Preface

This book is divided into two parts, Part I, *Description, Illustration, and Comments of the Results* (Chaps. 1–3), presenting the results obtained without mathematics, which are postponed in Part II, *Mathematical Proofs* (Chaps. 4 and 5).

Chapter 1, *The Viabilist Portfolio Performance and Insurance Approach*, describes in detail the VPPI robot-insurer guaranteeing the hedging of the floor. It defines the data and the conclusions of the Asset-Liability Management problem, proposes a *tychastic viability measure of risk* described by the minimum guaranteed investment (MGI) and, for smaller investments, the duration of the hedging. Knowing the price after the investment date, the VPPI management rule of the VPPI Robot-Insurer computes the number of shares of the risky asset, and thus, the value of the portfolio. Knowing "historical" discrete time series, we can replay the use of the VPPI management rule at each date after investment and measure the performance of the portfolio. Other similar problems are investigated: the *VPPImpulse Robot-Forecaster* assumes that, instead of computing a "Minimum Guaranteed Investment" associated with a forecast mechanism of the lower bounds of risky assets, a "provisonned" value above the floor is given and computes the lower bounds of the risky asset for which the provisonned value allows to hedge the floor. Chapter 2, *Technical and Qualitative Analysis of Tubes* is devoted to the design of a class of forecasting mechanisms of lower bounds of risky returns and the study-related issues. We start from what is provided at each date by the brokerage firms: the price tube, bounded by the High and Low prices, in which the Last Price belongs. The distance between High and Low prices, called the *tychastic gauge* of the price tube (*spread* in financial terminology), is another measure of the polysemous concept of volatility. Its velocity provides an accessible indicator of the evolution of tychastic volatility, as well as velocities and accelerations of the prices that range over the price tube.

Section 2.2, *Forecasting the Price Tube*, deals with the VIMADES extrapolator used in the VPPI robot insurer, for extrapolating both single-valued evolutions and tubes, such as the price tube.

Detecting and/or forecasting the *trend reversals* of evolutions, when markets go from bear to bull and back for example, are the issues of Sect. 2.4. We introduce a "trendometer"

1. sequencing time series by detecting dates at which trend reversal (minima and maxima) emerge delineating congruence periods when the time series increases (as bull markets) or decreasing (as bear market);
2. measuring the shock of the trend reversal by a *jerkiness index*.
   We apply these results to our favorite discrete time series (prices, MGI, Value of the portfolio, market alarms, etc.). Section 2.6 tackles the issue of the detection of generators of patterns recognizing whether a dynamical system (generator) provides evolutions remaining in the price tube around the last price.
   However, the "volatility issue" should not be confused with the question of prediction, dealt with in Sect. 2.2, in which we define the concept of extrapolator and present the example of the VPPI robot-insurer involving the VIMADES Extrapolator. Section 2.3 is devoted to the *sensitivity to tychastic gauges* of the MGI and the value of the portfolio.

Chapter 3, *Uncertainty on Uncertainties*, deals briefly with the mathematical translation of the polysemous concepts of uncertainty. Section 3.1, *Heterodox Approaches*, explains why we do not use the cushion management rules such as the variants of the CPPI, widely known for not hedging the floor for certain evolutions of prices governed by stochastic processes. In the stochastic approach, the MGI is not computed (but, at best, estimated), and there is no regulation rule associating with the *revealed price* of the risky asset, the amount of shares of the portfolio. These were the drawbacks which triggered this VPPI study. Section 3.2, *Forecasting Mechanism Factories*, briefly summarizes other forecasting techniques, statistical methods based on expectations and different measures of deviations such as the (conditional) VaR, fractals, black swans and black duals, trends and fluctuations provided by nonstandard analysis, analytical methods, etc. Section 3.3, *The Legacy of Ingenhousz*, examines different mathematical translations of "uncertainty": stochastic uncertainty, naturally, but also tychastic uncertainty, contingent uncertainty and its redundancy, impulse uncertainty. This section ends with further explanations showing how to correct stochastic viability by tychastic viability because stochastic viability is a (much too) particular case of tychastic viability.

Chapter 4, *Why Viability Theory? A Survival Kit* provides a sketchy summary, rather, a glossary of concepts of viability theory used in this analysis. Why? Because, finance, as well as economics, involve scarcity constraints (on shares), viability constraints (on the agents) and financial or monetary constraints, among many other ones. Optimization under constraints exits since *Lagrange*, having extensively being developed ever since and taught in mathematics, physics, engineering, and economics and finance curricula.[8] Viability Theory is the dynamical counterpart, dealing with *uncertain dynamics under constraints*. It was introduced by *Nagumo* in 1944 and practically ignored until the middle of the 1970s. For uncertain systems under constraints, motivated by economics and biological evolution, the story started at the end of the 1970s in the framework of differential inclusions (the case of stochastic differential equations and inclusions waited to be investigated in the 1990s).

Chapter 5, *Portfolio Insurance in the General Case*, uses these concepts to define the value of the portfolio and the management of shares and their transactions to hedge a floor depending not only on time, but also on the price of assets (as in portfolio replicated options) and the shares. Section 5.1, *Tychastic Viability Portfolio Insurance*, explains how to describe the value of the portfolio in terms of "guaranteed tubular viability kernels of capture basins." This being done, the viability algorithms carry over the computations illustrated in the first chapter. Section 5.2, *Mathematical Metaphors of the VPPI Management Rule*, translates the mathematical properties of viability theory in the context of insurance and regulation of portfolio. They are not useful to compute the insurance and manage the portfolio in a guaranteed way, but they provide mathematical metaphors analogous to the ones we see in the financial literature. The (financial) Greeks pop up, we can derive Hamilton-Jacobi-Bellman partial differential equations governing the evolution of the portfolio, describe the management rules in terms of Greeks, etc. In summary, they tell tales about the portfolio in an esoteric mathematical language.

Section 5.3, *Viability Multipliers to Manage Order Books*, briefly mentions how the theory of "viability multipliers" leads to Hamilton-Jacobi- Bellman partial differential equation providing the "transition time function" needed to conclude a deal of "bid-ask" sizes at "bid-ask" prices, subjected to lower ask constraints and upper bid constraints. This is a capture problem (bid and ask variables are equal) under the above constraints. The "viability multipliers," here the "bid weights" and "ask weights," correcting the dynamics of the order book for providing viable evolutions, are involved in the Hamilton-Jacobi-Bellman equation. They are the missing controls allowing to guide the bid-ask variables towards a deal.

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[8] The idea of optimizing utility functions goes back to 1728 when *Gabriel Cramer*, the discoverer of the Cramer rule in 1750, wrote that *the mathematicians estimate money in proportion to its quantity, and men of good sense in proportion to the usage that they may make of it* in a letter about the Saint-Petersburg paradox raised in the correspondence between *Pierre Rémond de Montmort* and *Nicolas Bernoulli*, patriarch the Bernoulli family, father of *Jean et Jacques Bernoulli* and grandfather of *Daniel Bernoulli* who published *Cramer*'s letter. This was the beginning of the "log saga" since this first utility function was $U(x) = k\log(x/c)$ which find a bright future in the entropy function $E(x) = x\log(1/x)$. The history of maximization of utility functions or mathematical expectation was punctuated by dissident views from *d'Alembert* to *Keynes* and not that so many other authors.

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