# Preface

This book is based on a one-semester course taught since 2002 at Instituto Superior Técnico (Lisbon) to mathematics, physics and engineering students. Its aim is to provide a quick introduction to differential geometry, including differential forms, followed by the main ideas of Riemannian geometry (minimizing properties of geodesics, completeness and curvature). Possible applications are given in the final two chapters, which have themselves been independently used for one-semester courses on geometric mechanics and general relativity. We hope that these will give mathematics students a chance to appreciate the usefulness of Riemannian geometry, and physics and engineering students an extra motivation to learn the mathematical background.

It is assumed that readers have basic knowledge of linear algebra, multivariable calculus and differential equations, as well as elementary notions of topology and algebra. For their convenience (especially physics and engineering students), we have summarized the main definitions and results from this background material at the end of each chapter as needed.

To help readers test and consolidate their understanding, and also to introduce important ideas and examples not treated in the main text, we have included more than 330 exercises, of which around 140 are solved in Chap. 7 (the solutions to the full set are available for instructors). We hope that this will make this book suitable for self-study, while retaining a sufficient number of unsolved exercises to pose a challenge.

We now give a short description of the contents of each chapter.

Chapter 1 discusses the basic concepts of differential geometry: differentiable manifolds and maps, vector fields and the Lie bracket. In addition, we give a brief overview of Lie groups and Lie group actions.

Chapter 2 is devoted to differential forms, covering the standard topics: wedge product, pull-back, exterior derivative, integration and the Stokes theorem.

Riemannian manifolds are introduced in Chap. 3, where we treat the Levi–Civita connection, minimizing properties of geodesics and the Hopf–Rinow theorem.

Chapter 4 addresses the notion of curvature. In particular, we use the powerful computational method given by the Cartan structure equations to prove the Gauss–Bonnet theorem. Constant curvature and isometric embeddings are also discussed.

Chapter 5 gives an overview of geometric mechanics, including holonomic and non-holonomic systems, Lagrangian and Hamiltonian mechanics, completely integrable systems and reduction.

Chapter 6 treats general relativity, starting with a geometric introduction to special relativity. The Einstein equation is motivated via the Cartan connection formulation of Newtonian gravity, and the basic examples of the Schwarzschild solution (including black holes) and cosmology are studied. We conclude with a discussion of causality and the celebrated Hawking and Penrose singularity theorems, which, although unusual in introductory texts, are very interesting applications of Riemannian geometry.

Finally, we want to thank the many colleagues and students who read this text, or parts of it, for their valuable comments and suggestions. Special thanks are due to our colleague and friend Pedro Girão.

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