Criticality on Rainfall: Statistical Observational Constraints for the Onset of Strong Convection Modelling

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1 Introduction

A better understanding of convection is crucial for reducing the intrinsic errors present in climate models [4]. Many atmospheric processes related to precipitation have large scale correlations in time and space, which are the result of the coupling between several non-linear mechanisms with different temporal and spatial characteristic scales. Despite the diversity of individual rain events, a recent array of statistical measures presents surprising statistical regularities giving support to the hypothesis that atmospheric convection and precipitation may be a real-world example of Self-Organised Criticality (SOC) [2, 16]. The usual approach consists of looking at the occurrence of rain by days or months. For "episodic" rain events, similar to avalanches in cellular-automaton models, scale-free rain event distributions are found [13]. However, a power-law distribution (i.e., scale-free) of the observable is not sufficient evidence for SOC dynamics, as there are many alternative mechanisms that give rise to such behaviour (see for example [7, 11]).

Further support for the SOC hypothesis was given by Peters and Neelin [14], who found a relation between satellite estimates of rain rate and water vapour over the tropical oceans compatible with a phase transition, in which large parts

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of the troposphere would be in a convectively active phase. In addition, it was shown that the system was close to the transition point. They also related it to the concept of atmospheric quasi-equilibrium [1], which argues that, since driven processes are generally slow compared to convection, the system should typically be in a far-from equilibrium statistically stationary state, where driving and dissipation are in balance. In addition, recent works have shown that local event size distributions present signs of universality in the system, as was expected in the SOC framework [5, 6, 12]. The resulting rain event size distributions were found to be well approximated by power laws of similar exponents over broad ranges, with differences in the large-scale cutoffs of the distributions. The possible consequences of this framework for the prediction of atmospheric phenomena still remain unclear.

2 Data and Methods

In this contribution we use very high-resolution (1 min) local rain intensities across different climates described in [12], stochastic convective models [15] and SOC models such as the BTW model and the Manna model, for investigating how predictable the time series of rain activity and rain event sizes are [2, 10].

We use the hazard function H_q as a decision variable, which is sensitive to clustering or repulsion between events in the time series. The conventional precursor pattern technique requires a large amount of data, does not capture long memory and has been found to perform worse than the hazard function in similar analysis [3]. The function H_q is defined as the probability that a threshold-crossing event will occur in the next Δt, conditional on no previous event within the past t_w, i.e.,

$$H_q(t_w; \Delta t) = \frac{\int_{t_w}^{t_w+\Delta t} P_q(\tau)\, \mathrm{d}\tau}{\int_{t_w}^{\infty} P_q(\tau)\, \mathrm{d}\tau}, \qquad (1)$$

where q corresponds to the different thresholds on sizes and Δt is set to 1 min for the rain data and one parallel update for the SOC models. The various quantities are illustrated in Fig. 1.

Note that the hazard function gives us a probabilistic forecast and in order to perform a deterministic prediction we will need to consider a discrimination threshold.

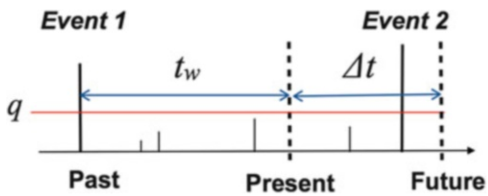

Fig. 1 Sketch of the hazard function variables

We also evaluate the quality of the prediction with the receiver operating characteristics method (ROC) [8]. For any binary prediction (occurrence or non-occurrence of an event) four possible outcomes can occur: true positive (TP), false positive (FP), true negative (TN) and false negative (FN); see Fig. 2.

ROC curves compare *sensitivity* and *specificity*. The *sensitivity* is defined as the number of correctly predicted occurrences divided by the total number of actual occurrences, and the *specificity* as the number of correctly predicted non-occurrences divided by the total number of actual non-occurrences:

$$\mathit{sensitivity} = \frac{\mathrm{TP}}{\mathrm{TP}+\mathrm{FN}}, \qquad \mathit{specificity} = \frac{\mathrm{TN}}{\mathrm{FP}+\mathrm{TN}}. \tag{2}$$

Each threshold on the decision variable will give a different point on the ROC curve. If we consider the minimum possible threshold we will always predict the occurrence of an event, for which the *sensitivity* is one and the *specificity* zero. The diagonal in Fig. 3 corresponds to random prediction. Points above the diagonal represent good predictions (better than random) and points below poor predictions.

		Actual value	
		Positive	**Negative**
Predicted value	**Positive**	True Positive (TP)	False Positive (FP)
	Negative	False Negative (FN)	True Negative (TN)

Fig. 2 Four possible outcomes of a binary prediction in a contingency table

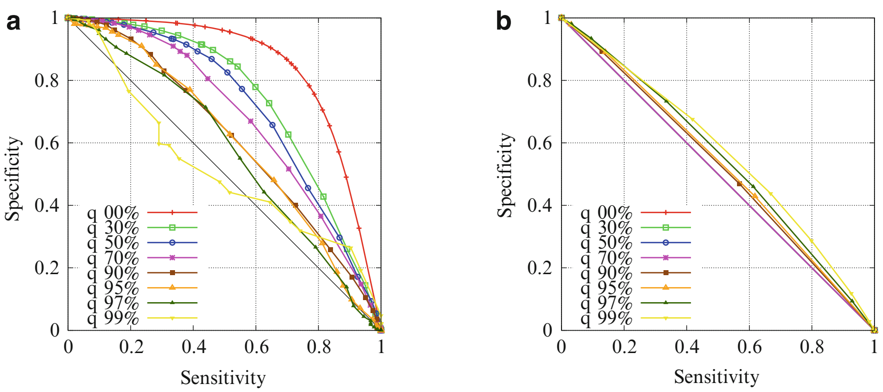

Fig. 3 Example of ROC curves data on the slow time scale for rainfall data (**a**) and for the 2D BTW SOC model simulated data (**b**)

3 Results

We find that on the events scale (slow time scale), rain data renormalise to a trivial Poisson point process for large thresholds, while for small thresholds events cluster. This is in contrast to the anti-clustering of high-threshold events in the 2D BTW model as a result of finite-size effects and the building up of correlations, seen previously by Garber et al. [9]; see Fig. 3a, b.

However, rain data has an unavoidable threshold on intensity due to the device resolution that blurs the interpretation of the results on the event scale. At the level of intensities (slow time scale), we find that prediction is insensitive to all but very high thresholds.

References

1. A. Arakawa, W.H. Schubert, Interaction of a cumulus cloud ensemble with the large-scale environment, Part I. J. Atmos. Sci. **31**(3), 674–701 (1974)
2. P. Bak, C. Tang, K. Wiesenfeld, Self-organized criticality: An Explanation of 1/f noise. Phys. Rev. Lett. **59**(4), 381–384 (1987)
3. M.I. Bogachev, I.S. Kireenkov, E.M. Nifontov, A. Bunde, Statistics of return intervals between long heartbeat intervals and their usability for online prediction of disorders. New J. Phys. **11**, 1–19 (2009)
4. Core Writing Team, R.K. Pachauri, A. Reisinger (eds.), *Fourth Assessment Report of the Intergovernmental Panel on Climate Change* (IPCC, Geneva), p. 104 (2007)
5. A. Deluca, A. Corral, Scale invariant events and dry spells for medium resolution local rain data (submitted)
6. A. Deluca, P. Puig, A. Corral, Testing universality in critical exponents: the case of rainfall (in preparation)
7. R. Dickman, Rain, power laws, and advection. Phys. Rev. Lett. **90**(10), 1–4 (2003)
8. J.P. Egan, *Signal Detection Theory and ROC Analysis* (Academic, New York, 1975)
9. A. Garber, S. Hallerberg, H. Kantz, Predicting extreme avalanches in self-organized critical sandpiles. Phys. Rev. E **80**, 1–5 (2009)
10. S.S. Manna, Two-state model of self-organized criticality. J. Phys. A: Math. Gen. **24**(8), 363–369 (1991)
11. M. Mitzenmacher, A brief history of generative models for power law and lognormal distributions. Internet Math. **1**(2), 226–251 (2004)
12. O. Peters, A. Deluca, A. Corral, J.D. Neelin, C.E. Holloway, Universality of rain event size distributions. J. Stat. Mech. P11030 (2010). http://iopscience.iop.org/1742-5468/2010/11/P11030/fulltext/
13. O. Peters, C. Hertlein, K. Christensen, A complexity view of rainfall. Phys. Rev. Lett. **88**(1), 1–4 (2002)
14. O. Peters, J.D. Neelin, Critical phenomena in atmospheric precipitation. Nat. Phys. **2**(6), 393–396 (2006)
15. S.N. Stechmann, J.D. Neelin, A stochastic model for the transition to strong convection. J. Atmos. Sci. **68**, 2955–2970
16. C. Tang, P. Bak, Critical exponents and scaling relations for self-organized critical phenomena. Phys. Rev. Lett. **60**(23), 2347–2350 (1988)

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