Chapter 2
Technical and Quantitative Analysis of Tubes

We assumed in the first chapter that a forecasting mechanism of the lower bounds of the risky returns was given for computing the minimum guaranteed investment and the value of the portfolio for hedging a floor. This chapter is devoted to the design of such mechanisms and the study related issues.

The underlying approach is to start from what is known in the past and provided at each date by the brokerage firms: the price tube, bounded by the High and Low prices, in which the Last Price belongs. We regard the distance between High and Low prices, called the *tychastic gauge* of the price tube, as another measure of the polysemous concept of volatility. The tychastic gauge vanishes for riskless assets. The larger the tychastic gauge, the more "tychastically volatile" the risky asset (see Sect. 3.3, *The Legacy of Ingenhousz*, for explanations justifying this choice). By using tools of set-valued analysis, we can "differentiate tubes" and compute the tube of velocities in which range the derivatives of the Last prices, the tube of returns as well as other ones.

Detecting and/or forecasting the trend reversals of evolutions, when markets go from bear to bull and back, or minimum guaranteed investments, or market alarms and tychastic gauges, etc., is mandatory. Section 2.4, brings original answers to this question by applying the general study of *reversal dates when trend reverses and congruent periods during which time series increase or decrease on one hand, and a measure of the violence or intensity of the time reversal by a nonlinear indicator, the jerkiness indicator*.

However, the "volatility issue" should not be confused with the question of prediction: the tychastic gauge of a riskless asset vanishes, but the question of forecasting its future remains open. This issue will be dealt with in Sect. 2.2, in which we define the concept of extrapolator, examples of which are obtained by combining history dependent differential equations and the regularization of Dirac combs of discrete time series for extrapolating them. The VPPI robot-insurer involves the VIMADES Extrapolator for extrapolating price tubes and thus, forecast lower bounds of risky returns. Next, we study the *sensitivity to tychastic gauges* of the minimum guaranteed investment and the value of the portfolio in Sect. 2.3.

The last issue studied in this chapter is the detection of generators of patterns recognizing whether a dynamical system (generator) provides evolutions remaining in the price tube around the last price. This is neither the case of exponential evolutions

J.-P. Aubin et al., *Tychastic Measure of Viability Risk*,
DOI: 10.1007/978-3-319-08129-8_2,

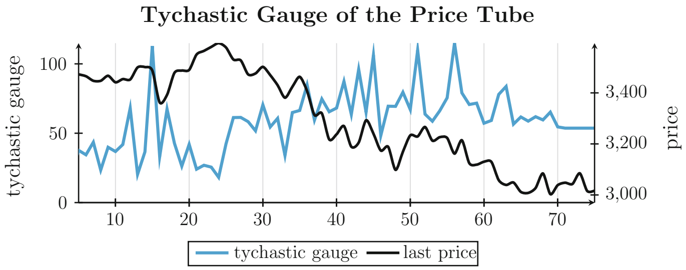
nor second-order polynomials ones, as the adequate algorithms show. It disclaims the possibility for price candidates to be generated by geometric models[1] (deterministic as well as stochastic). However, detection by the VIMADES Extrapolator performs better (see Sect. 2.6).

2.1 Tychastic Gauge and Derivatives of the Price Tubes

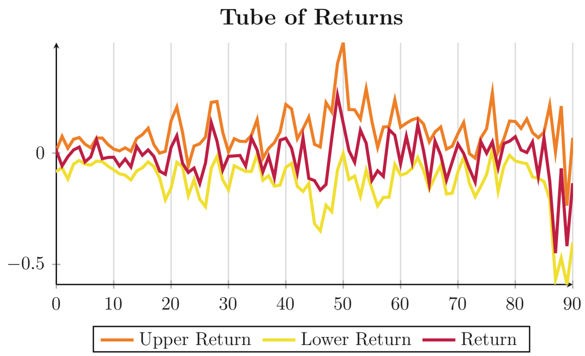
Recall that brokerage firms provide at each date t *lower bounds* $S^\flat(t)$ and *upper bounds* $S^\sharp(t)$ defining the *price interval* $\Sigma(t) := [S^\flat(t), S^\sharp(t)]$ of the risky asset inside which the price $S(t)$ evolves.

Definition 2.1.1 (*Price Tubes and their Tychastic Gauge*) The length $S^\sharp(t) - S^\flat(t) \geq 0$ of the price interval is called its *tychastic gauge*. The *price tube* is the set-valued map $t \rightsquigarrow \Sigma(t)$ inside which remain the evolutions of prices $t \mapsto S(t) \in \Sigma(t)$, called *selections* of the price tube.

Intuitively, *the larger the tychastic gauge of the price tube, the more uncertain* the evolution of prices. *Gauging price tubes and forecasting then are two problems, linked but different.* The *tychastic gauge* of the price tube oscillates during this crisis period:

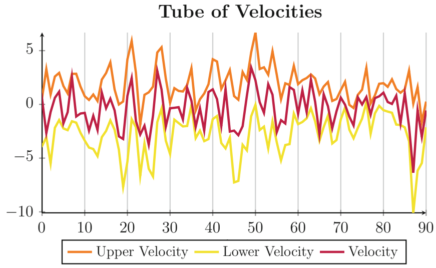


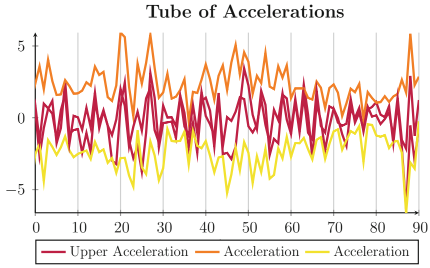
We can compute the return price tube surrounding the returns of the Last Prices ranging over the price tube (see Sect. 4.2 for the definition of derivatives of tubes):



[1] See [24, Sect. 1.3, p. 23], for further comments on this crucial issue.

Actually, the *return price tube* is deduced from the velocity price tube, which, together with the acceleration tube, are displayed in the following figure:





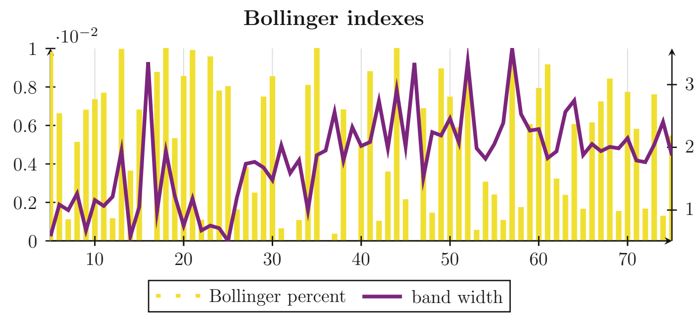
The tychastic gauge of the price tube is compared with the acceleration of the Last Price in the figure below:

"Technical analysis", the set of statistical methods used by chartists, studies evolutions with respect to a tube surrounding them. Among them, the relation of the last price located in the price tube have been studied by chartists. For instance, *John Bollinger* introduced in the 1980s *Bollinger bands*, which are example of tubes, and relate the tychastic gauge of the price tube to the last price:

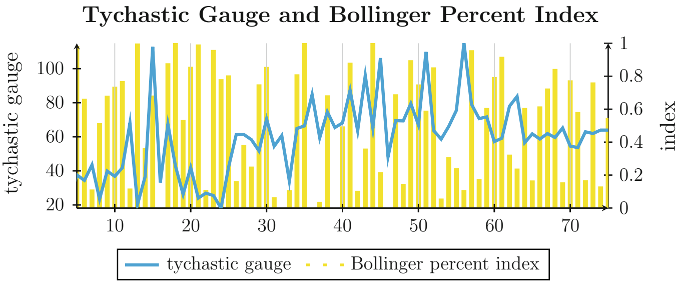
Definition 2.1.2 (*Bollinger Indexes*) The *Bollinger percent index* of the last price $S(t)$ in the price tube $\Sigma(t) := [S^{\sharp}(t), S^{\flat}(t)]$ is used to measure the uncertainty of a selection $S(t) \in \Sigma(t)$ which is measured by the ratio %b (pronounced "percent b") $\dfrac{S^{\sharp}(t) - S(t)}{S^{\sharp}(t) - S^{\flat}(t)}$, which can be regarded as a *relative tychastic gauge*. The *Bollinger band width* $\dfrac{S^{\sharp}(t) - S^{\flat}(t)}{S(t)}$ is the tychastic gauge relative to the last price.

We do not need this kind of information for computing the insurance tube, which requires only to forecast the whole tube, not the behavior of one of its selections. The Last Prices are only used for computing the value of performance.

We compute below the Bollinger indexes for the price tube and the display of the tychastic gauge of the price tube and its Bollinger percent index:



The following graphic displays the tychastic gauge of the price tube and its Bollinger percent index:



2.2 Forecasting the Price Tube

In the description of the uncertainty on prices described by price tubes, we thus have to distinguish two facets: the *predictability of the evolution of the price tube* on one hand, the "*thickness*" *of the price tube*, on the other hand, which could correspond to the concept of volatility in some sense. This thickness summarizes the tychastic uncertainty on the "risky" asset prices, and provides a measure of this uncertainty. We call it its *tychastic gauge* (see Definition 2.1.1) and we compute it for the forecast price tube.

For operating the VPPI robot-insurer by using price tubes for describing the uncertainty, we need to forecast the lower bounds of the risky returns.

We have seen how to differentiate price tubes and, in particular, how to provide the tube of returns, and thus, its lower bound. The task which remains to underdo is to forecast the price tube for forecasting the lower bound of its tube of returns. For that purpose, we need to define the concept of extrapolation and to chose one extrapolator to integrate in the VPPI robot-insurer.

Definition 2.2.1 (*Extrapolator*) Let us fix a duration $\delta \geq 0$, an integer $p \geq 0$ and a constant $c > 0$. Let us consider any (chronological) time $t \in \mathbb{R}$, a temporal window $[t-\delta, t]$ of aperture δ and an evolution $S(\cdot) : t \in [t-\delta, t] \mapsto S(t) \in \mathbb{R}$. We denote by $\mathcal{E}_c^p(t-\delta, t)$ the subset of *future evolutions* $t \geq 0 \mapsto A(t) \in \mathbb{R}$ such that

$$\sup_{\tau \in [t-\delta, t]} |S(\tau) - A(t-\delta+\tau)| \leq c\delta^p \tag{2.1}$$

An extrapolator of order p and duration δ is a map $\mathbb{E}xtr$ from $\mathcal{C}(t-\delta, 0; \mathbb{R}) \mapsto \mathcal{C}(0, t+\delta; \mathbb{R})$ such that

$$\forall\ S(\cdot) \in \mathcal{C}(t-\delta; \mathbb{R}), \quad \mathbb{E}xtr(S(\cdot)) \in \mathcal{E}_c^p(t-\delta, t) \tag{2.2}$$

There are many classical and less classical examples of extrapolators which fit this definition. The most classical are *Peano* and *Riemann* "high order derivatives" (see *Applicazioni geometriche del calcolo infinitesimale*, [145] by *Giuseppe Peano* and [8, 9]). We shall review briefly how we can combine:

1. differential equations or inclusions providing extrapolators of continuous evolutions and, next, how we can pass from discrete time series to evolutions

2. imbedding procedures mapping time series to "*Dirac combs*" and regulating procedures mapping them into functions to be extrapolated (we return to extrapolated time series by taking the values of the extrapolation at future discrete times).

Each of these steps are subject to "model error" and there is no scientific criterion enabling us to decide which one is the best. At most, we can compute or estimates the constant c and the order p to check whether it is an extrapolator in the sense of Definition 2.2.1.

1. ***Prediction: Historic Differential Inclusions***
   The knowledge of the past may allow us to *extrapolate it* by adequate history dependent (or path dependent, memory dependent, functional) differential inclusions associating with the history of the evolution up to each time t a set of velocities. "Histories" are evolutions $\varphi \in \mathcal{C}(-\infty, 0; X)$ defined for negative times. The history space $\mathcal{C}(-\infty, 0; X)$ is a "storage" space in which we place at each $t \geq 0$ any evolution $x(\cdot)$ defined on $]-\infty, T]$ up to time T thanks to the translation operator $\kappa(-T)$:

   Definition 2.2.2 (*Translations*) For any $T \in \mathbb{R}$, the *translation* $\kappa(T)x(\cdot) : \mathcal{C}(-\infty, +\infty; X) \mapsto \mathcal{C}(-\infty, +\infty; X)$ of an evolution $x(\cdot)$ is defined by

   $$(\kappa(T)x(\cdot))(t) \;:=\; x(t-T) \tag{2.3}$$

   It is a translation to the right if T is positive and to the left if T is negative. Regarding $T \geq 0$ as an *evolving present time*, we can regard the translation $\kappa(-T) : \mathcal{C}(-\infty, +\infty; X) \mapsto \mathcal{C}(-\infty, 0; X)$ as a *recording operator* and translation $\kappa(+T) : \mathcal{C}(-\infty, +\infty; X) \mapsto \mathcal{C}(0, +\infty; X)$ as a *recalling operator* in the sense that

   a. $\kappa(-T)(x(\cdot))_- \in \mathcal{C}(-\infty, 0; X)$ can be regarded as the history of the evolution up to time T of the evolution $x(\cdot)$;
   b. $\kappa(+T)(x(\cdot))_+ \in \mathcal{C}(0, \infty; X)$ can be regarded as the future of the evolution from time T of the evolution $x(\cdot)$.

   This operation is needed to define concatenation of evolutions:

   Definition 2.2.3 (*Concatenations*) Let $T \in \mathbb{R}$. The *concatenation* $(x(\cdot) \diamond_T y(\cdot))(\cdot)$ at T of an evolution $x(\cdot) \in \mathcal{C}(-\infty, +\infty; X)$ and of an evolution $y(\cdot) \in \mathcal{C}(0, +\infty; X)$ such that $y(0) = x(T)$ is defined by

   $$\begin{cases} (x(\cdot)\Diamond_T y(\cdot))(t) := x(t) & \text{if} \quad t \leq T \\ (x(\cdot)\Diamond_T y(\cdot))(t) := y(t-T) & \text{if} \quad t \geq T \end{cases}$$

   Observe that these two operations are independent of the algebraic structure of the state space and are sufficient to define general *evolutionary systems* (see [28, Definition 2.8.2, p. 70]. Hence, instead of studying evolutions $t \mapsto x(t) \in X$, we associate evolutions $t \mapsto (\kappa(-t)x(\cdot)) \in \mathcal{C}(-\infty, 0; X)$ in the history space. Viability Theorems and their applications for history dependent dynamics and environment require a specific *Clio analysis*[2] of history dependent maps introduced in [46] (for studying portfolios where stochastic differential equations are replaced by differential equations with memory). For instance, let a history dependent functional $\mathbf{v} : \varphi \in \mathcal{C}(-\infty, 0; \mathbb{R}) \mapsto \mathbf{v}(\varphi) \in \mathbb{R}$. The addition operator $\varphi \mapsto \varphi + h\psi$ used in differential calculus in vector spaces is replaced by the *translation and concatenation operators* $\Diamond_h$ associating with each history $\varphi \in \mathcal{C}(-\infty, 0; X)$ the function $\varphi \Diamond_h \psi \in \mathcal{C}(-\infty, 0; \mathbf{R}^n)$ defined by

$$(\varphi \Diamond_h \psi)(\tau) \ := \ \begin{cases} \varphi(\tau + h) & \text{if} \quad \tau \in]-\infty, -h] \\ \varphi(0) + \psi(\tau + h) & \text{if} \quad \tau \in [-h, 0] \end{cases}$$

Definition 2.2.4 (*Clio Derivatives*) The *Clio derivative* $D\mathbf{v}(\varphi)(\psi)$ of a history dependent functional $\mathbf{v} : \varphi \in \mathcal{C}(-\infty, 0; X) \mapsto \mathbf{v}(\varphi) \in X$ is the limit

$$D\mathbf{v}(\varphi)(\psi) \ := \ \liminf_{h \to 0+} \nabla_h \mathbf{v}(\varphi)(\psi) \ \in \ X \tag{2.4}$$

of "*Clio differential quotients*"

$$\nabla_h \mathbf{v}(\varphi)(\psi) \ := \ \frac{\mathbf{v}((\varphi \Diamond_h \psi)) - \mathbf{v}(\varphi)}{h} \ \in \ X$$

Histories are the inputs of differential inclusions with memory

$$x'(t) \ \in \ F(\kappa(-t)x(\cdot)) \tag{2.5}$$

where $F : \mathcal{C}(-\infty, 0; X) \rightsquigarrow \mathbb{R}^n$ is a set-valued map defining the dynamics of history dependent differential inclusion.

One can also use history dependent differential equations or inclusions depending on functionals on past evolutions,[3] such as their derivatives up to a given order m:

$$x'(t) \ \in \ F\left(\left(D^p(\kappa(-t)x(\cdot))\right)_{|p| \leq m}\right) \tag{2.6}$$

in order to take into account not only the history of an evolution, but its "trends". For instance, these history dependent differential inclusions have been be used for forecasting the asset prices and manage portfolios.

The history dependent environments are subsets $\mathcal{K} \subset \mathcal{C}(-\infty, 0; X)$ of histories. Actually, the first "general" viability theorem was proved by Georges Haddad in

[2] The two sisters Mnemosyne and Lesmosyne, daughter of Heaven (Ouranos) and Earth (Gaia), are respectively the goddesses of memory and forgetting. Clio, muse of history, and the eight other muses, were born of the same breath out of the love between Zeus and Mnenosyne.

[3] See *Nonoscillation Theory of Functional Differential Equations with Applications*, [4] by Agarwal, Berezansky, Braverman and Domoshnitsky (of the *Nikolai Viktorovich Azbelev's* school) for a recent account of this field.

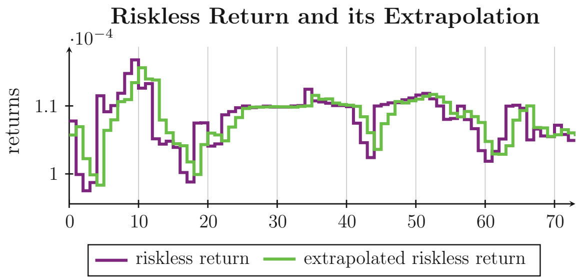
the framework of history dependent differential inclusions at the end of the 1970s (see [115, 116, 117] summarized in [15]). Since their study, motivated by the evolutionary systems in life sciences, including economics and finance, is much more involved than the one of differential inclusions, most of the viability studies rested on the case of differential inclusions.

2. ***Dirac Combs of Discrete Time Series***
   Let us consider a discrete time series (chroniques) $(x_j)_{j \in \mathbb{Z}}$. Using Dirac measures δ_j at dates $j \in \mathbb{Z}$, we can imbed the discrete time series in the space of distributions by associating with it its "*Dirac comb*"

$$\mathbb{D}\left((x_j)_{j \in \mathbb{Z}}\right) := \sum_{j \in \mathbb{Z}} x_j \delta_j \tag{2.7}$$

   Dirac combs are only measures, but we can "regularize them" by taking their convolution product $(\lambda \star x)(t) = \int_{-\infty}^{+\infty} \lambda(\tau) x(t-\tau) d\tau$. It inherits the differentiability properties of λ, being as much differentiable than λ is (see *Applied Functional Analysis*, [14], for instance). These functions λ are assumed to be integrable with compact support $[0, p]$ and total mass equal to one, not necessarily positive (if λ is positive, we recover classical sliding average techniques). Therefore, combining a regularization procedure of the Dirac tube of a discrete time series, we obtain a smooth functions to which we can apply a given extrapolator. Hence, there are as many extrapolation methods as such functions λ. The VIMADES Extrapolator which is integrated in some versions of the VPPI robot-insurer (however, the user is free to choose her or his forecasting mechanism) belongs to this class for *non negative* functions λ with compact support $[0, p]$. It is based on techniques used in numerical analysis (see [10]): it takes into account the extrapolation of all derivatives up to order p. One can check that it is an extrapolator of order p and constant c applying to the class of time series the pth difference of which are smaller than the constant c (in the sense of Definition 2.2.1).
   For instance, taking $p = 4$, we obtain an extrapolator of order 4 which *captures the trends of the regularized time series: its values, its velocities, its accelerations and its jerks* (see [33, 34, 37]). In this case, the VIMADES Extrapolator needs to know the four preceding dates of the time series to extrapolate. We first test the performance of the VIMADES Extrapolator by using at each date the return of the riskless asset.

The riskless tube is given directly by the brokerage firms, and not derived from a price tube reduced to a simple curve. Even though it is regarded as deterministic in this sense, its future is not known, and needs also to be forecast (being a single-valued evolution, its tychastic gauge is equal to 0). The VIMADES Extrapolator is used to extrapolate the riskless return:



For forecasting the lower bounds of the risky returns, we need first to extrapolate and forecast the price tube. In the example below, the discrete time series and price tube are still those of the CAC 40 index used in Chap. 1, p. 17.

The VIMADES Extrapolator needs historical data during the four preceding dates, which are displayed below:

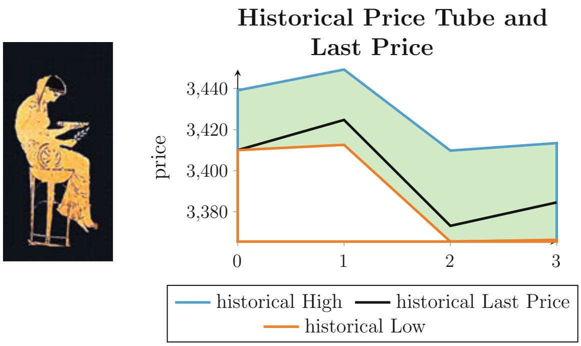
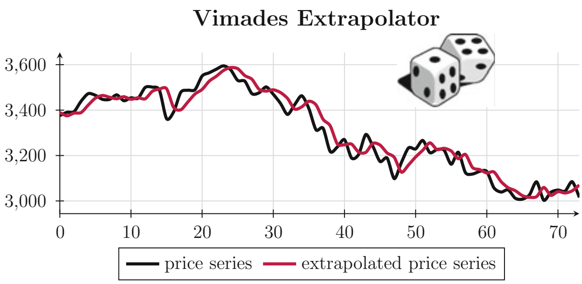


Fig. 2.1 Historical Price Tube. Pythia gives a look into the historical price tube for preparing her mantic process for extrapolating it, leaving to Tyche (see Fig. 3.2) the task of using this extrapolation for computing the hedging exit time function. Nowadays, Pythia would use without doubt the VIMADES Extrapolator!

Knowing them, the VIMADES Robot-Extrapolator[4] provides the extrapolation of the Last Price during the exercise period:



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[4] The software of the Robot-Extrapolator of VIMADES has been registered on May 21, 2010, at the INPI, the French Institut National de la Propriété Industrielle.

The VIMADES Extrapolator forecasts the price tube the Highs of which are the suprema of the extrapolated prices when the prices range over the price tube and the Lows are the infima of those extrapolated prices (forecast Highs and Lows may differ from the extrapolations of the Highs and Laws because the Extrapolator takes into account past velocities, accelerations and as many derivatives as needed).

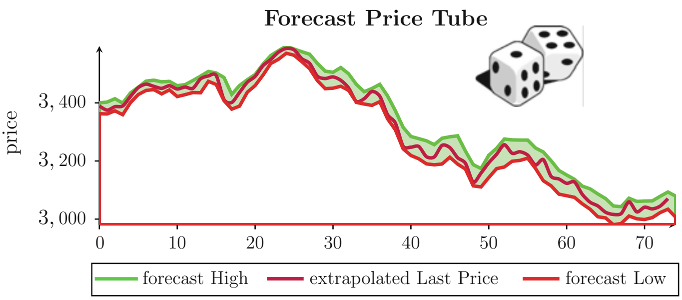


Fig. 2.2 Forecast Price Tube. The uncertainty is described by a tube $t \mapsto \Sigma(t)$: for instance, this price tube has been forecast by the VIMADES Extrapolator from the past or historical price tube of the CAC 40 index defined in Fig. 2.1 which forecasts the ex-post actual tube Fig. 2.3. Since we shall deduce the computation of the lower bounds displayed in Fig. 2.8 from the price tubes, we moved the dices of this figure to place them in the price tube for locating precisely where the uncertainty is described and thus, *the model risk*

However, to take into account at each date the new information, we use it to refresh the data of the four preceding dates by "moving"[5] or "sliding" the VPPI extrapolator. The VIMADES Extrapolator then provides the extrapolation of the price tube during the exercise period: see Fig. 2.2. We may compare it with the actual one obtained ex-post (Fig. 2.3).

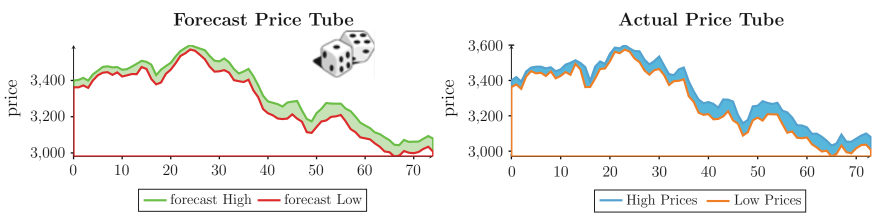


Fig. 2.3 The Extrapolated Price Tube and the ex-post Actual One

The Fig. 2.4 displays the errors produced by the VIMADES Extrapolator comparing the actual and the forecast price tubes.

[5] This terminology is used for describing moving averages of all kinds. Here, this is the tube itself which is moved instead of an average of one of its unknown evolution.

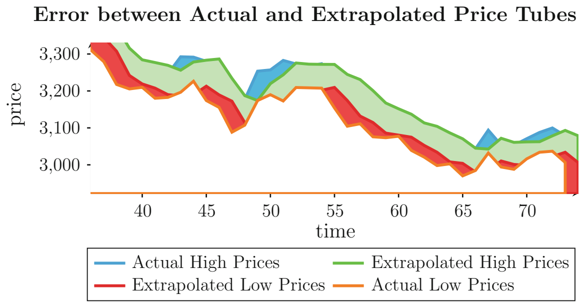


Fig. 2.4 Error between Actual and Forecast Price Tubes. The errors between the forecast tube computed ex-ante and the actual tube observed ex-post in this historical back testing are represented in this figure. We observe that the errors concern the high prices when the prices increase and the low prices in the opposite case

One can take this opportunity for testing the VIMADES Extrapolator and check whether the extrapolation of the Last Prices series remains in the price tube (this is not a theorem, but an *a posteriori* experimental observation). Figure 2.5 displays the price tube, both the Last Price evolution and its extrapolation. The histogram displays detection errors when the extrapolation does not belong to the price tube.

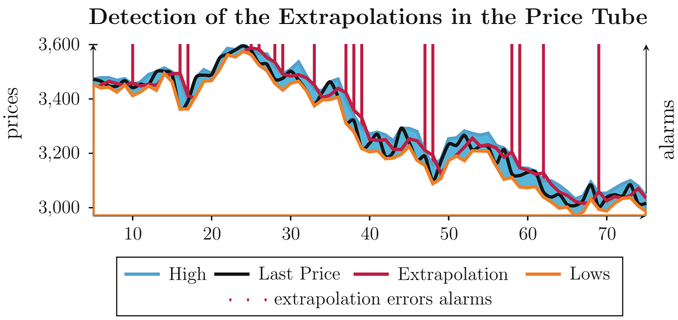


Fig. 2.5 Detection of the extrapolation of the last price in the price tube. We apply the detection of extrapolation patterns combining the detection techniques of patterns of the last price by its extrapolation in its price tube (see Sect. 2.6 for other examples, such as detection on second-degree polynomials (Fig. 2.13) and exponentials (Fig. 2.14) in the price tube)

We can compute the forecast return price tube by taking the upper and lower bounds of the returns of the extrapolated prices ranging over the forecast tube. We obtain the following tube bounded below by the lower bound of forecast risky returns which was used in the examples provided in this book (Fig. 2.6):

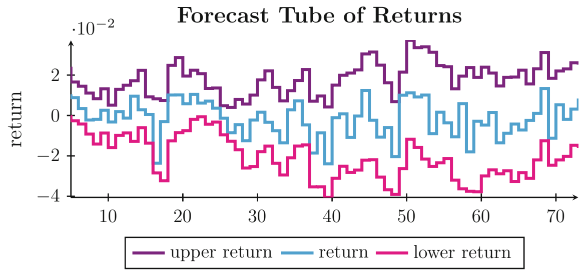


Fig. 2.6 Forecast returns. This figure displays the forecast return of the last price and the forecast tube of price returns

In summary, knowing the price tube provided by the brokerage firms, we compute the forecast price tube from which we deduced the forecast lower bounds $R^{\flat}(t)$ of the risky returns displayed in Fig. 2.8: we can thus operate the VPPI robot-insurer which compute the insurance tube, the VPPI measure of risk and the VPPI management rule (Fig. 2.7).

The above example assumes that the future $x(t+h)$ is known on some interval $[t, t+\delta]$ for $h \leq \delta$. When this is not the case, we can use one of the many available extrapolation procedure to deduce *from the history of the evolution up to time t and the extrapolation* $\widehat{x}(t+h)$ which are known on some interval $[t, t+\delta]$. We then can compute the extrapolated jerkiness indicator for *forecasting* trend reversals: integrating the VIMADES Extrapolator in the VIMADES Trendometer, we can forecast the reversal dates at each date.

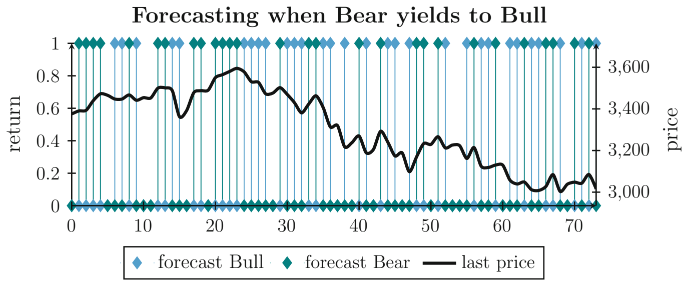


Fig. 2.7 Forecasting trend reversals. This figure provides the time reversal when the prospective derivatives is predicted by the VIMADES extrapolator (compare with Fig. 2.9)

2.3 Sensitivity to the Tychastic Gauge

As we have seen, the apprehension of uncertainty involves several aspects which interfere: the concept of *tychastic gauge*, measuring the thickness of the price tube, and its *forecasting*. Using price tubes and their forecasting, we compute the minimum guaranteed investment and the VPPI management rule. Naturally, *the size of the tychastic gauge influences both of them*. A way to measure the influence of the tychastic gauge is to compare it with the case without tychasticity (tychastic gauge equal to 0), where the price tube is reduced to the actual price. We compute the insurance and performance tubes obtained in this case with the same variable annuities floor. However, we use the extrapolation of the actual price regarded as the price tube without tychasticity, from which we forecast the lower bounds of the future risky return:

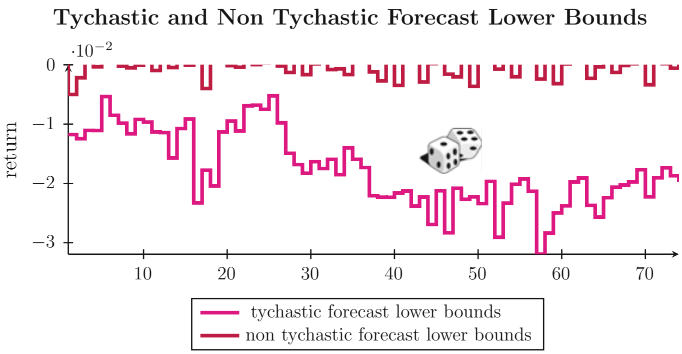
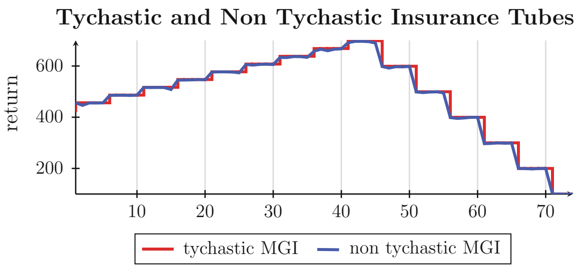


Fig. 2.8 Tychastic and non tychastic forecast lower bounds of returns of the CAC 40 returns. Since the larger the price tube, i.e., the larger the tychastic gauge, the smaller the forecast lower bounds of the risky return, the more tychastic is the uncertainty. This fact is illustrated by choosing the least tychastic case when the price tube is reduced to the last price series (the non tychastic case)

However, there is no simple relation between the respective minimum guaranteed investment between the tychastic and non tychastic cases. The tychastic minimum guaranteed investment can be both above or below the non tychastic one:



The situation is akin to the sensitivity of the value of the portfolio to small changes in volatility, called Vega, a (pseudo-Greek) in option theory.

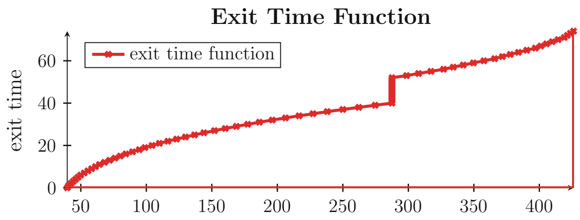
The *Key Risk Indicator* (KRI) at investment date and the *Key Performance Indicators* (KPI) at exercise date are summarized in this table:

minimum guaranteed investment (MGI)	409.18
minimum guaranteed cushion (MGC)	369.55
actualized exercise value	98.47
cumulated prediction penalties	−239.89

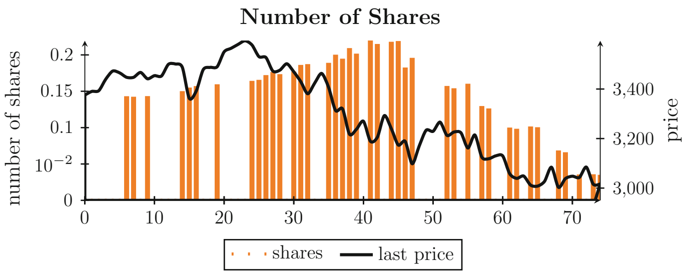
For the sake of comparison, we compare it with the one we obtained under the tychastic case:

minimum guaranteed investment (MGI)	426.13
minimum guaranteed cushion (MGC)	386.5
actualized exercise value	109.12
cumulated prediction penalties	−54.77

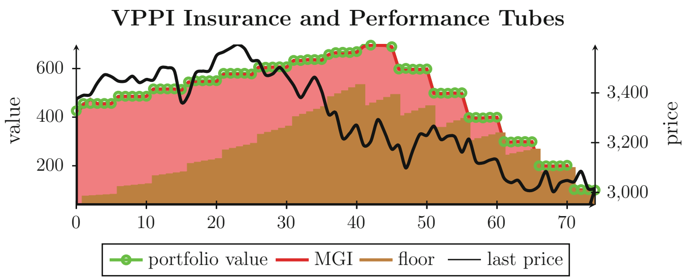
The hedging exit time function is displayed below:



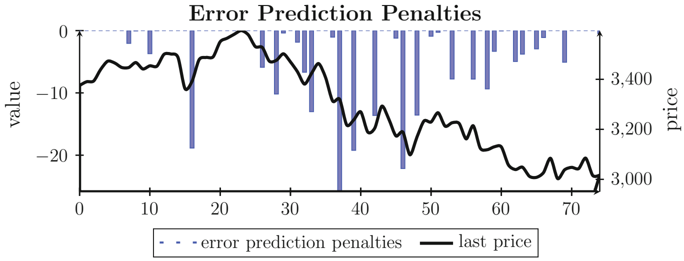
The number of shares is provided in



The performance tube is depicted in



and the error prediction penalties in



2.4 Trend Reversal: From Bear to Bull and Back

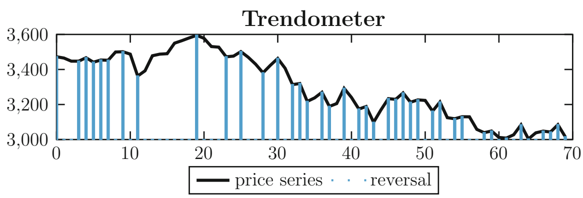
Knowing when at some date a function reverses its trend from increasing behavior to decreasing behavior provides alarms whenever the trend of the price of the assets changes: from "bear markets" when the prices are falling, to "bull markets", when they are "rising", and back. This problem is tackled at the level of technical analysis of time series.

At each date, the VIMADES Trendometer

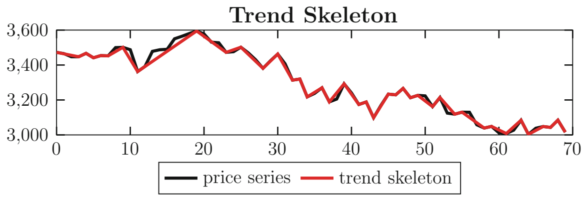
1. detects automatically whether it is a *trend reversal date* at which the function achieves either a local minimum or a local maximum;
2. computes the (nonlinear) *jerkiness indicator* measuring the frequency and the violence of the trend reversal at the aftermath of monotone periods when they blow up, since *bear* and *bull* markets periods delineated by the transversal dates are not jerky by definition (see [27]).

2.4.1 Trendometer

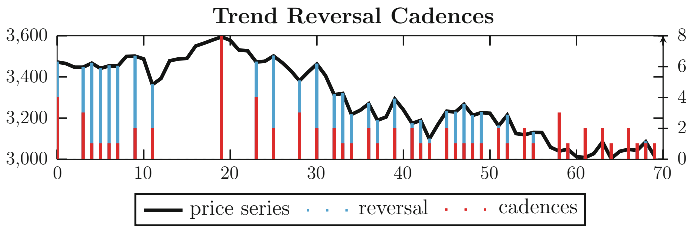
The trendometer detects all local extrema of a time series:

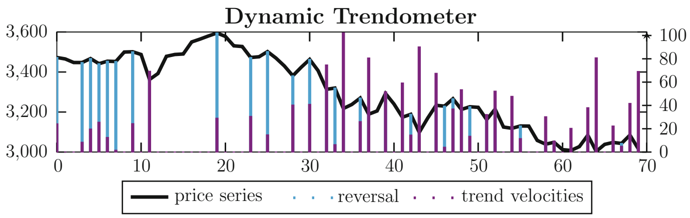


It allows time series analysts to extract from a time series a trend skeleton summarizing the time series by interpolating the trend reversal values and thus, *cadences* (difference between successive trend reversal dates) and average *trend velocities* between successive trend reversal values:



Cadences and trend velocities can be displayed for providing dynamical indicators on the time series:





2.4.2 Trend Jerkiness and Eccentricities

The VIMADES Trendometer measures also the jerkiness function of the time series at every date:

The trend reversal dates of a time series are classified in chronological order, or by decreasing jerkiness, or by increasing duration of their congruence periods (since high jerkiness and short durations of congruence periods are two indicators of a jerky situation):

The *VIMADES Trendometer* computes and classifies the dates in the four trigonometric quadrants: the North West quadrant $\mathbb{R}_{++}$, the North East quadrant $\mathbb{R}_{+-}$, the South West quadrant $\mathbb{R}_{--}$ and the South East quadrant $\mathbb{R}_{-+}$. Definition 4.2.1 of trend reversibility indexes provides in the lychee framework the following particular case:

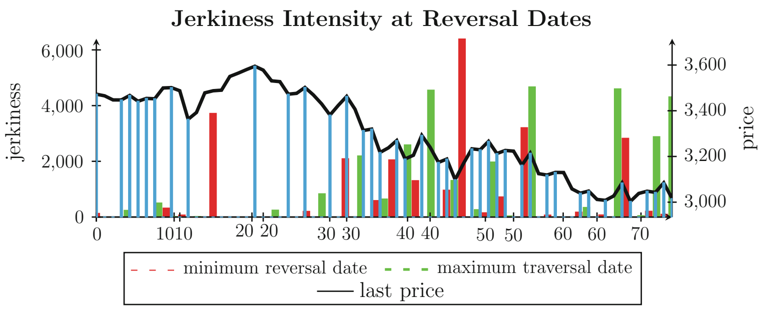
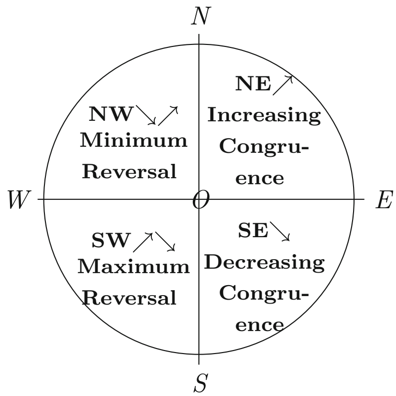


Fig. 2.9 From bear to bull and back. The *thin bars* display the reversal values triggering alarms at the reversal dates. The height of the *thicker bars* underlines the trend jerkiness index of the time series at trend reversal dates: the *colors* distinguish the minimum reversal dates $t_{\searrow\nearrow}$ from bear to bull markets at which the price achieves a local minimum and maximum reversal dates $t_{\nearrow\searrow}$ from bull to bear markets

Definition 2.4.1 (*The Trend Compass*) The *trend compass* classifies the prices in four qualitative cells:



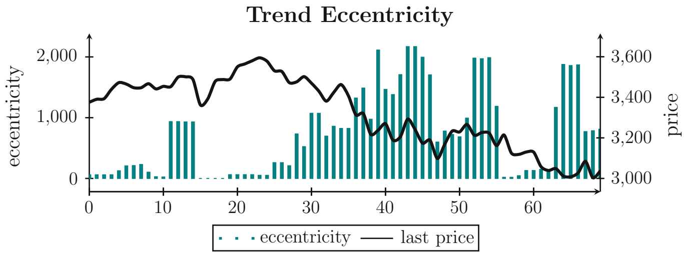
1. *minimum time reversal* cell (North East quadrant);
2. *maximum time reversal* cell (South East quadrant);
3. *decreasing time congruence* cell when the function decreases, or a “bear” period (South West quadrant);
4. *increasing time congruence* cell when the function increases, or a “bull” period (North West quadrant).

The trend compass classifies the dates in these four classes between reversal and congruence phases, distinguishing the ascending ones (bear markets) and the descending ones (bull market):

Date class. of	Date class. of durat.	Date class. of jerk.	Jerk. class. of date	Jerk. class. of durat.	Jerk. class. of	Durat. class. of date	Durat. class. of	Durat. class. of jerk.
03/08/12	0	240	03/10/12	0	6400	12/03/00	0	240
08/08/12	2	1	16/10/12	0	4672	09/10/01	0	504
09/08/12	0	504	31/10/12	1	4602	14/08/05	0	321
10/08/12	0	321	27/09/12	1	4555	29/07/03	0	19
13/08/12	0	19	09/11/12	0	4314	05/08/03	0	73
14/08/12	0	73	20/08/12	1	3721	09/07/17	0	648
16/08/12	1	20	15/10/12	1	3208	31/05/00	0	2053
20/08/12	1	3721	07/11/12	0	2884	28/05/05	0	1307
30/08/12	7	248	01/11/12	0	2827	23/12/01	0	1313
05/09/12	3	204	24/09/12	1	2590	09/02/00	0	6400
07/09/12	1	836	14/09/12	1	2195	15/10/12	0	153
12/09/12	2	2093	12/09/12	2	2093	04/01/00	0	1976
14/09/12	1	2195	20/09/12	0	2053	09/12/00	0	723
18/09/12	1	592	09/10/12	0	1976	27/09/07	0	40
19/09/12	0	648	02/10/12	0	1313	25/07/00	0	4672
20/09/12	0	2053	25/09/12	0	1307	23/11/07	0	5
24/09/12	1	2590	01/10/12	1	965	07/04/00	0	345
25/09/12	0	1307	07/09/12	1	836	23/10/11	0	2827
27/09/12	1	4555	10/10/12	0	723	19/01/00	0	207
01/10/12	1	965	19/09/12	0	648	08/03/10	0	2884
02/10/12	0	1313	18/09/12	1	592	14/04/02	0	99
03/10/12	0	6400	09/08/12	0	504	02/01/06	0	4314
05/10/12	1	266	25/10/12	0	345	13/08/01	1	20
08/10/12	0	153	10/08/12	0	321	02/02/07	1	3721
09/10/12	0	1976	05/10/12	1	266	20/06/12	1	836
10/10/12	0	723	30/08/12	7	248	21/08/02	1	2195
11/10/12	0	40	03/08/12	0	240	21/09/00	1	592
15/10/12	1	3208	06/11/12	0	207	11/10/08	1	2590
16/10/12	0	4672	05/09/12	3	204	10/03/00	1	4555
18/10/12	1	71	24/10/12	2	176	16/03/00	1	965
19/10/12	0	5	08/10/12	0	153	05/08/12	1	266
24/10/12	2	176	08/11/12	0	99	17/02/00	1	3208
25/10/12	0	345	29/10/12	1	76	01/01/00	1	71
29/10/12	1	76	14/08/12	0	73	23/09/05	1	76
31/10/12	1	4602	18/10/12	1	71	23/06/00	1	4602
01/11/12	0	2827	05/11/12	1	48	22/07/00	1	48
05/11/12	1	48	11/10/12	0	40	03/09/00	2	1
06/11/12	0	207	16/08/12	1	20	12/09/12	2	2093
07/11/12	0	2884	13/08/12	0	19	24/10/12	2	176
08/11/12	0	99	19/10/12	0	5	05/09/12	3	204
09/11/12	0	4314	08/08/12	2	1	30/08/12	7	248

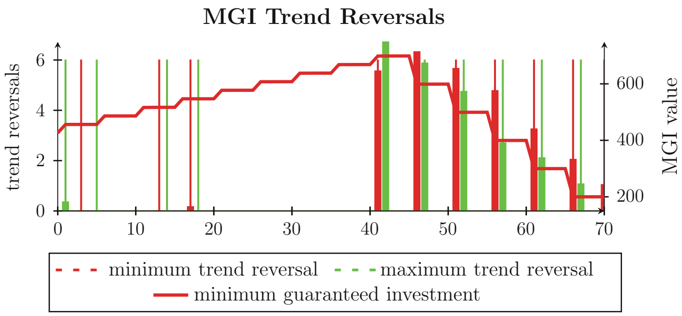
The ***eccentricity index*** associates at each time the average of trend jerkiness during a given period. This provides another indicator of a volatile behavior of the prices: the higher this eccentricity index, the more "volatile" the evolution. For instance, if the period is four dates, we obtain the following graph of the eccentricity of the price:

Reversal dates	Jerkiness intensity	Reversal dates	Jerkiness intensity	Reversal dates	Jerkiness intensity
03/10/12	6399, 3995	09/10/12	1974, 7165	06/11/12	206, 3981
16/10/12	4671, 195	02/10/12	1312, 4576	05/09/12	203, 2652
31/10/12	4600, 8908	25/09/12	1305, 533	24/10/12	174, 968
27/09/12	4554, 1506	01/10/12	963, 8912	08/10/12	151, 973
09/11/12	4313, 3243	07/09/12	834, 662	08/11/12	97, 8416
20/08/12	3719, 6546	10/10/12	722, 4491	29/10/12	75, 4085
15/10/12	3206, 6275	19/09/12	647, 0114	14/08/12	71, 8112
07/11/12	2883, 3776	18/09/12	590, 5854	18/10/12	69, 848
01/11/12	2826, 0836	09/08/12	503, 4376	05/11/12	47, 1593
24/09/12	2589, 47	25/10/12	343, 792	11/10/12	39, 1841
14/09/12	2193, 683	10/08/12	319, 7732	16/08/12	18, 6272
12/09/12	2092, 0907	05/10/12	264, 5396	13/08/12	18, 3116
20/09/12	2052, 3989	30/08/12	246, 809	19/10/12	3, 608
				08/08/12	0, 1832

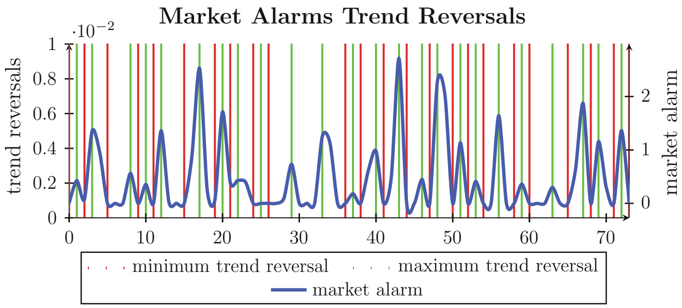


The VIMADES Trendometer provides automatically alarms warning investors of the need to make an *urgent* qualitative assessment of the causes triggering jerky periods, economic, financial, political, Panurgic (or mimetic behavior detecting a collective erratic decision process by lack of trust in the forecast future, etc.).

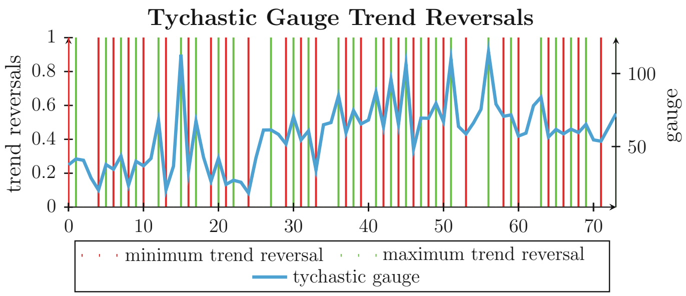
The VIMADES Trendometer can be used for sequencing time reversals of other series. For instance, to detect the trend reversal dates of the insurance tube:



The VIMADES Trendometer detects the trend reversals of market alarms:



It is interesting to compare the trend reversal of the market alarms with the ones of the tychastic gauge:



2.4.3 Detecting Extrema and Measuring Their Jerkiness

For individual continuous time evolutions, the trendometer detects all their local extrema and measures their jerkiness (Fig. 2.10):

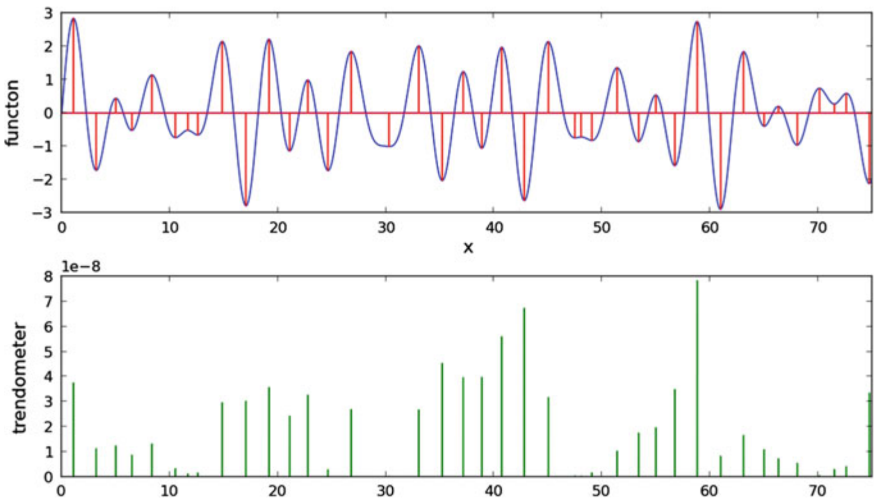
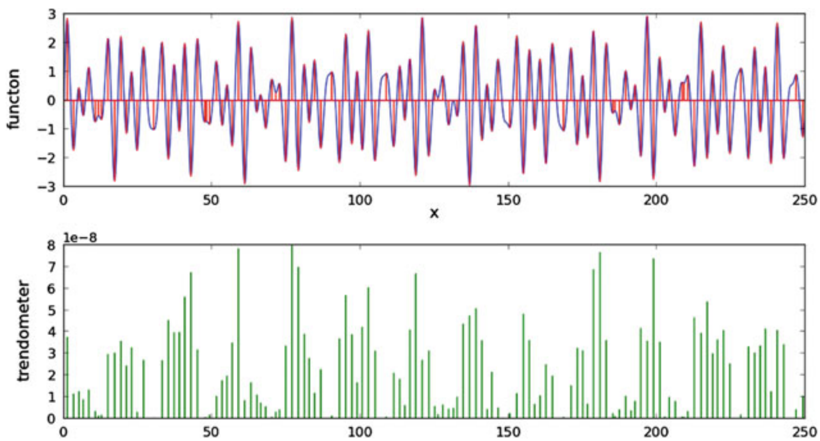
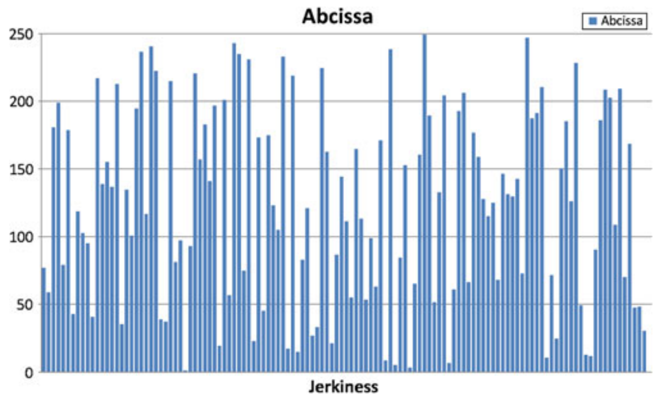


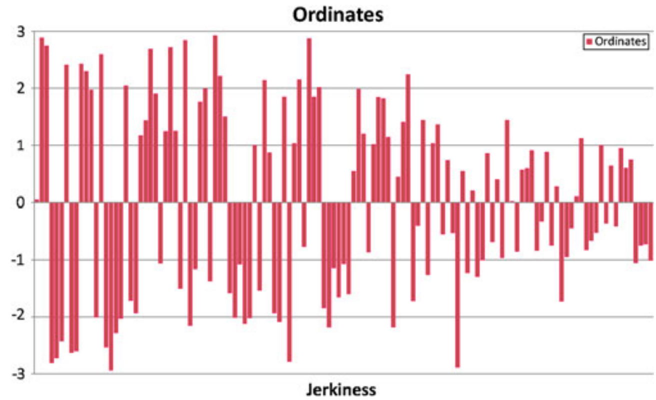
Fig. 2.10 Applications of the trendometer to trigonometric functions. The trendometer can be applied to detect and measure the strength of minima and maxima of differentiable functions, such as the sum $t \in [0, 75] \mapsto \sin(x) + \sin(\sqrt{2}x) + \sin(\sqrt{3}x)$ of three trigonometric functions, as suggested on page 146 of the book *A New Kind of Science*, [177], by Stephen Wolfram displaying two regularly spaced families. They thus detect the zeros of its derivative $t \mapsto \cos(x) + \sqrt{2}\cos(\sqrt{2}x) + \sqrt{3}\cos(\sqrt{3}x)$. The figure *above* displays the graph of this function and the *vertical bars* indicate the values at which the function reaches its extrema. The figure *below* displays the jerkiness of the extrema at the dates when they are reached

For the sake of comparison with the example of the Wolfram book, we display the trendometer applied to this function on the interval [0, 250]:

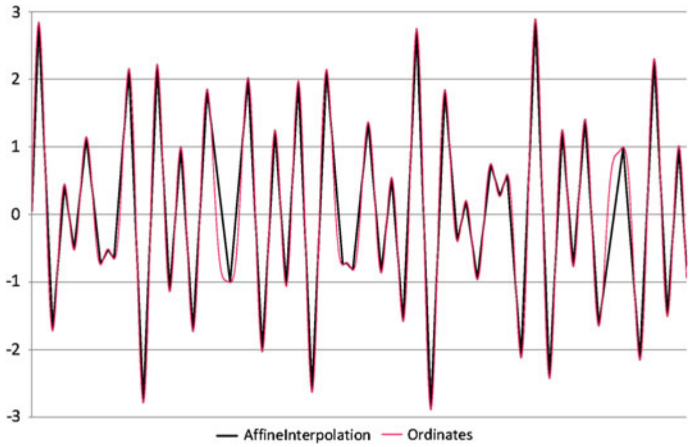


The two next figures display the abscissa and ordinates of the function in terms of decreasing jerkiness of their extrema:





By using a piecewise interpolation between the extrema, we obtain a "*trend skeleton*" *summarizing the function*:



Stephen Wolfram states: "*Among all the mathematical functions defined, say, in* Mathematica*, it turns out that there are also a few—not traditionally common in* *natural sciences—which yield complex curves which do not appear to have any explicit dependence on representations of individual numbers*". This complexity, such as chaos produced by iterated maps, is linked to the fact that viability kernels of compact spaces under disconnecting maps (inverses of *Hutchinson maps*) are uncountable Cantor sets (see Theorem 2.9.10, p. 80, [28]).

The trendometer provides a trend reversal of the Fermat rule:

Trend reversal of the Fermat rule *The trendometer provides a "trend reversal" of the Fermat rule. Instead of using the zeros of the derivative for finding all the local extrema of any numerical function of one variable, the extrema of the primitive of a function detected by the trendometer allows us to find zeros of the function.*

2.4.4 Differential Connection Tensor of a Family of Series

Differential connection tensor of a family of series have emerged from two different, yet, connected, motivations. The first one follows the observation that the classical definition of derivatives involves prospective (or forward) difference quotients. Actually, the available and known derivatives are retrospective (or backward). They coïncide whenever the functions are differentiable in the classical sense, but not in the case of non smooth maps, single-valued or set-valued.

The later ones are used in differential inclusions (and thus, in uncertain control systems) governing evolutions in function of time and state. We follow the plea of some physicists for taking also into account the retrospective derivatives to study prospective evolutions in function of time, state and retrospective derivatives, a particular, but specific, example of historical of "path dependent" evolutionary systems. This is even more crucial in life sciences, in the absence of experimentation of uncertain evolutionary systems. The second motivation emerged from the study of networks with junctions (cross-roads in traffic networks, synapses in neural networks, banks in financial networks, etc.), an important feature of "complex systems". At each junction, the velocities of the incoming (retrospective) and outgoing (prospective) evolutions are confronted. This leads to the introduction of the "differential connection tensor" of two evolutions, defined as the tensor product of retrospective and prospective derivatives, which can be used for controlling evolutionary systems governing the evolutions through networks with junctions (see [27]).

Given a family of temporal series (the prices of the 40 assets of the stock market index CAC 40, for instance, as we shall see later), the *differential connection tensor* is *the tensor product*[6] *of retrospective and prospective velocities* which measures the *jerkiness* between two functions, smooth or not smooth (temporal series) providing the trend reversal dates of the differential connection tensor. The differential

[6] Recall that the tensor product $p \otimes q$ of two vectors $p := (p_i)_i \in \mathbb{R}^\ell$ and $q := (q_j)_j \in \mathbb{R}^\ell$ is the rank one linear operator

$$p \otimes q \in \mathcal{L}(\mathbb{R}^\ell, \mathbb{R}^\ell) : x \mapsto \langle p, x \rangle\, q$$

the entries of which (in the canonical basis) are equal to $(p_i q_j)_{i,j}$.

connection tensor plays the role of *covariance matrices* of families of random variables: statistical events in the sample space are replaced by dates and random variables by temporal series.

The question arises whether it is possible to detect the connection dates *when the monotonicity of a series of a family of temporal series is followed by a reversal of the monotonicity of other series*, in order to detect the influence of each series on the dynamic behavior of other ones. When the two series are the same (diagonal entries), we recover their reversal dates. The *differential connection tensor* measures the jerkiness between two series, providing the other entries of the differential connection tensor.

The VIMADES Tensor Trendometer[7] software provides at each date the coefficients of the differential connection matrix.

2.4.4.1 Differential Connection Tensor Between Prices and Volumes

We describe the results obtained when we consider only two series for displaying meaningful figures.

The entry of the first row and the first column is the jerkiness of the trend reversal of the price, the first row and second column, the monotonicity jerkiness between price and volume, the second line the first column, the monotonicity jerkiness volume and price and the second row and second column, the jerkiness of the trend reversal of the volume.

The selected series are those of an asset price and volume of securities exchange during a daily session[8] (Fig. 2.11).

At each date, the connection matrix displays the jerkiness measures among and between the two series. For instance, on December 7, 2004, three weeks before the big discontinuity, all four coefficients of the differential connection matrix are different from zero:

$$\begin{pmatrix} 0,39 & 33 \\ 1,80 & 153 \end{pmatrix} \tag{2.8}$$

At the discontinuity date, a small decrease of prices was followed by a large increase in volume, as indicated by the differential connection:

$$\begin{pmatrix} 0,2 & 0 \\ 2{,}654 & 0 \end{pmatrix} \tag{2.9}$$

[7] The software of the Tensor Trendometer of VIMADES has been registered on November 25, 2013, at the INPI, the French Institut National de la Propriété Industrielle.

[8] It is calculated daily volume (number of shares traded) or by value of transactions. The volume used here is the volume of securities, not their values. The volume is an important activity indicator because it measures the interest of investors.

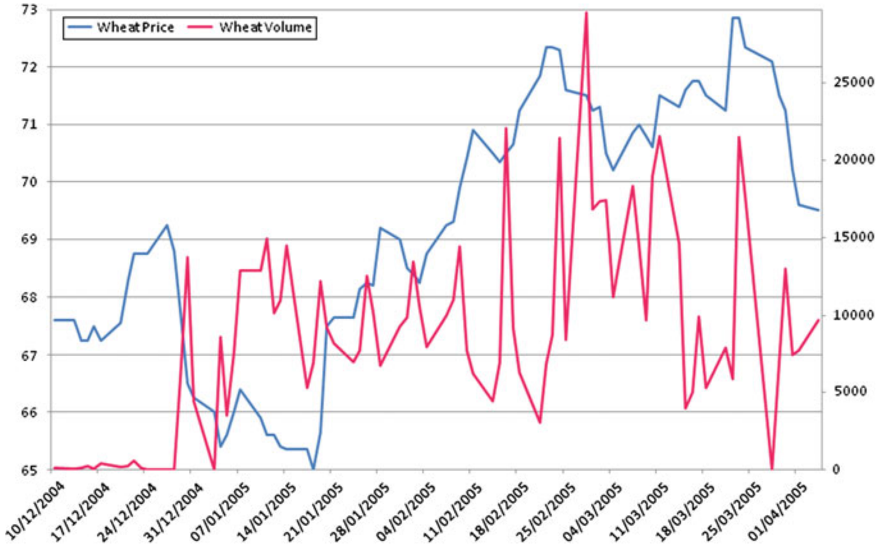


Fig. 2.11 Price and volume series of wheat. This figure displays the series of "settlement prices" of wheat and the volume of exchanges on the London Commodity Market from December 19, 2004 to April 4, 2005 around the date of January 10, 2005, when an important discontinuity of the volume happened (from 7,534 to 12,842 units). The number of dates is reduced for the visibility of this graphical representation of the series of differential connection matrices

Figure 2.12 displays the dates at which at least the monotonicity of a series is followed by the reversal of itself and/or another one:

A statistical study over the period from 05/01/2000 to 30/09/2013 shows the proportions between the following dates:

1. trend reversal dates of the price series: 26 %
2. monotonicity reversal dates between price and volume series: 24 %
3. monotonicity reversal dates between volume and price series: 22 %
4. trend reversal dates of the price series: 28 %

2.4.4.2 Case of the Price Series of the CAC 40

We use the tensor trendometer for detecting the dynamic correlations between the forty price series of the CAC 40. For instance, on August 6, 2010, the prices are displayed in the following figure

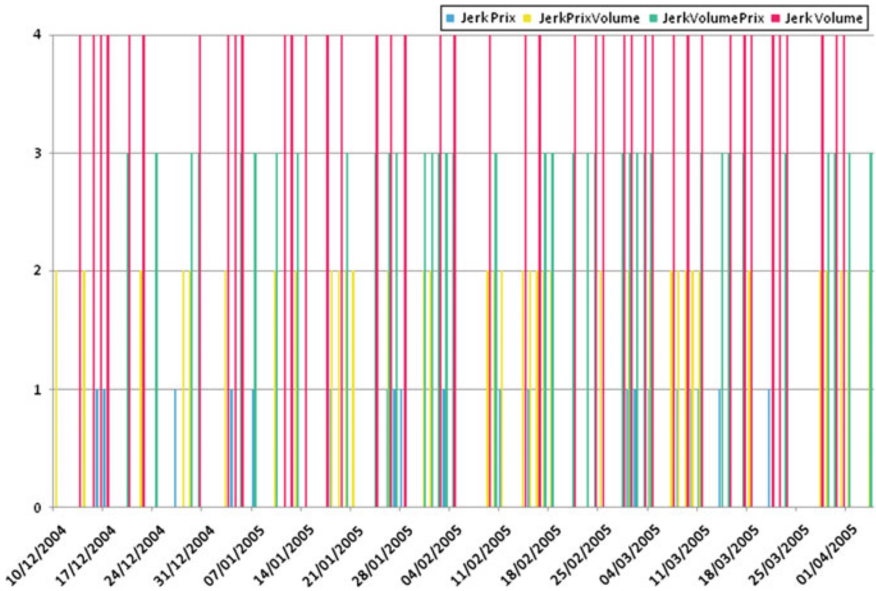
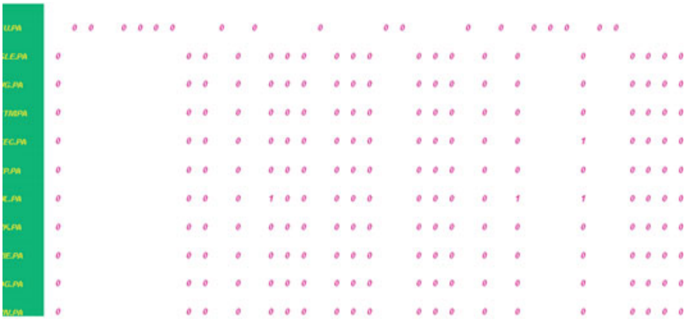


Fig. 2.12 Differential connection tensor between price and volume. In order to represent the detection of the different entries of the differential connection matrix between the price and volume series at each date of the temporal window, we indicate by *vertical bars* between 0 and 1 the trend reversal dates of the price series and by *vertical bars* between 0 and 4 the trend reversal dates of the volume series, which occupy the diagonal of the differential connection matrix. The *vertical bars* between 0 and 2 detect the dates when monotonicity behavior of the price precedes the monotonicity behavior of the volume whereas *vertical bars* between 0 and 3 detect the dates when monotonicity behavior of the volume is followed by the monotonicity behavior of the price

AC.PA	AI.PA	ALO.PA	MT.PA	CS.PA	BNP.PA	EN.PA	CAP.PA	CA.PA	ACA.PA
26,03	90,84	27,02	18,85	9,95	32,13	19,67	29,72	15,21	3,7
BN.PA	EAD.PA	EDF.PA	GTO.PA	EI.PA	GSZ.PA	LG.PA	LR.PA	OR.PA	MC.PA
48,22	30,38	15,78	64,50	68,82	17,02	37,30	26,26	99,88	126,1
ML.PA	ORA.PA	RI.PA	PUB.PA	PP.PA	RNO.PA	SGO.PA	SAN.PA	SU.PA	GLE.PA
55,58	9,97	86,10	40,35	123,02	35,42	24,14	21,58	49,64	18,6
UG.PA	STM.PA	TEC.PA	FP.PA	UL.PA	VK.PA	VIE.PA	DG.PA	VIV.PA	
6,37	4,12	85,45	35,15	79,32	35,75	7,74	32,24	14,59	

At each date, it provides the 40 × 40 matrix displaying the qualitative jerkiness for each pair of series when the trend of the first one is followed by the opposite trend of the second one. At each entry, the existence of a trend reversal by a circles:



The quantitative version replaces the circles by the values of the jerkiness:

	AC.PA	AI.PA	ALO.PA	MT.PA	CS.PA	BNP.PA	EN.PA	CAP.PA	CA.PA	ACA.PA
AC.PA	0,000	0,008	0,001	0,000	0,001	0,002	0,002	0,003	0,000	0,000
AI.PA	0,182	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,111	0,004
ALO.PA	0,023	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,014	0,001
MT.PA	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
CS.PA	0,038	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,023	0,001
BNP.PA	0,041	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,025	0,001
EN.PA	0,054	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,033	0,001
CAP.PA	0,000	0,060	0,004	0,000	0,011	0,016	0,012	0,024	0,000	0,000
CA.PA	0,045	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,028	0,001
ACA.PA	0,024	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,015	0,001

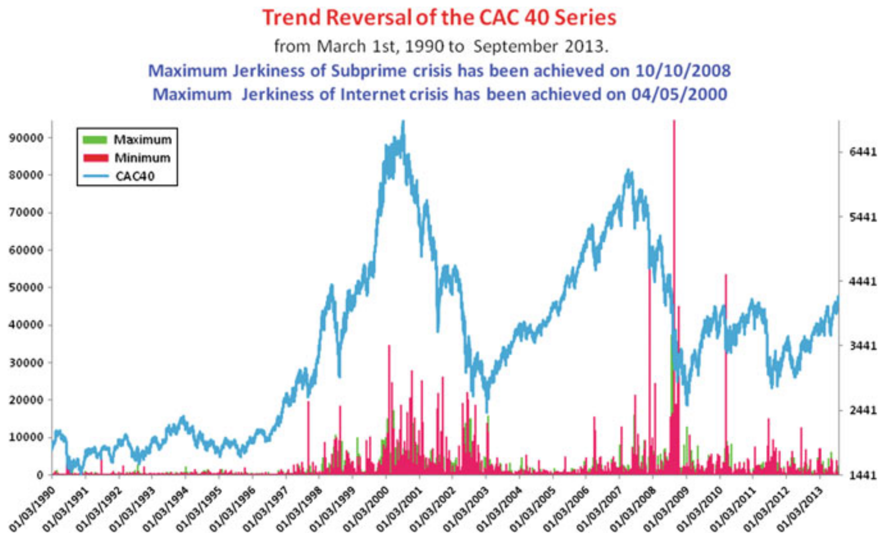
We turn our attention to the CAC 40 series for detecting the financial crisis of the beginning of our century.

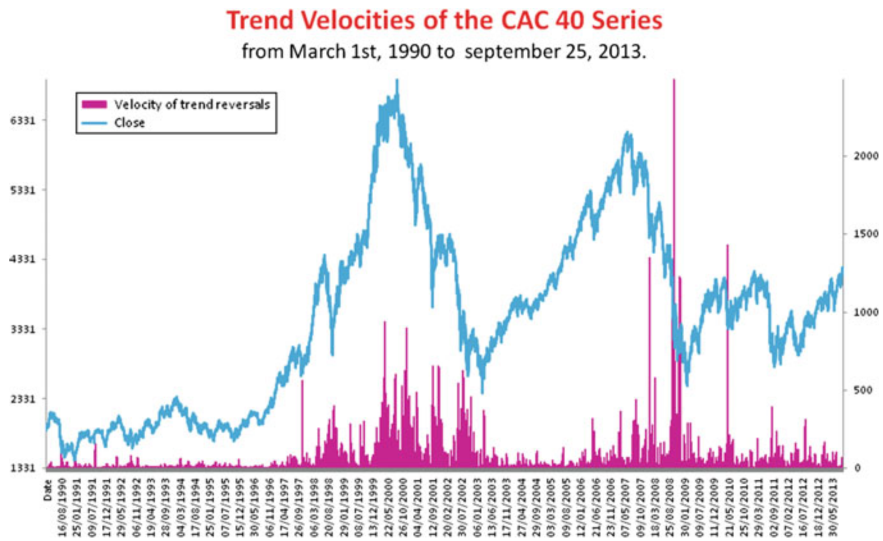
The temporal window is from du 03/01, 1990 to 09/25, 2013.

The first figure displays the series of the CAC 40 indexes (close). The vertical bars indicate the reversal dates and their height displays their jerkiness.

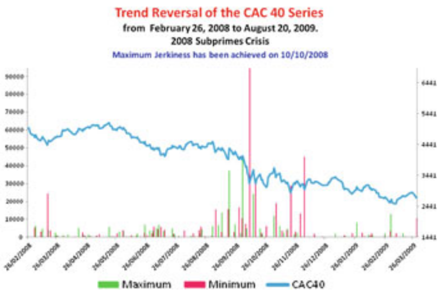
The second figure displays the velocities of the jerkiness between two consecutive trend reversal dates, a ratio involving the variation of the jerkiness and the duration of the congruence period (bull and bear). It is a dynamic view of the agitation of the temporal series.

The analysis of this series, as other time series of asset prices, shows that often, the jerkiness at minima (bear periods) is higher than the ones at maxima (bull periods). For the CAC 40, the proportion of "bear jerkiness" (57 %) over "bull jerkiness" (43 %). A possible explanation is a mimetic one: the fear of bear periods propagates and amplifies for selling the shares whereas investors may wait to regain confidence in bull phases.



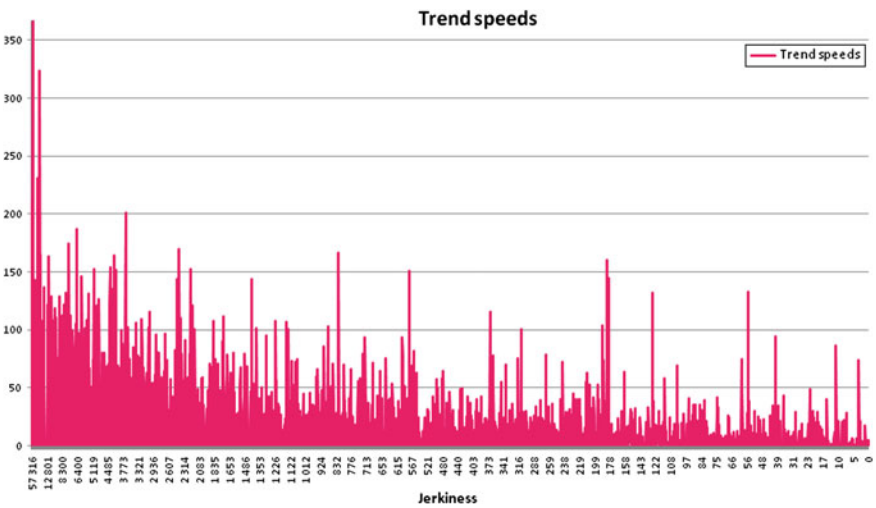


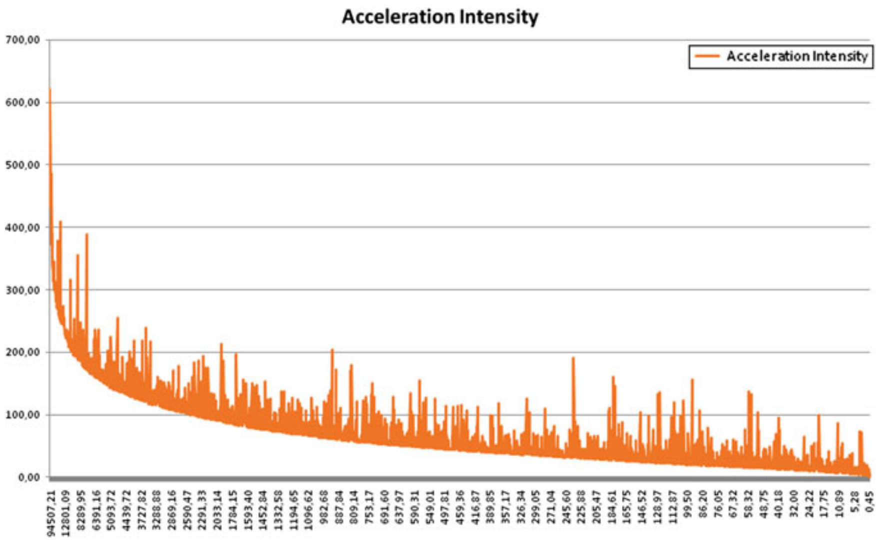
The third and fourth figures zoom on the 2000 Internet crisis (around May 4, 2000) and the 2008 subprime crisis (around October 10, 2008), which are detected thanks to the trendometer but not observed on simple examination of the time series of the CAC 40.

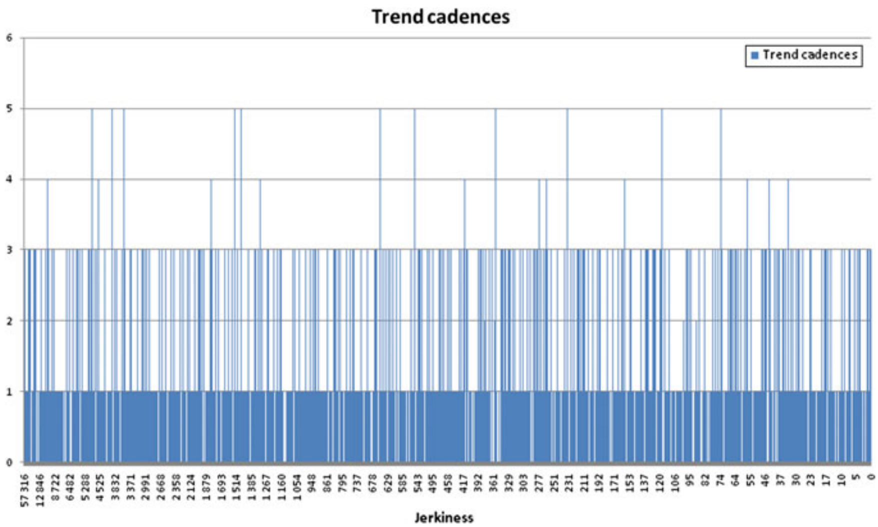


The next figure displays the classification by decreasing jerkiness of

1. trend speeds. They are absolute values of the velocities of the jerkiness between two consecutive trend reversal dates, a ratio involving the variation of the jerkiness and the duration of the congruence period (bull and bear);
2. the absolute value of the "acceleration";
3. the "cadences", duration of the congruence period (bull and bear).



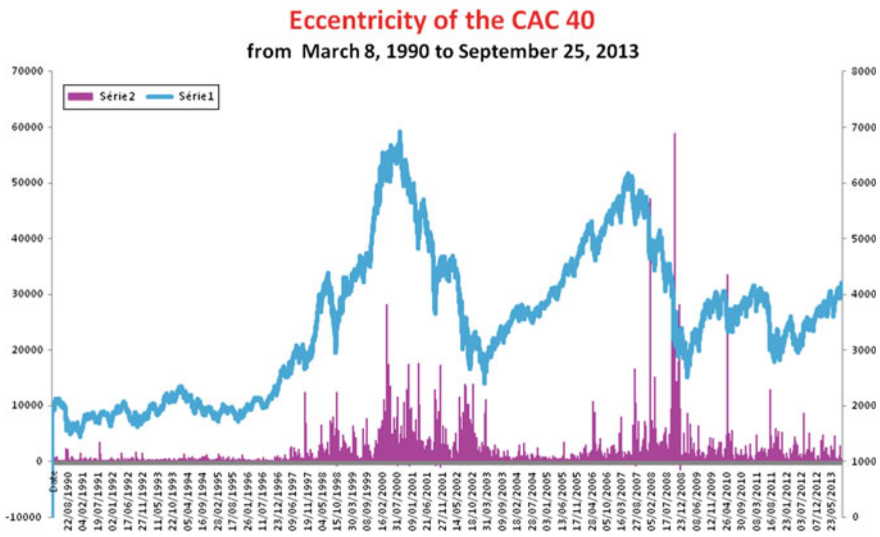




The next table provides the first dates by decreasing jerkiness. The most violent are those of the subprime crisis (in bold), then the ones of the year 2006 and, next, the dates of the Internet crisis (in italics).

Date	Jerkiness	Date	Jerkiness	Date	Jerkiness
10/10/**2008**	94507, 21	03/01/*2001*	15153, 31	17/02/*2000*	10025, 57
23/01/**2008**	57315, 90	11/09/*2002*	15111, 43	28/10/*2002*	9962, 69
07/05/2010	53585, 50	10/03/*2000*	15055, 45	01/09/1998	9917, 22
05/12/**2008**	44927, 23	10/08/2011	15011, 24	15/02/**2008**	9905, 51
03/10/**2008**	43319, 41	27/08/*2002*	14958, 41	19/04/1999	9887, 67
19/09/**2008**	37200, 13	22/11/*2000*	14768, 91	26/10/*2001*	9556, 17
05/04/*2000*	34609, 80	03/04/*2000*	14280, 35	29/06/*2000*	9470, 44
21/01/**2008**	34130, 42	03/04/*2001*	14003, 47	25/02/*2000*	9438, 07
16/10/**2008**	29794, 42	18/07/*2002*	13813, 67	27/03/*2001*	9436, 84
21/11/**2008**	28840, 69	19/12/*2000*	13743, 01	15/05/*2000*	9411, 84
04/12/*2000*	27861, 03	12/03/2003	13707, 93	04/10/2011	9409, 14
12/11/*2001*	26039, 07	12/09/**2008**	13682, 85	17/01/*2000*	9398, 39
22/03/*2001*	25128, 11	01/12/**2008**	13207, 66	11/08/1998	9320, 83
27/04/*2000*	24577, 70	29/10/1997	13085, 95	20/11/**2007**	9291, 91
17/03/**2008**	24416, 22	04/03/2009	12845, 84	05/10/1998	9277, 96
14/10/**2008**	24007, 60	14/03/**2007**	12801, 09	29/07/1999	9253, 97
05/08/*2002*	22021, 61	24/06/*2002*	12658, 98	04/12/**2007**	9200, 48
14/09/*2001*	21658, 15	02/08/2012	12628, 14	04/02/*2000*	9093, 25
10/08/**2007**	21252, 50	24/05/*2000*	12456, 94	02/10/*2002*	8959, 94
13/11/*2000*	20662, 32	10/05/*2000*	12411, 27	13/09/*2000*	8897, 37
22/01/**2008**	20184, 96	28/07/*2000*	12145, 83	10/05/2010	8877, 39
14/08/*2002*	20052, 16	23/02/*2001*	11960, 59	30/09/*2002*	8845, 61
28/10/1997	19720, 61	04/11/**2008**	11904, 50	04/11/1998	8843, 75
14/06/*2002*	19114, 56	08/06/***2006***	11773, 65	09/08/2011	8833, 20
06/11/**2008**	18900, 51	30/10/*2001*	11733, 86	11/06/*2002*	8832, 22
03/08/*2000*	18621, 37	15/10/*2001*	11630, 50	07/07/*2000*	8797, 60
29/10/*2002*	18550, 19	24/03/2003	11294, 44	16/01/*2001*	8778, 74
08/10/1998	18307, 12	15/03/*2000*	11232, 52	27/04/1998	8721, 52
02/05/*2000*	18087, 38	17/09/**2007**	10948, 51	19/02/**2008**	8327, 20
21/09/*2001*	17771, 78	13/08/**2007**	10933, 30	20/11/*2000*	8299, 90
11/09/*2001*	17660, 69	25/10/*2001*	10809, 42	03/07/*2002*	8289, 95
16/08/**2007**	17398, 86	02/10/**2008**	10720, 31	28/06/*2000*	8258, 67
16/05/*2000*	17228, 62	23/10/*2002*	10675, 86	28/06/2010	8137, 05
04/04/*2000*	16958, 95	25/08/1998	10673, 02	31/01/*2000*	8093, 58
18/10/*2000*	16761, 07	30/03/2009	10672, 64	21/11/*2000*	8074, 23
29/09/**2008**	16502, 34	24/01/**2008**	10352, 96	28/01/2009	8049, 26
08/08/**2007**	16048, 09	20/03/*2001*	10294, 67	26/02/**2007**	8038, 76
21/03/2003	15703, 11	14/12/*2001*	10253, 40	31/01/*2001*	8033, 95
18/09/**2008**	15506, 17	31/07/**2007**	10134, 80	26/11/*2002*	7933, 90
22/05/***2006***	15470, 19	26/04/*2000*	10093, 65	08/08/2011	7821, 87
05/09/**2008**	15406, 87	02/09/1999	10080, 12	18/05/2010	7793, 80

The next figure displays the eccentricity of the CAC 40 series, which also detects the Internet and Subprime bubbles, but takes into account the previous velocities, accelerations and jerks of the preceding jerkiness.



2.5 Dimensional Rank Analysis

It is tempting to compare several indicators, such as, for instance, acceleration of prices and velocities of gauges. They do not take their values in the same space and so, are not really comparable, except if we modify their values in such a way *they range over the same space of values*.

In physics, since *Isaac Newton* and its "principle of similitude", the purpose of *dimensional analysis* is to compare physical quantities by "homogenizing" them in terms of their "basic physical dimensions", such as length, mass, time, electric charges, etc., thanks to the *Buckingham* π *Theorem* (1914), rediscovering a theorem due to *Joseph Bertrand* in 1878. They are used to define homogeneous measures (without dimensions) of the form $\frac{\sum_{i=1}^{n} p_i x_i}{\prod_{i=1}^{n} x_i^{a_i}}$ where $\sum_{i=1}^{n} a_i = 1$.

We borrow the same strategy whenever financial discrete time series are observed or evaluated using a battery of "indicators" taking different values (returns, averages, VaR, Sharpe ratios, etc.). Pattern recognition, segmentation, clustering and many other techniques are used to detects the relevant indicators for detecting alarms, anomalies or signals (see for instance *Mc Queen*'s *k-means*, *Diday*'s *dynamical clustering*, *Vapnik*'s *support vector machines and networks*, *neural networks*, Pernot's *Choix d'un classifieur en discrimination* [148]), *Diday*'s *symbolic data analysis* (see the bibliography of *Symbolic Data Analysis: Conceptual Statistics and Data Mining*, [72]), etc.

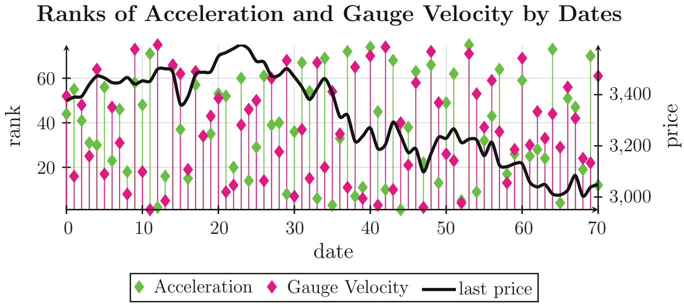
In the case when evolutions described by time series are concerned, they "*mine the trajectory of a time vectorial series*" for detecting the rôles of each component of the vectorial series and classifying the trajectory (regarded as a cloud) in *a posteriori* discovered classes.

This “transversal” approach can be complemented by a joint study of time series as evolutions, associating with them other indicators, classifying them according several dynamic criteria.

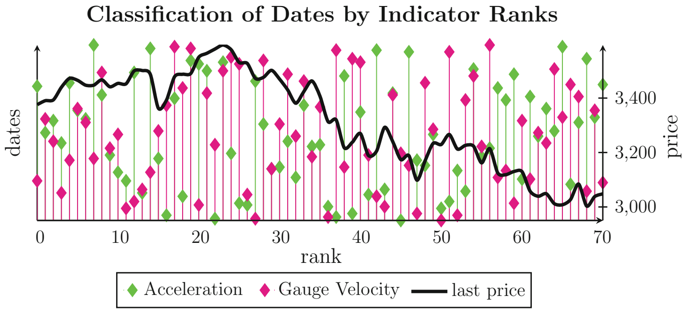
In statistics, “*ranking*” refers to the data transformation in which numerical or ordinal values are replaced by their rank for sorting them (*Milton Friedman*[9] used this procedure in his non-parametric statistical tests). This is a systematic way to perform this task by replacing the incomparable ratings provided by different indicators by the comparable ranks of their images, taking values in the same rank space.

Here, we consider the very special preliminary case when time series are *defined on a same time-interval* (for examples, different indicators on a given time series). Once ranked, the time series take their values in the same vector-space, the dimension of which is the number of dates of the time interval. Once sorted either by rank (in function of dates) or by date (in function of ranks), the homogeneous results are sent as inputs to a time series classifier.

As an illustration, we used this approach for comparing the acceleration of the price and the velocity of the price tube:



Since the ranks are common, we can invert such ranking classification providing for each rank, the dates at which the indicators achieve their ranks.



[9] The non-parametric statistical Friedman test was developed in 1937 by the *Milton Friedman* for detecting differences in treatments of several discrete-time series, which was integrated in many statistical software packages. It is related to the *Durbin* test and the *Kruskal-Wallis* analysis of variance by ranks (see for instance *Rank Correlation Methods*, [125] by *Maurice Kendall*).

Classification by ranks allows us to single-out the dates at which the ranks lie in given classes. For instance, we choose in the following tables to detect the dates at which the three first and last ranks are achieved.

Dates	First	Three	Ranks	Last	Three	Ranks
Acceleration	04/10/2012	21/08/2012	21/09/2012	20/09/2012	20/09/2012	08/11/2012
Gauge velocity	20/08/2012	09/10/2012	01/10/2012	26/10/2012	26/10/2012	28/09/2012

2.6 Detecting Patterns of Evolutions

The question arises to single out dynamical systems regarded as "*pattern generators*": they *govern well identified time series* regarded as *patterns* of interest. For instance, linear or polynomial of fixed degree, piecewise polynomials of fixed degree, exponentials, periodic functions, etc., among the thousands examples studied for many centuries.

The problem is to deliver a differential equation, if any, which provides evolutions viable in a tube, hints at *laws explaining* the evolution they govern, providing more information than pattern recognition mechanisms which may reproduce patterns (such as statistical models, interpolation by spline functions, the VPPI extrapolator, etc.) without providing interpretations of the phenomenon involved, if any.

We may also look at this problem in an *inverse way* by "detecting" the exponential evolutions viable in the "tube" delimited at each date by low and high prices surrounding the evolution of the CAC-40.[10]

A *generator of detectors of patterns* should provide

1. a viable *pattern generator* in a given class of dynamical systems;
2. the *pattern regulator* providing at each time the adequate parameters kept constant as long as the recognition of a pattern is possible (such evolutions are called "heavy", in the sense of heavy trends);
3. the *largest window* on which pattern recognition occurs;
4. the *detected pattern*.

Once detected, the pattern generator and regulator may allow us to explain and reproduce the underlying dynamics concealed in the time series as a *prediction mechanism*. Hence, it is relevant to design generators of detectors which provide

[10] One can take other tubes, such as the tube made of a "snake" of a given (large) "radius" around it. For instance, the radius can be an error or a relative threshold imposed *a priori*. For instance, the *Keltner channels*, introduced in the 1960s by *Chester Keltner* is the tube surrounding a time series of "radius" equal to twice the average of the High, Low and Last Prices which could be used as a tube instead of the price tube.

1. the sequence of impulse or punctuation dates providing the ending date of the largest window over which the time series is recognized by a pattern generated by the pattern generator. Such instants are regarded as "*anomaly dates*";
2. the length or duration of the window between two successive anomaly dates, denominated by their "*cadence*";
3. on each window, the restriction of the time series and its recognizing pattern. The sequence of patterns on the successive windows constitute the "punctuated evolution" generated by the impulse differential inclusions describing the pattern generator.

We provide the examples of detection by second-degree polynomial and exponentials to test whether there patterns consistent with the price tube (Fig. 2.14):

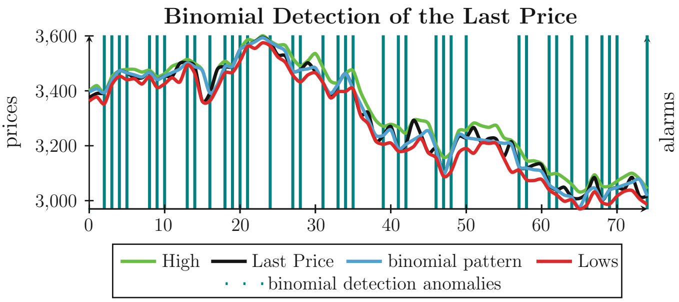


Fig. 2.13 Binomial detection of the last price in the price tube. This figure displays the price tube, the last price and its detection by an second degree polynomial pattern (see Fig. 2.5 for the detection by extrapolation)

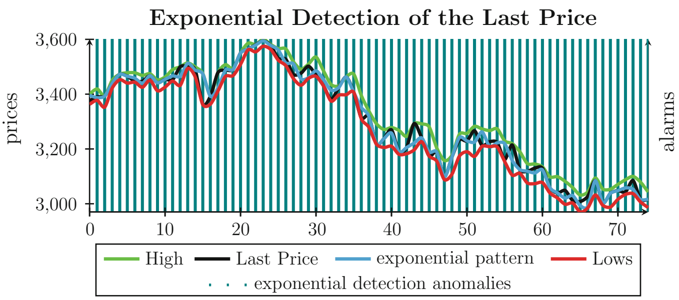


Fig. 2.14 Exponential detection of the last price in the price tube. This figure displays the price tube, the last price and its detection by an exponential pattern. Contrary to the binomial detection (see Fig. 2.13). In this example, *there is never more exponential detection than between two consecutive dates*, so that no geometric model of price evolution is consistent with the observation of the price tube

2.7 Classification of Indicators Used in Technical Analysis

It is time to conclude this short introduction to some chartist and/or technical analysis of time series. The situation becomes complicated since there are many series to study by ... associating with them other time series ... to which we can apply several operators: the jerkiness indicator for detecting dates or trend reversals and their jerkiness, and the congruent periods they delineate, the extrapolated or forecast series, etc.

1. With any series (close, MGI, the portfolio value) are associated
   (a) Dynamic indicators (yields, velocities, accelerations);
   (b) Integral indicators (sum and average between two dates), during congruent periods, for instance;
   (c) Indicators specific to the nature of the series;
2. ***The VIMADES Extrapolator*** which extrapolates
   (a) the series of extrapolations without sliding and its limit, which can be regarded as an "asymptotic index", replacing or complementing standard averages (the extrapolation without sliding of the returns from the current date to the exercise date is used for computing the MGI);
   (b) the series of sliding extrapolations and forecasts of the returns (and derivatives, acceleration, etc.);
   (c) the series of relative errors of the sliding extrapolation
3. With any pair of series (riskless and risky assets), the market alarms, MGI, Value Portfolio, etc.
4. With any tube (price tube, for example):
   (a) Gauge of the tube, the tubes of returns, velocities, accelerations;
   (b) Velocity of gauge, etc.;
5. With any tube and a series therein, Bollinger percent and Bandwidth, the VPPI insurance/performance ratio (see Definition 1.3.1) (which is associated with Bollinger percent of the MGI between the floor and the value of the portfolio);
6. ***The VIMADES Trendometer*** which "sequences" series by computing the trend reversal dates at which extrema are achieved, and thus delineate the congruence period between two consecutive trend reversal dates, and classifies the dates in four classes (trend compass): dates at which a minimum is achieved, a maximum, at which the series is increasing (bear market) and at which it is decreasing (bull market).

(a) At each date, the VIMADES Trendometer combines the values of the series, the reversal dates, the congruence duration, the jerkiness.
(b) Classifies the four dimensional series (value, jerkiness, reversal date, congruence duration) sorted by increasing or decreasing values of the jerkiness, the reversal dates, and congruence duration.

Note that the VIMADES Extrapolator and Trendometer may be applied to each series, and that the trajectories of vectors of indicators regarded as "clouds of data" can be "mined" by data analysis techniques.

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