerical nonsmooth optimization. In addition, the most common cases where nonsmoothness is involved in practical computations are introduced. In preparing this book, all efforts have been made to ensure that it is self-contained.

The book is organized into three parts: Part I deals with nonsmooth optimization theory. We first provide an easy-to-read introduction to convex and non-convex analysis with many numerical examples and illustrative figures. Then we discuss nonsmooth optimality conditions from both analytical and geometrical viewpoints. We also generalize the concept of convexity for nonsmooth functions. At the end of the part, we give brief surveys of different generalizations of subdifferentials and approximations to subdifferentials.

In Part II, we consider nonsmooth optimization problems. First, we introduce some real-life nonsmooth optimization problems, for instance, the molecular distance geometry problem, protein structural alignment, data mining, hemivariational inequalities, the power unit-commitment problem, image restoration, and the nonlinear income tax problem. Then we discuss some formulations which lead to nonsmooth optimization problems even though the original problem is smooth (continuously differentiable). Examples here include exact penalty formulations. We also represent the maximum eigenvalue problem, which is an important component of many engineering design problems and graph theoretical applications. We refer to these problems as semi-academic problems. Finally, a comprehensive list of test problems—that is, academic problems—used in nonsmooth optimization is given.

Part III is a guide to nonsmooth optimization software. First, we give short descriptions and the pseudo-codes of the most commonly used methods for nonsmooth optimization. These include different subgradient methods, cutting plane methods, bundle methods, and the gradient sampling method, as well as some hybrid methods and discrete gradient methods. In addition, we introduce some common ways of dealing with constrained nonsmooth optimization problems. We also compare implementations of different nonsmooth optimization methods for solving unconstrained problems. At the end of the part, we provide a table enabling the quick selection of suitable software for different types of nonsmooth optimization problems.

The book is ideal for anyone teaching or attending courses in nonsmooth optimization. As a comprehensible introduction to the field, it is also well-suited for self-access learning for practitioners who know the basics of optimization. Furthermore, it can serve as a reference text for anyone—including experts—dealing with nonsmooth optimization.

*Acknowledgments:* First of all, we would like to thank Prof. Herskovits for giving the reason to write a book on nonsmooth analysis and optimization: He once asked why the subject concerned is elusive in all the books and articles dealing with it, and pointed out the lack of an extensive elementary book.

In addition, we would like to acknowledge Prof. Kuntsevich and Kappel for providing Shor's  $r$ -algorithm on their web site as well as Prof. Lukšan and Vlček for providing the bundle-Newton algorithm.

We are also grateful to the following colleagues and students, all of whom have influenced the content of the book: Annabella Astorino, Ville-Pekka Eronen, Antonio Fuduli, Manlio Gaudioso, Kaisa Joki, Sami Kankaanpää, Refail Kasimbeyli, Yury Nikulin, Gurkan Ozturk, Rami Rakkolainen, Julien Ugon, Dean Webb and Outi Wilppu.

The work was financially supported by the University of Turku (Finland), Magnus Ehrnrooth Foundation, Turku University Foundation, Federation University Australia, and Australian Research Council.

Ballarat, April 2014  
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Introduction to Nonsmooth Optimization  
Theory, Practice and Software

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2014, XVIII, 372 p. 72 illus., Hardcover

ISBN: 978-3-319-08113-7