

Preface

This is a monograph intended for people interested in a more complete understanding of the relation between quantum and classical mechanics, here explored in the particular context of spin systems, that is, $SU(2)$ -symmetric mechanical systems. As such, this book is aimed both at mathematicians with interest in dynamics and quantum theory and physicists with a mathematically oriented mind.

The mathematical prerequisites for reading this monograph are mostly elementary. On the one hand, some knowledge of Lie groups and their representations can be helpful, but since the group considered here is $SU(2)$, with its finite dimensional representations, not much more than lower-division college mathematics is actually required. In this sense, Chaps. 2 and 3 can almost be used as a particular introduction to some aspects of Lie group representation theory, for students of mathematical sciences. On the other hand, some knowledge of symplectic differential geometry can be helpful, but again, since the symplectic manifold considered here is the two-sphere with its standard area form, some of Chap. 4 can be seen as a concise presentation of the concepts of Hamiltonian vector field and Poisson algebra. As the remaining chapters of the book are built on these first three, the very diligent upper-division undergraduate student in mathematics, physics or engineering, with sufficient knowledge of calculus and linear algebra already acquired, should not find many serious difficulties in reading throughout most of this book.

Motivation is another story, however, and this monograph may not appeal to those not yet somewhat familiar with standard quantum mechanics and classical Hamiltonian mechanics. Rather, in fact, for those who have already taken a regular course in quantum mechanics with its standard treatment of angular momentum, some of the contents in Chap. 3 will look familiar, although most likely they will find the material covered in this chapter more precisely and explicitly treated than in other available texts. In particular, our detailed presentation of the rotationally invariant decomposition of the operator algebra of spin systems is not so easily found elsewhere. Similarly, a person with minimal knowledge of symplectic geometry and Hamiltonian dynamics may also find Chap. 4 too straightforward,

and perhaps somewhat amusing when he looks at the detailed presentation of the rotationally invariant decomposition of the Poisson algebra on the two-sphere, but again, we were not able to find this last part in any other book.

Thus, this book is mainly addressing a person – student or researcher – who has pondered on the question of whether or how quantum (Heisenberg) dynamics and classical Hamiltonian dynamics (also called Poisson dynamics) can be precisely related. However, since this rather general question has already been asked and studied in various ways in many books and research papers, it is important to clarify how this question is addressed here and what is actually new in this book.

To this end, we must first emphasize that most ways of studying the above question are of two major types: the first one, proceeding from quantum dynamics to Poisson dynamics, and the second, proceeding in reverse order from Poisson dynamics to quantum dynamics, the latter also widely known as the process of “quantization” of a Poisson manifold. This monograph proceeds in the first way, which in its own turn can be approached according to two distinct methodologies: one which studies the so-called semiclassical limit directly from the quantum formalism, and another and more detailed approach, which first translates the quantum formalism to the classical formalism, in a process also referred to as “dequantization”, and then studies the semiclassical limit in the dequantized setting. This more detailed methodology is the one we shall follow in the present monograph, albeit here we are limiting ourselves to the context of spin systems.

The advantage of working in the context of spin systems for this purpose is twofold: on the one hand, all quantum spaces are finite dimensional, which greatly simplifies the quantum formalism, and, on the other hand, since each process of dequantization depends on a choice of symbol correspondence, it is important to classify and make explicit all such possible choices, which is easier in the finite dimensional context, and this is fully done for the first time in Chap. 6.

The comparison of all such choices of symbol correspondences is continued in Chap. 7, which presents a detailed study of spherical symbol products, and Chap. 8, which starts the study of their asymptotic limit of high spin numbers. Together, Chaps. 6–8 of this book present for the first time in the literature a systematic study of general symbol correspondences for spin systems, with their symbol products and asymptotic limit of high spin numbers. In so doing, as far as we know these chapters also present the first systematic study, in books as well as in research papers, of how classical mechanics may or may not emerge as an asymptotic limit of quantum mechanics, in a rather simple and precise context.

Symbol Correspondences for Spin Systems

de M. Rios, P.; Straume, E.

2014, IX, 200 p., Hardcover

ISBN: 978-3-319-08197-7

A product of Birkhäuser Basel