Chapter 2
Analytical Methods for Analysis of Integral Cellular Networks

The classical theory of multidimensional queueing systems is based on many assumptions, one of them is basic: one call—one channel. However, in modern integral (multiservice) communication networks, this assumption is not carried out. Thus, in them, for example, a video information requires the wider band in a digital transmission line than data or voice information. In the teletraffic theory the calls, requiring a large number of channels in a transmission line, are called wideband, and the calls that require a smaller number of channels—narrowband. As it was noted in the previous chapter, the multi-flow system, in which heterogeneous calls require for simultaneous maintenance a random number of channels, is called multi-rate queue (MRQ).

Since in MRQ with inelastic calls in the absence of the required number of free channels call service cannot be started, one would expect that wideband calls will be lost more often than narrowband. Therefore, in such systems to maintain the quality of service (QoS) of heterogeneous calls at the desired level, it is necessary to determine the appropriate CAC scheme.

The alternative way for satisfying the desired QoS level is determining the appropriate schemes to partition of common pool of channels among heterogeneous calls.

In this chapter the new method to study the MRQ model with the randomized access scheme is proposed, and on its basis the efficient algorithms to calculate the QoS metrics in specific telecommunication network models are developed. In addition both isolated and virtual schemes to partition of common pool of channels in integral telecommunication networks are proposed, and exact formulas to calculate the QoS metrics of such networks are obtained. The results of numerical experiments, performed with the help of the developed algorithms, and the meaningful analysis of these results are given.

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2.1 Model of Multi-rate Queue with Randomized Access Scheme

Let the input of the multichannel system, containing $N > 1$ channels, receive a Poisson stream of heterogeneous calls with the total intensity Λ. Every new incoming call with the probability σ_i requires for service simultaneously b_i channels, $1 \le b_i \le N$, $i = 1, \ldots, K$; meanwhile $\sigma_1 + \ldots + \sigma_k = 1$. It is believed that at the moment of call arrival the number of channels, requested by it for service, is known. Then, subject to the known properties of the Poisson flow, one can argue that at the input of N channel system K types of the Poisson streams of calls enter, the intensity of the ith stream being $\lambda_i = \Lambda\sigma_i$, $i = 1, \ldots, K$; meanwhile the calls of the ith type (i-calls) require simultaneously b_i channels, $i = 1, \ldots, K$. The holding time of the i-calls is a random variable subjected to an exponentially distribution law with the parameter μ_i, $i = 1, \ldots, K$.

The system uses a randomized scheme for access. For this, the access matrix of the dimension $K \times N$ is determined, in which elements define the rules of reception of heterogeneous calls, depending on their type and the number of busy channels. More precisely, the element $\alpha_i(n)$ of the matrix indicates the probability of reception of the i-call for service, if at the time of its arrival the number of busy channels equals n; with the complementary probability $1 - \alpha_i(n)$, this call is lost. In this model interruption of service process of the call of any type is not allowed, i.e., it is assumed that $\alpha_i(n) = 0$ for any $i = 1, \ldots, K$, if $n > N - b_i$. Note that the condition $b_i = b_j$ at $i \ne j$ not in the least means that the equality $\alpha_i(n) = \alpha_j(n)$ is to be fulfilled.

Let us consider the problem of determining the QoS metrics of the studied model while using the proposed access scheme. The main QoS metrics are stationary loss (blocking) probability of i-calls (PB_i, $i = 1, \ldots, K$) and the average number of the busy channels (N_{av}).

The state of the system at arbitrary time instant is described by the K-dimensional vector $\boldsymbol{m} = (m_1, \ldots, m_K)$, where m_i is a number of the i-calls in the system (i.e., in the channels), $i = 1, \ldots, K$. In other words, the functioning of the given MRQ is described by K-dimensional Markov chain with the following state space:

$$S = \{\boldsymbol{m} : \; m_i = 0, 1, \ldots, [N/b_i]; \; (\boldsymbol{m}, \boldsymbol{b}) \le N\}, \tag{2.1}$$

where $\boldsymbol{b} = (b_1, \ldots, b_k)$; $[x]$ is an integer part of x; $(\boldsymbol{m}, \boldsymbol{b})$ is a scalar product of the vectors $\boldsymbol{m}$ and $\boldsymbol{b}$.

Note that from this scheme one can obtain in particular cases the well-known deterministic access schemes. Let us consider some of them:

1. If $\alpha_i(n) = 1$ for every $i = 1, \ldots, K$ at $n \le N - b_i$, then one gets the model with full available group of channels, i.e., model with the complete sharing (CS) scheme [13, 29].
2. Assume that the parameters $\alpha_i(n)$ for every $i = 1, \ldots, K$ are defined as follows:

$$\alpha_i(n) = \begin{cases} 1 & \text{if } n \le N - b, \\ 0 & \text{in other cases,} \end{cases} \tag{2.2}$$

where $b = \max\{b_i : i = 1, \ldots, K\}$. Then one gets the complete sharing with equalization (CSE) scheme [7], i.e., the received call of any type is served, if at this moment the number of free channels is not less than b.

3. Assume that the parameters $\alpha_i(n)$ for every $i = 1, \ldots, K$ are defined as follows:

$$\alpha_i(n) = \begin{cases} 1 & \text{if } n \le N - b_i - r_i, \\ 0 & \text{in other cases,} \end{cases} \tag{2.3}$$

where $0 \le r_i \le N - b_i$. Then one gets the trunk reservation (TR) scheme [28], i.e., if at the time of receiving the i-call the number of free channels is not less than $b_i + r_i$, then it is served; otherwise the received call is lost with probability 1. The parameter r_i is called the backup parameter of the channels for i-calls, $i = 1, \ldots, K$.

Let us state the proposed method of solving the problem. Transitions between the states $\boldsymbol{m}$ and $\boldsymbol{m}' \in S$ occur only at the moment of receiving calls and their leaving the system after completion of service. In view of this, the nonnegative elements of the Q-matrix of the given multidimensional Markov chain are determined from the following relationships:

$$q(\boldsymbol{m}, \boldsymbol{m}') = \begin{cases} \lambda_i \alpha_i((\boldsymbol{m}, \boldsymbol{b})) & \text{if } \boldsymbol{m}' = \boldsymbol{m} + \boldsymbol{e}_i, \\ m_i \mu_i & \text{if } \boldsymbol{m}' = \boldsymbol{m} - \boldsymbol{e}_i, \\ 0 & \text{in other cases,} \end{cases} \tag{2.4}$$

where $\boldsymbol{m}, \boldsymbol{m}' \in S$, $\boldsymbol{e}_i$ is the ith orthogonal vector in K-dimensional Euclidean space, $i = 1, \ldots, K$.

For any positive values of the parameters of incoming traffics, all the states are communicating and, consequently, the system is ergodic. Let us denote the stationary probability of state $\boldsymbol{m} \in S$ as $p(\boldsymbol{m})$. The desired QoS metrics are determined in terms of the steady-state probabilities. Thus the stationary probability of blocking the i-calls is calculated as follows:

$$\mathrm{PB}_i = \sum_{n=0}^{N} (1 - \alpha_i(n)) \sum_{\boldsymbol{m} \in S_n} p(\boldsymbol{m}), \quad i = 1, \ldots, K, \tag{2.5}$$

where $S_n = \{\boldsymbol{m} \in S : (\boldsymbol{m}, \boldsymbol{b}) = n\}$, $n = 0, 1, \ldots, N$, i.e., the sets S_n combine the microstates from state space (2.1) with the same number of busy channels.

Note 2.1 From formula (2.5), one gets that if $\alpha_i(n) = \alpha_j(n)$ at $b_i = b_j$ for some i, j, $i \ne j$, then $\mathrm{PB}_i = \mathrm{PB}_j$, for any values of the model load parameters.

The average number of busy channels is defined as

$$N_{\mathrm{av}} = \sum_{n=1}^{N} n \sum_{\boldsymbol{m} \in S_n} p(\boldsymbol{m}). \tag{2.6}$$

The main problem in finding the QoS metrics (2.5) and (2.6) is the calculation of $p(\boldsymbol{m})$, $\boldsymbol{m} \in S$, which satisfies the corresponding system of global balance equations (SGBE):

$$\begin{aligned}
&\left(\sum_{i=1}^{K} \lambda_i \alpha_i((\boldsymbol{m}, \boldsymbol{b})) I((\boldsymbol{m}, \boldsymbol{b}) \le N - b_i) + \sum_{i=1}^{K} m_i \mu_i\right) p(\boldsymbol{m}) \\
&= \sum_{i=1}^{K} \lambda_i \alpha_i((\boldsymbol{m} - \boldsymbol{e}_i, \boldsymbol{b})) p(\boldsymbol{m} - \boldsymbol{e}_i) I(m_i > 0) \\
&\quad + \sum_{i=1}^{K} (m_i + 1) p(\boldsymbol{m} + \boldsymbol{e}_i) I(\boldsymbol{m} + \boldsymbol{e}_i \in S),
\end{aligned} \tag{2.7}$$

$$\sum_{\boldsymbol{m} \in S} p(\boldsymbol{m}) = 1. \tag{2.8}$$

The given SGBE has no explicit solution, and this fact complicates the solution of the considered problem for large state space dimensions (2.1).

In this regard, we propose another approach based on the use of the fact that the QoS metrics (2.5) and (2.6) are defined in terms of the probabilities of the merged states S_n, $n = 0, 1, \ldots, N$. Since the sets S_n, $n = 0, 1, \ldots, N$ define some splitting of the state space (2.1), then the desired QoS metrics can be calculated using the probabilities of merged states. Indeed, it is clear that the probabilities of the merged states are defined as follows:

$$\pi(n) = \sum_{\boldsymbol{m} \in S_n} p(\boldsymbol{m}), \qquad n = 0, 1, \ldots, N. \tag{2.9}$$

It is obvious that (see normalizing condition (2.8))

$$\sum_{n=0}^{N} \pi(n) = 1. \tag{2.10}$$

Consequently, taking into account (2.5), (2.6), (2.9), and (2.10), one finds that

$$\mathrm{PB}_i = \sum_{n=0}^{N} (1 - \alpha_i(n)) \pi(n), \quad i = 1, \ldots, K, \tag{2.11}$$

$$N_{\mathrm{av}} = \sum_{n=1}^{N} n\pi(n). \tag{2.12}$$

Thus, without determining the stationary distribution of the original (initial) model, one can calculate the QoS metrics (2.5) and (2.6), if it is possible to determine the values of the probabilities of merged states $\pi(n)$, $n = 0, 1, \ldots, N$. With the help of the following statement, one can solve this problem.

Proposition 2.1 If $\mu_i = \mu_j$, $i, j = 1, \ldots, K$, then QoS metrics (2.5) and (2.6) are defined as follows:

$$\mathrm{PB}_i = \left(\sum_{n=0}^{N} (1 - \alpha_i(n))g_n\right) \Big/ \left(\sum_{n=0}^{N} g_n\right), \quad i = 1, \ldots, K, \tag{2.13}$$

$$N_{\mathrm{av}} = \left(\sum_{n=1}^{N} n g_n\right) \Big/ \left(\sum_{n=0}^{N} g_n\right). \tag{2.14}$$

Henceforward, the following notations are used: $v_i = \lambda_i/\mu_i$, $i = 1, \ldots, K$;

$$\tilde{v}_\iota(n-i) = \sum_{j \in A(i)} v_j \alpha_j(n-i), \tag{2.15}$$

where $A(i) = \{j : j\text{-calls demand } i \text{ channels}\}$, $i = 1, \ldots, N$;

$$g_n = \begin{cases} 1, & n = 0, \\ \dfrac{1}{n}\displaystyle\sum_{i=1}^{n} i\tilde{v}_\iota(n-i)g_{n-i}, & n = 1, \ldots, N. \end{cases} \tag{2.16}$$

To prove Proposition 2.1, let us first prove the following fact.

Proposition 2.2 If $\mu_i = \mu_j$, $i, j = 1, \ldots, K$, then probabilities of merged states are defined as

$$\pi(n) = g_n \pi(0), \quad n = 0, 1, \ldots, N, \tag{2.17}$$

where $\pi(0) = \left(\sum_{n=0}^{N} g_n\right)^{-1}$.

Note 2.2 In special case, if $\alpha_i(n) = \alpha_j(n)$ for $b_i = b_j$, from Eqs. (2.15) and (2.16) one finds that the parameters g_n, $n = 0, 1, \ldots, N$ are determined as follows:

$$g_n = \begin{cases} 1, & n = 0 \\ \dfrac{1}{n}\sum_{i=1}^{n} i\widetilde{v}_\imath \alpha_i(n-i) g_{n-i}, & n = 1, \ldots, N, \end{cases}$$

where $\widetilde{v}_\imath = \begin{cases} \sum_{j \in A(i)} v_j & \text{if } A(i) \neq \varnothing, \\ 0 & \text{if } A(i) = \varnothing. \end{cases}$

Proposition 2.2 is a direct consequence of a following one.

Proposition 2.3 If $\mu_i = \mu_j$, $i, j = 1, \ldots, K$, then the following equalities hold true:

$$\sum_{i=1}^{K} v_i b_i \alpha_i (n - b_i) \pi(n - b_i) = n\pi(n), \quad n = 1, \ldots, N, \tag{2.18}$$

where $\pi(x) = 0$ for $x < 0$.

Proof of Proposition 2.3 For simplicity, we present the proof of this fact for a single-rate model, i.e., for a model in which $b_i = 1$ for all $i = 1, \ldots, K$. Generalization for a multi-rate model is straightforward.

Let us use the scheme proposed in [13]. Taking into account relationship (2.4), one obtains that the SGBE for the states $\boldsymbol{m} \in S_{n-1}$ has the following form:

$$\left(\sum_{i=1}^{K} \lambda_i \alpha_i (n-1) + \sum_{i=1}^{K} m_i \mu_i \right) p(\boldsymbol{m}) = \sum_{i=1}^{K} \lambda_i \alpha_i (n-2) p(\boldsymbol{m} - \boldsymbol{e}_i)$$
$$+ \sum_{i=1}^{K} (m_i + 1) \mu_i p(\boldsymbol{m} + \boldsymbol{e}_i). \tag{2.19}$$

For simplicity, it is assumed that the states $\boldsymbol{m}$, $\boldsymbol{m} - \boldsymbol{e}_i$, $\boldsymbol{m} + \boldsymbol{e}_i$ participating in Eq. (2.19) are in state space (2.1); otherwise the corresponding members are zeroed.

Summing both sides of Eq. (2.19) over all possible $\boldsymbol{m} \in S_{n-1}$, after collecting similar terms and taking into account structure of SGBE, one gets

$$\sum_{i=1}^{K} \lambda_i \alpha_i (n-1) \sum_{\boldsymbol{m} \in S_{n-1}} p(\boldsymbol{m}) = \sum_{\boldsymbol{m} \in S_n} m_i \mu_i p(\boldsymbol{m}). \tag{2.20}$$

In latter transformations, while rearranging the terms in the sum, the two facts essentially have been taken into account: relationship (2.9) as well as the following fact: for all states $\boldsymbol{m} \in S_n$, $n = 1, \ldots, N$ the value $\sum_{j=1}^{K} v_j \alpha_j(n)$ is the same.

Taking into account Eq. (2.9), Eq. (2.20) might be rewritten as follows:

$$\pi(n-1)\sum_{i=1}^{K}\lambda_i\alpha_i(n-1) = \sum_{\boldsymbol{m}\in S_n} m_i\mu_i p(\boldsymbol{m}). \tag{2.21}$$

From Eq. (2.21) for $\mu_i = \mu_j$, $i, j = 1, \ldots, K$, we have

$$\pi(n-1)\sum_{i=1}^{K}v_i\alpha_i(n-1) = \sum_{\boldsymbol{m}\in S_n} m_i p(\boldsymbol{m}). \tag{2.22}$$

The right side of Eq. (2.22) can be represented as follows:

$$\sum_{\boldsymbol{m}\in S_n} m_i p(\boldsymbol{m}) = \sum_{\boldsymbol{m}\in S_n} m_i \frac{p(\boldsymbol{m})}{\pi(n)}\pi(n). \tag{2.23}$$

From the definition of the conditional probability, one has

$$P(\boldsymbol{m}|n) = P\left(\boldsymbol{m}\middle|\sum_{i=1}^{K} m_i = n\right) = \begin{cases} \dfrac{p(\boldsymbol{m})}{\pi(n)} & \text{if } \boldsymbol{m}\in S_n \\ 0 & \text{in other cases,} \end{cases} \tag{2.24}$$

where $P(\cdot\,|\,\cdot)$ is a sign of conditional probability.

Then from Eq. (2.23), taking into account Eq. (2.24), one obtains

$$\begin{aligned} \sum_{i=1}^{K}\sum_{m\in S_n} m_i p(\boldsymbol{m}) &= \sum_{i=1}^{K}\left(\sum_{\boldsymbol{m}\in S_n} m_i P(\boldsymbol{m}|n)\right)\pi(n) \\ &= \sum_{i=1}^{K} E(m_i|n)\pi(n) = E\left(\sum_{i=1}^{K} m_i\middle| n\right)\pi(n) = n\pi(n), \end{aligned} \tag{2.25}$$

where $E(\cdot\,|\,\cdot)$ is a sign of conditional expectation.

Consequently, taking into account (2.22) and (2.25), one concludes that relationships (2.18) are valid for single-rate model. As it has been noted above, the generalization of this proof for a multi-rate model is straightforward.

Now one can prove *Proposition* 2.2. Indeed, after some algebraic transformations one finds that the system of equations (2.18), taking into account the normalization condition (2.10), has the following augmented matrix:

$$\begin{pmatrix} \widetilde{v}_1(0) & -1 & 0 & \dots & 0 & 0 & 0 \\ 2\widetilde{v}_2(0) & \widetilde{v}_1(1) & -2 & \dots & 0 & 0 & 0 \\ . & . & . & . & . & . & . \\ N\widetilde{v}_N(0) & (N-1)\widetilde{v}_{N-1}(1) & (N-2)\widetilde{v}_{N-2}(2) & \dots & \widetilde{v}_1(N-1) & -N & 0 \\ 1 & 1 & 1 & \dots & 1 & 1 & 1 \end{pmatrix}$$

Hence, one finds that the stationary probabilities of merged states, while using randomized access scheme, are determined from Eq. (2.17).

Consequently, taking into account (2.11) and (2.12), one finds that QoS metrics of model (2.5) and (2.6) are calculated from relations (2.13) and (2.14). In other words, *Proposition* 2.1 is proved.

An important advantage of this algorithm is that its computational complexity does not depend on the total number of types of calls (i.e., on K) and is estimated as $O(N)$. Such invariance is achieved through merging of flows by the number of required channels (see the definition of the sets $A(n)$, $n = 1, \ldots, N$).

Note that in special cases we exactly obtain results from the above-indicated well-known access schemes based on CS, CSE, and TR schemes (see *Propositions* 1.3, 1.4, and 1.5 in Sect. 1.1).

To calculate the QoS metrics of the system, the computational procedure, described above, can be used even in cases where the service intensities of heterogeneous calls are unessentially different from one another. And in the cases, where the service intensities of heterogeneous calls are essentially different, one can use different schemes of "unification" ("averaging") of their values. So, from a practical standpoint, the use of the following three general values is most interesting:

(1) $\mu = \max\{\mu_1, \ldots, \mu_K\}$; (2) $\mu = \min\{\mu_1, \ldots, \mu_K\}$; (3) $\mu = \frac{1}{\Lambda}\sum_{i=1}^{K} v_i$, where $\Lambda = \sum_{i=1}^{K} \lambda_i$.

Note that for every "averaging" scheme (this and others), the accuracy of the used approximations can be studied numerically, since analytical solution does not exist. For the models of small dimension, the exact solution can be found from SGBE.

2.1.1 Model of Integral Wireless Network with Multi-parametric Access Scheme

The multi-rate queueing model in the problems of calculating the QoS metrics of the integral wireless communication network of a cellular structure was used in [25]. In mentioned work, four types of calls are considered: handover voice calls (hv-calls), new voice calls (ov-calls), handover data calls (hd-calls), and new data calls (od-calls). The network uses a fixed channel allocation scheme (FCA scheme), and every cell has $N > 1$ radio channels. The intensity of the x-calls is λ_x, $x \in \{\text{hv}, \text{ov}, \text{hd}, \text{od}\}$.

To service one voice call (v-call), it is required only one free channel, and one nonelastic data call (d-call) requires $b > 1$ channels simultaneously. The distribution functions of channels holding time by heterogeneous calls are exponential, the average intensity of processing one v-call (new or handover) is μ_v, and the corresponding parameter for d-calls (new or handover) is μ_d.

In the system the following multi-parametric access scheme is used (see [25]). For defining the success scheme, three parameters N_1, N_2, and N_3 are introduced. It is assumed that parameters N_1 and N_2 are multiples of b. These parameters satisfy the inequality $0 < N_1 \le N_2 \le N_3 \le N$. The proposed access scheme is determined by the following rules of receiving heterogeneous calls:

- If upon arrival of an od-call the number of busy channels is no more than $N_1 - b$, it is served; otherwise, it is rejected.
- If upon arrival of an hd-call the number of busy channels is no more than $N_2 - b$, it is served; otherwise, it is rejected.
- If upon arrival of an ov-call the number of busy channels is less than N_3, it is served; otherwise, it is rejected.
- If upon arrival of an hv-call there is at least one free channel, it is served; otherwise, it is rejected.

To calculate the QoS metrics of the pointed model in [25], a recursive method is developed. This method faces the well-known computational difficulties for models of a large dimension. The approximate method of solving this problem for certain ratios of loads of heterogeneous traffics is developed in [16]. Here we develop an accurate and computationally efficient method to solving this problem [17, 23].

It is easy to see that the given model of an integral network with the multi-parametric access scheme is a special case of the model, studied in Sect. 2.1, with a randomized access scheme. Indeed, we obtain the given model if in the studied above model one will set $K = 4$ and the parameters $\alpha_i(n)$ will be determined as follows:

$$\alpha_{od} = \begin{cases} 1 & \text{if } n \le N_1 - b, \\ 0 & \text{in other cases;} \end{cases} \tag{2.26}$$

$$\alpha_{hd} = \begin{cases} 1 & \text{if } n \le N_2 - b, \\ 0 & \text{in other cases;} \end{cases} \tag{2.27}$$

$$\alpha_{ov} = \begin{cases} 1 & \text{if } n < N_3, \\ 0 & \text{in other cases;} \end{cases} \tag{2.28}$$

$$\alpha_{hv} = \begin{cases} 1 & \text{if } n < N, \\ 0 & \text{in other cases.} \end{cases} \tag{2.29}$$

In view of Eqs. (2.26)–(2.29), one finds that the sets $A(i)$, $i = 1, \ldots, N$ (see formula (2.15)) are defined as follows:

$$A(i) = \begin{cases} \{\mathrm{ov}, \mathrm{hv}\} & \text{if } i = 1, \\ \{\mathrm{od}, \mathrm{hd}\} & \text{if } i = b, \\ \varnothing & \text{in other cases.} \end{cases}$$

After certain transformations from Eqs. (2.15), (2.16), one gets that in this model for $\mu_{\mathrm{v}} = \mu_{\mathrm{d}}$ the parameters g_n, $n = 0, 1, \ldots, N$ are determined from the following simple formulas:

$$g_0 = 1,$$

$$g_n = \frac{1}{n}((v_{\mathrm{v}} I(n-1 < N_3) + v_{\mathrm{hv}} I(n-1 \geq N_3))g_{n-1} + b(v_{\mathrm{d}} I(n \leq N_1)$$

$$+ v_{\mathrm{hd}} I(N_1 < n \leq N_2))g_{n-b}),$$

where $n = 1, \ldots, N$ and $g_x = 0$ if $x < 0$. Here the following notation is taken: $v_{\mathrm{ov}} = \lambda_{\mathrm{ov}}/\mu_{\mathrm{v}}$, $v_{\mathrm{hv}} = \lambda_{\mathrm{hv}}/\mu_{\mathrm{v}}$, $v_{\mathrm{v}} = v_{\mathrm{ov}} + v_{\mathrm{hv}}$; $v_{\mathrm{od}} = \lambda_{\mathrm{od}}/\mu_{\mathrm{d}}$, $v_{\mathrm{hd}} = \lambda_{\mathrm{hd}}/\mu_{\mathrm{d}}$, $v_{\mathrm{d}} = v_{\mathrm{od}} + v_{\mathrm{hd}}$.

Consequently, the desired QoS metrics are calculated as follows:

$$\mathrm{PB}_{\mathrm{hv}} = \pi(N); \mathrm{PB}_{\mathrm{ov}} = \sum_{n=N_3}^{N} \pi(n); \mathrm{PB}_{\mathrm{od}}$$

$$= \sum_{n=N_1-b+1}^{N} \pi(n);\ \mathrm{PB}_{\mathrm{hd}} = \sum_{n=N_2-b+1}^{N} \pi(n). \tag{2.30}$$

In other words, the computation of the QoS metrics of the given model by formulas (2.30) does not offer difficulties for models of any dimension, and it is much easier than the known algorithms [16, 25].

In the case $\mu_{\mathrm{v}} \neq \mu_{\mathrm{d}}$, as it is indicated above, one can use different schemes of approximate solution of the problem.

Note that problems of obtaining prescribed QoS level for heterogeneous calls are of definite scientific and practical interest. In this case, some adjustable parameters should exist for the solution of such problems. In this connection, note that, in networks with FCA schemes, only threshold parameters of the CAC scheme can be controlled since the control of load parameters is a rather complicated and sometimes even an unsolvable problem from the practical viewpoint.

Here a problem of finding the set of values of threshold parameters of the described above CAC scheme is considered for which a prescribed QoS level for heterogeneous calls is satisfied. We call this set (if it is not empty) the set of efficient values (SEVs) of threshold parameters.

For the models being investigated, there exist great possibilities of solution of these problems since there are three degrees of freedom (i.e., the thresholds N_1, N_2, and N_3) in them. Hence, various statements of problems of finding the set of efficient values of threshold parameters are possible.

A verbal definition of the problem being considered is as follows. In an FCA scheme under fixed loads, upper bounds are prescribed for possible values of loss probabilities of heterogeneous calls. It is required to find values of threshold parameters N_1, N_2, and N_3 that satisfy the prescribed constraints.

For small values of N, the solution of this problem can be found by a simple exhaustive search for all possible combinations of parameters N_1, N_2, and N_3. However, this approach becomes inefficient with the growth in N and sometimes is simply impossible. Therefore, an algorithmic approach is proposed below to the solution of the mentioned problem without using an exhaustive search for variants.

For simplicity, assume that new and handover calls are not distinguished in a data traffic, i.e., assume that $N_1 = N_2$. Then, according to relationship (2.30), we have $\mathrm{PB}_{\mathrm{od}} = \mathrm{PB}_{\mathrm{hd}}$.

We denote $\mathrm{PB}_{\mathrm{d}} = \mathrm{PB}_{\mathrm{od}} = \mathrm{PB}_{\mathrm{hd}}$. Then the problem is mathematically written as follows: it is required to find pairs (N_2, N_3) where $N_2 \le N_3$, for which the following constraints are satisfied:

$$\mathrm{PB}_{\mathrm{hv}} \le \varepsilon_{\mathrm{hv}}, \tag{2.31}$$

$$\mathrm{PB}_{\mathrm{ov}} \le \varepsilon_{\mathrm{ov}}, \tag{2.32}$$

$$\mathrm{PB}_{\mathrm{d}} \le \varepsilon_{\mathrm{d}}, \tag{2.33}$$

where $\varepsilon_{\mathrm{hv}}$, $\varepsilon_{\mathrm{ov}}$, and ε_{d} are given values.

A possible algorithm for solution of problem (2.31)–(2.33) using monotonic property of QoS metrics being investigated is presented below.

The main idea of such an iterative algorithm is as follows: for each fixed value of the parameter N_3, the search for the set of efficient values is performed due to the choice of the corresponding values of the parameter N_2. For convenience, this argument is shown in the notation of these functions.

For generality, we consider the kth iteration, $k = 1, 2, \ldots, N$.

Step 1 Set $N_3 = k$ and check the following conditions:

$$\mathrm{PB}_{\mathrm{hv}}(1) \le \varepsilon_{\mathrm{hv}}, \tag{2.34}$$

$$\mathrm{PB}_{\mathrm{ov}}(1) \le \varepsilon_{\mathrm{ov}}, \tag{2.35}$$

$$\mathrm{PB}_{\mathrm{d}}(N_3) \le \varepsilon_{\mathrm{d}}. \tag{2.36}$$

If all conditions (2.34)–(2.36) are satisfied, go to the next step. Otherwise, for the prescribed value of N_3, the problem has no solution.

Note 2.3 Since the function $\mathrm{PB}_{\mathrm{hv}}$ does not decrease with respect to the parameter N_3, the nonfulfillment of condition (2.34) for a definite value of N_3 implies its nonfulfillment for all $k > N_3$. Allowance for this fact considerably accelerates the operation of the algorithm.

Step 2 Solve the following problem:

$$\underline{N_2} = \arg\min_{N_2 \in [1, N_3]} \{\mathrm{PB_d}(N_2) \le \varepsilon_\mathrm{d}\}. \tag{2.37}$$

Step 3 If $\mathrm{PB_{hv}}\left(\underline{N_2}\right) \le \varepsilon_\mathrm{hv}$ and $\mathrm{PB_{ov}}\left(\underline{N_2}\right) \le \varepsilon_\mathrm{ov}$, then go to the next step. Otherwise, for this value of N_3, the problem has no solution.

Step 4 Simultaneously solve the following problems:

$$N_2^\mathrm{hv} = \arg\max_{N_2 \in [\underline{N_2}, N_3]} \{\mathrm{PB_{hv}}(N_2) \le \varepsilon_\mathrm{hv}\}, \tag{2.38}$$

$$N_2^\mathrm{ov} = \arg\max_{N_2 \in [\underline{N_2}, N_3]} \{\mathrm{PB_{ov}}(N_2) \le \varepsilon_\mathrm{ov}\}. \tag{2.39}$$

Step 5 Determine the sought-for interval of appropriate values of N_2 for a given value of N_3 as $\left[\underline{N_2}, \overline{N_2}\right]$ where $\overline{N_2} = \min\left(N_2^\mathrm{hv}, N_2^\mathrm{ov}\right)$.

Step 6 If $N_3 < N$, then set $N_3 = N_3 + 1$ and go to step 1. Otherwise, terminate the algorithm.

Note 2.4 Based on monotonic property of the functions being investigated, the dichotomy (binary search) method can be used for the solution of problems (2.37)–(2.39).

Hence, for each fixed value of the threshold N_3, the set of admissible values of N_2 is found (if they exist), and the set of efficient values of threshold parameters is found by uniting all the solutions obtained.

Numerical experiments were performed using the developed algorithm. For a sample model, the following initial data for test problems (2.31)–(2.33) were used: $N = 50$, $v_\mathrm{ov} = 8/9$, $v_\mathrm{hv} = 1/3$, $v_\mathrm{od} = 1/2$, $v_\mathrm{hd} = 1/4$. The corresponding SEVs for the problem under various constraints on the values of loss probabilities of heterogeneous calls are shown in Table 2.1. Here, the Cartesian product $[a, b] \times [c, d]$ means that $N_2 \in [a, b]$ and $N_3 \in [c, d]$.

As is obvious from Table 2.1, the weakening of requirements on the QoS metrics of d-calls leads to an extension of SEVs owing to the decrease in inefficient values of the parameter N_2 (see rows 1–4 in Table 2.1). This would be expected since the loss probability of d-calls decreases with increasing the parameter N_2. In this case, an SEV is rather smoothly extended with respect to the change in the upper bound of the loss probability of d-calls (i.e., ε_d). It should also note that, for a fixed value of ε_d, an SEV retains its form for rather wide range of varying the other bounds ε_hv and ε_ov (see rows 5–8 in Table 2.1).

In practice, loads of heterogeneous traffics are changed in time. Therefore, problems of investigating the sensitivity of efficient values of threshold parameters with respect to a change in loads are topical questions. In this connection, we note that any analytical investigation of this problem is impossible in principle; it can be investigated only by means of numerical experiments. In particular, performed numerical experiments show that efficient values of threshold parameters of

Table 2.1 Results of solution of problem (2.31)–(2.33)

Parameter values			SEV
$\varepsilon_{\mathrm{hv}}$	$\varepsilon_{\mathrm{ov}}$	ε_{d}	
10^{-4}	10^{-5}	10^{-6}	[12,30] × [31,50]
10^{-4}	10^{-5}	10^{-5}	[10, 30] × [31,50]
10^{-4}	10^{-5}	10^{-4}	[9, 30] × [31,50]
10^{-4}	10^{-5}	10^{-3}	[8, 30] × [31,50]
10^{-4}	10^{-4}	10^{-3}	[8, 30] × [31,50]
10^{-4}	10^{-3}	10^{-3}	[8, 30] × [31,50]
10^{-4}	10^{-2}	10^{-3}	[8, 30] × [31,50]
10^{-2}	10^{-4}	10^{-3}	[8, 30] × [31,50]

problem (2.31)–(2.33) are preserved within a sufficiently wide load variation interval. This is explained by a rather smooth change in the QoS metrics being investigated with respect to loads of heterogeneous traffics.

2.1.2 Numerical Results

Let us consider the results of numerical experiments for the model of multi-rate queue with three types of traffics. The appropriate algorithm to calculate the QoS metrics is quite simple and allows us to study their behavior in all ranges of changing the values of the load and structural parameters of the system. To keep it brief only dependency of QoS metrics on the number of channels is shown in two schemes of determining the access probabilities of heterogeneous calls. In both schemes the bandwidth and load parameters are fixed and chosen as follows: $b_1 = 2$, $b_2 = 5$, $b_3 = 8$; $v_1 = 0.03$ Erl, $v_2 = 0.02$ Erl, $v_3 = 0.01$ Erl.

In the first scheme it is assumed that $\alpha_i(j) = b_i/(j + b_i)$, and in the second one, $\alpha_i(j) = (j + 1)/(j + b_i)$, $i = 1, 2, 3$. In other words, in the first scheme access probabilities of heterogeneous calls are determined by decreasing function, while in the second one the indicated parameters are defined by increasing function with respect to the number of busy channels. Besides these properties, the introduced access probabilities have the following properties: for the first scheme $\alpha_1(j) < \alpha_2(j) < \alpha_3(j)$; for the second scheme $\alpha_1(j) > \alpha_2(j) > \alpha_3(j)$ for any j, $j = 1, 2, \ldots, N$. It means that in the first scheme for the given number of busy channels, access probabilities are determined by increasing function with respect to their bandwidth, while in the second scheme we have inverse situation.

Here our goal is comparison of QoS metrics of the system under different access schemes. Corresponding results are summarized in Figs. 2.1, 2.2, and 2.3 where labels 1 and 2 denote loss probabilities for the first scheme and second scheme, respectively. Their analysis enables us to make the following conclusions. First of all, note that all the QoS metrics under study are decreasing functions with respect to the total number of channels. They completely confirmed all theoretical expectations. However, unlike the function PB_1 the rates of change of the functions PB_2

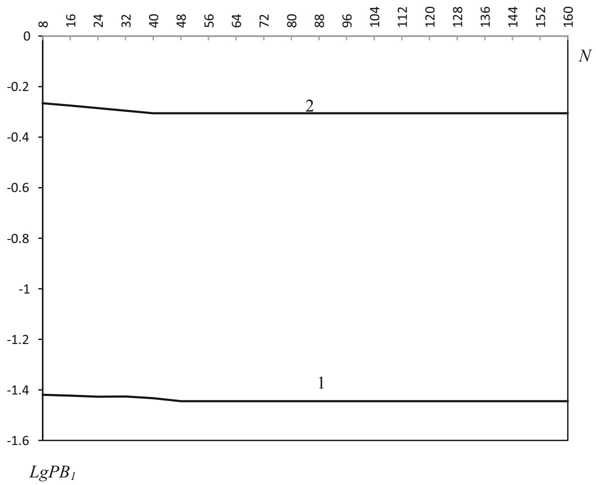


Fig. 2.1 Blocking probability of calls of the first type versus *N*

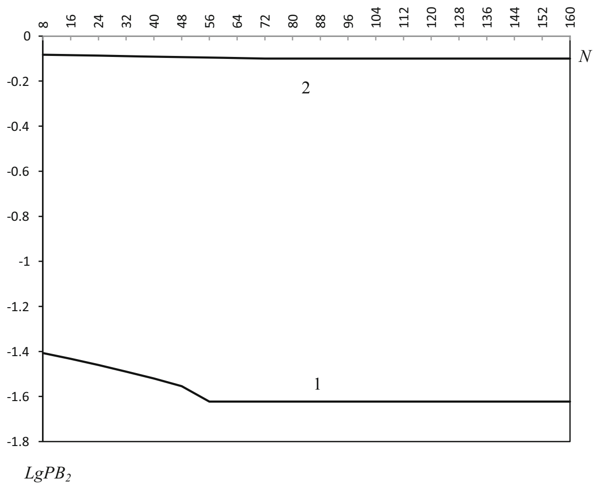


Fig. 2.2 Blocking probability of calls of the second type versus *N*

and PB_3 in first scheme are sufficiently high for small values of channels, i.e., for $N \geq 56$ both functions PB_2 and PB_3 become almost constant.

It is worth noting that for given initial data the first scheme is preferable. However, quite probably, for other values of initial data of the QoS metrics (either all or some of them), the second scheme will be better than the first one. Note that finding the optimal (in known sense) values of access probabilities is not a trivial problem especially for large-size models with many types of heterogeneous calls. However, note that for solving such kind of problems, the methods of Markov decision processes are useful.

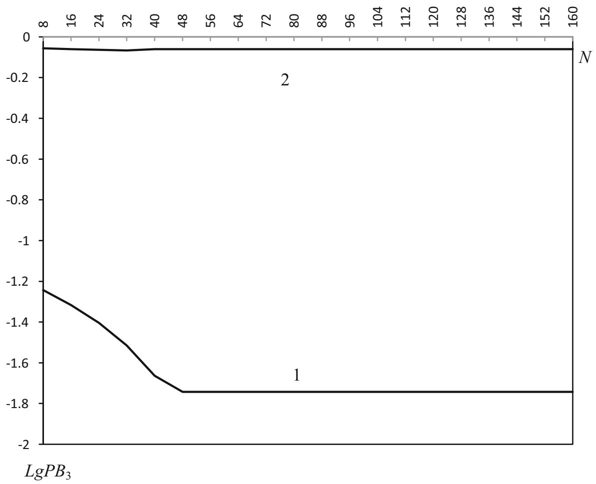


Fig. 2.3 Blocking probability of calls of the third type versus N

2.2 Models of Integral Cellular Networks with Partition of Channels

In majority of known CAC schemes all channels of a cell are equally accessible to a call of any type. However, to reduce the possibility of occurrence of conflict situations using the appropriate schemes of partition of the entire pool of channels between heterogeneous calls is effective also. The analysis of the accessible literature has shown that models of integral cellular networks with such kind of access schemes are insufficiently investigated. Note that the isolated (rigid) partition of channels not always is effective [9], so other partition schemes are required. Thereupon note that non-isolated schemes of partition of channels in network with single traffic (networks of the second generation) have been offered in [20] and in [27], Chap. 1. Feature of these schemes consists that in them partition of channels is not rigid, i.e., the scheme of virtual partition of channels (virtual partitioning, VP) is used.

Here two multi-parametric schemes to partition of entire pool of channels between heterogeneous calls in models of integral wireless networks with voice and data calls (see Sect. 2.1.1) are proposed. Exact algorithms to calculate the QoS metrics of heterogeneous calls under given partition schemes are developed. Results related to determining the efficient partition scheme are carried out.

2.2.1 Model with Complete Partition

At first consider the complete (or isolated) partition scheme (CP scheme) for distribution of channels between zones in which reassignment of the channel from one zone to another is not allowed.

The entire pool of $N > 1$ channels of an isolated cell of integral networks is divided into two groups: exactly N_v channels are assigned for voice calls only and remains $N_{vd} = N - N_v$; channels are used commonly by voice and data calls. In other words, pool of channels is divided into individual zone with N_v channels (v-zone only for voice calls) and common zone with N_{vd} channels (vd-zone for both voice calls and data calls). Disconnection (isolation) of division of channels means that any channel cannot be transferred from one zone to another.

For the sake of simplicity here assume that call of any type to service required only one free channel (i.e., $b = 1$).

The following rules to access of v-calls are used:

- If upon arrival of an ov-call there is at least one free channel in v-zone, this call seizes one of them; otherwise this call is rejected.
- If upon arrival of an hv-call there is at least one free channel in v-zone, this call seizes one of them; otherwise free channel is searched in vd-zone. At that there is a limit to the number of hv-calls in vd-zone, i.e., an hv-call is accepted to vd-zone only if the number of hv-calls in this zone is less than R_{hv}, $1 \le R_{hv} \le N_{vd}$; otherwise it is rejected.

Note that channel holding time of hv-calls in vd-zone has exponential distribution with the same average $1/\mu_v$.

Accesses of d-calls are controlled by the following rules:

- If upon arrival of an hd-call there is at least one free channel in vd-zone, this call seizes one of them; otherwise this call is rejected.
- Arrived od-call is accepted to vd-zone only if the number of d-calls in this zone is less than R_{od}, $1 \le R_{od} \le N_{vd}$; otherwise it is rejected.

Consider the problem of finding the main QoS metrics (i.e., loss probabilities of heterogeneous calls) of the network under given partition scheme of the channels.

From the description of the proposed CP scheme, we conclude that the loss probability of new voice calls is easily defined as loss probability in a classical Erlang's model $M/M/N_v/N_v$ with load v_v Erl, where $v_v = (\lambda_{ov} + \lambda_{hv})/\mu_v$. In other words, to calculate this QoS metrics the well-known Erlang's B-formula might be used:

$$\mathrm{PB}_{ov} = E_B(v_v, N_v) \tag{2.40}$$

where $E_B(v, n) = (v^n/n!)/(\sum_{i=0}^{n}(v^i/i!))$.

However, the loss probability of hv-calls cannot be defined by means of the formula (2.40) since the hv-calls not accepted in the v-zone under certain conditions are transferred to vd-zone. Thus, intensity of hv-calls to vd-zone $\left(\tilde{\lambda}_{hv}\right)$ is determined as $\tilde{\lambda}_{hv} = \lambda_{hv} PB_{ov}$.

Therefore, to calculate the remaining three QoS metrics, it is required to study the multi-flow Erlang's model $M/M/N_{vd}/N_{vd}$ with three types of calls, i.e., hv-calls (with intensity $\tilde{\lambda}_{hv}$), od-calls (with intensity λ_{od}), and hd-calls (with intensity λ_{hd}). Since the channel holding times of heterogeneous calls differ from each other, the state of the mentioned model is described by 2-D vector $\boldsymbol{n} = (n_d, n_v)$, where n_d (respectively, n_v) is the total number of data (respectively, handover voice calls) calls in the channels. Then the state space of the corresponding 2-D MC describing this model is defined thus

$$S = \{\boldsymbol{n} : \ n_d = 0, 1, \ldots, N_{vd}; \ n_v = 0, 1, \ldots, R_{hv}; \ n_d + n_v \le N_{vd}\}.$$

Taking into account the proposed CP scheme for heterogeneous calls, we conclude that the nonnegative elements of the Q-matrix of the appropriate 2-D MC in this model are determined as follows (see Fig. 2.4):

$$q(\boldsymbol{n}, \boldsymbol{n}') = \begin{cases} \lambda_d & \text{if } n_d < R_{od}, \ \boldsymbol{n}' = \boldsymbol{n} + \boldsymbol{e}_1, \\ \lambda_{hd} & \text{if } n_d \ge R_{od}, \ \boldsymbol{n}' = \boldsymbol{n} + \boldsymbol{e}_1, \\ \tilde{\lambda}_{hv} & \text{if } n_v < R_{hv}, \ \boldsymbol{n}' = \boldsymbol{n} + \boldsymbol{e}_2, \\ n_d \mu_d & \text{if } \boldsymbol{n}' = \boldsymbol{n} - \boldsymbol{e}_1, \\ n_v \mu_v & \text{if } \boldsymbol{n}' = \boldsymbol{n} - \boldsymbol{e}_2, \\ 0 & \text{in other cases,} \end{cases} \tag{2.41}$$

where $\lambda_d = \lambda_{od} + \lambda_{hd}$, $\boldsymbol{e}_1 = (1, 0)$, $\boldsymbol{e}_2 = (0, 1)$.

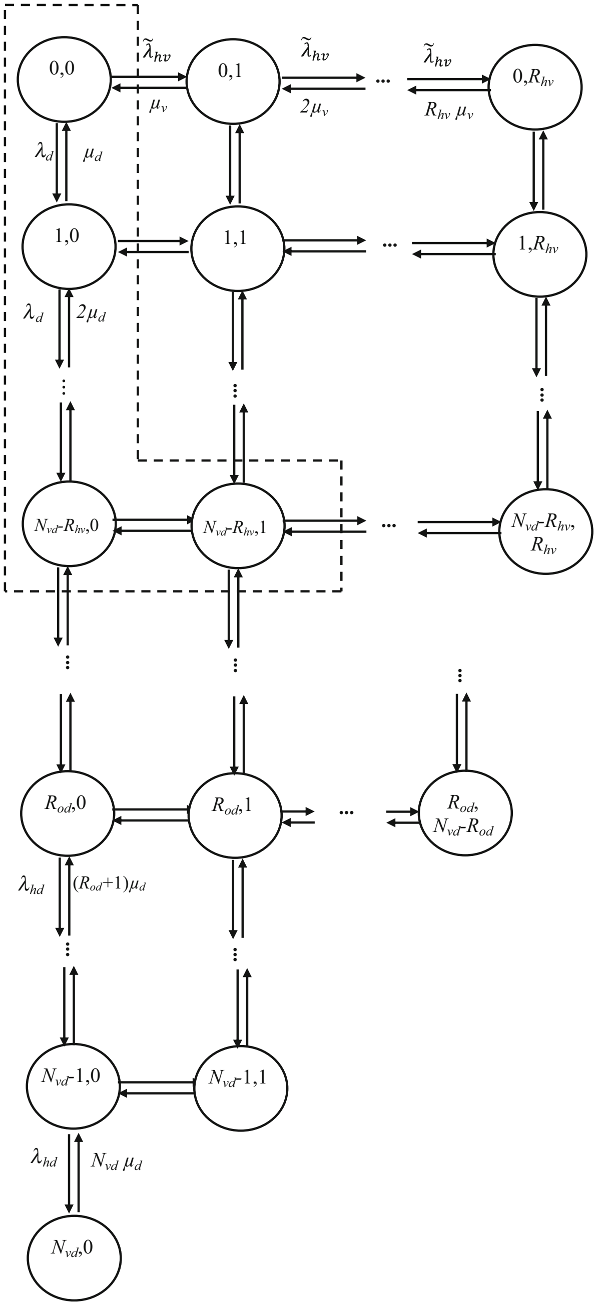
It is easy to show that all states of this 2-DMC are communicating, so in this chain stationary mode exists. Let $p(\boldsymbol{n})$ denote the stationary probability of state $\boldsymbol{n} \in S$.

Desired QoS metrics of the proposed CP scheme are determined via marginal distribution of the above-indicated 2-D MC. Indeed, in this scheme losses of hv-calls occur in the following cases: (a) upon arrival of hv-call, the number of calls of this type in the system is equal R_{hv} regardless of the number of busy channels, and (b) upon arrival of hv-calls, all channels are busy. Therefore by using PASTA theorem, we obtain

$$PB_{hv} = \sum_{\boldsymbol{n} \in S} p(\boldsymbol{n})(\delta(n_v, R_{hv})(1 - \delta(n_d + n_v, N_{vd})) + (1 - \delta(n_v, R_{hv}))\delta(n_d + n_v, N_{vd})). \tag{2.42}$$

Arguing similarly, we find that loss probabilities of od-calls (PB_{od}) and hd-calls (PB_{hd}) are determined as follows:

Fig. 2.4 State transition diagram of the model with CP scheme for partition of channels



$$\mathrm{PB_{od}} = \sum_{\boldsymbol{n} \in S} p(\boldsymbol{n}) I(n_{\mathrm{d}} \geq R_{\mathrm{od}}); \tag{2.43}$$

$$\mathrm{PB_{hd}} = \sum_{\boldsymbol{n} \in S} p(\boldsymbol{n})(n_{\mathrm{d}} + n_{\mathrm{v}}, N_{\mathrm{vd}}). \tag{2.44}$$

System of global balance equations (SGBE) for stationary probabilities is constructed by using relationship (2.41) (we left it to reader). However stationary probabilities can be determined analytically without the numerical solution of the indicated SGBE which for real system has a large dimension.

Proposition 2.4 Stationary distribution of the system at use CP scheme has the following multiplicative form:

$$p(i,j) = \begin{cases} \dfrac{v_{\mathrm{d}}^i}{i!}\dfrac{\tilde{v}_{\mathrm{hv}}^j}{j!} p(0,0) & \text{if } 0 \leq i \leq R_{\mathrm{od}},\ 0 \leq j \leq \min(R_{\mathrm{hv}}, N_{\mathrm{vd}} - i), \\ \left(\dfrac{v_{\mathrm{d}}}{v_{\mathrm{hd}}}\right)^{R_{\mathrm{od}}} \dfrac{v_{\mathrm{hd}}^i}{i!}\dfrac{\tilde{v}_{\mathrm{hv}}^j}{j!} p(0,0) & \text{if } R_{\mathrm{od}} + 1 \leq i \leq N_{\mathrm{vd}},\ 0 \leq j \leq \min(R_{\mathrm{hv}}, N_{\mathrm{vd}} - i), \end{cases} \tag{2.45}$$

where $p(0,0)$ is determined from the normalizing condition, i.e., $\sum_{\boldsymbol{n} \in S} p(\boldsymbol{n}) = 1$ and $\tilde{v}_{\mathrm{hv}} = \tilde{\lambda}_{\mathrm{hv}}/\mu_{\mathrm{v}}$.

Proof of this fact is based on Kolmogorov's theorem about reversibility of 2-D MC [18]. Indeed, it is easily shown that there is no circulation between states $\boldsymbol{n}, \boldsymbol{n}+\boldsymbol{e}_1, \boldsymbol{n}+\boldsymbol{e}_2, \boldsymbol{n}+\boldsymbol{e}_1+\boldsymbol{e}_2$ of the state diagram of the underlying 2-D MC, i.e., there is a general solution of the system of local balance equations (SLBE) for state probabilities. Thus by choosing the path $(0,0), (1,0), \ldots, (i,0), (i,1), \ldots, (i,j)$ from state (0, 0) to state (i, j), we find that multiplicative solution (2.45) is holding (see Fig. 2.4, area in a dashed line). Note that in this proof scheme it is required to take into account two cases which are indicated in the right side of formula (2.45).

Finally, after calculating the state probabilities, the QoS metrics (2.42)–(2.44) are determined from the following explicit formulas:

$$\mathrm{PB_{hv}} = \sum_{i=0}^{N_{\mathrm{vd}}-R_{\mathrm{hv}}} p(i, R_{\mathrm{hv}}) + \sum_{i=N_{\mathrm{vd}}-R_{\mathrm{hv}}+1}^{N_{\mathrm{vd}}} p(i, N_{\mathrm{vd}} - i), \tag{2.46}$$

$$\begin{aligned} \mathrm{PB_{od}} = {} & I(R_{\mathrm{od}} \leq N_{\mathrm{vd}} - R_{\mathrm{hv}}) \sum_{i=R_{\mathrm{od}}}^{N_{\mathrm{vd}}} \sum_{j=0}^{\min(R_{\mathrm{hv}}, N_{\mathrm{vd}}-i)} p(i,j) \\ & + I(R_{\mathrm{od}} > N_{\mathrm{vd}} - R_{\mathrm{hv}}) \left(\sum_{i=N_{\mathrm{vd}}-R_{\mathrm{hv}}}^{R_{\mathrm{od}}-1} p(i, N_{\mathrm{vd}} - i) + \sum_{i=R_{\mathrm{od}}}^{N_{\mathrm{vd}}} \sum_{j=0}^{N_{\mathrm{vd}}-i} p(i,j) \right), \end{aligned} \tag{2.47}$$

$$PB_{hd} = \sum_{i=N_{vd}-R_{hv}}^{N_{vd}} p(i, N_{vd} - i). \tag{2.48}$$

2.2.2 Model with Virtual Partition

Now consider similar model with virtual partition (VP scheme) of channels between two zones. The basic difference of the given scheme from the previous one consists in the following: upon completion of servicing of a v-call in v-zone, the relinquished channel is transferred to the vd-zone if there is a v-call present here, while the channel in the vd-zone that has servicing v-call is switched to the v-zone. In other words, partition is a virtual one, and this procedure is similar to channel reallocation scheme.

Note that at use VP scheme the loss probability of new voice calls cannot be calculated simply from classical Erlang's B-formula (2.40). It is explained by the fact that in this scheme reallocation of channels is allowed.

As abovementioned scheme, here the state of the model is described by 2-D vector $\boldsymbol{n} = (n_d, n_v)$ also, where n_d (respectively, n_v) is the total number of data (respectively, handover voice calls) calls in the channels. However, the state space of the corresponding 2-D MC is defined as follows:

$$S = \{\boldsymbol{n} : \; n_d = 0, 1, \ldots, N_{vd}; \; n_v = 0, 1, \ldots, N_v + R_{hv}; \; n_d + n_v \leq N\}.$$

In VP scheme the nonnegative elements of the Q-matrix of the appropriate 2-D MC is determined as follows (see Fig. 2.5):

$$q(\boldsymbol{n}, \boldsymbol{n}') = \begin{cases} \lambda_d & \text{if } n_d < R_{od}, \boldsymbol{n}' = \boldsymbol{n} + \boldsymbol{e}_1, \\ \lambda_{hd} & \text{if } n_d \geq R_{od}, \boldsymbol{n}' = \boldsymbol{n} + \boldsymbol{e}_1, \\ \lambda_v & \text{if } n_v < N_v, \boldsymbol{n}' = \boldsymbol{n} + \boldsymbol{e}_2, \\ \lambda_{hv} & \text{if } N_v \leq n_v < N_v + R_{hv}, \boldsymbol{n}' = \boldsymbol{n} + \boldsymbol{e}_2, \\ n_d \mu_d & \text{if } \boldsymbol{n}' = \boldsymbol{n} - \boldsymbol{e}_1, \\ n_v \mu_v & \text{if } \boldsymbol{n}' = \boldsymbol{n} - \boldsymbol{e}_2, \\ 0 & \text{in other cases,} \end{cases} \tag{2.49}$$

where $\lambda_v = \lambda_{ov} + \lambda_{hv}$.

Using the scheme of the proof of the Proposition 2.4, it is possible to show that the following fact is true (see Fig. 2.5, area in a dashed line).

Proposition 2.5 Stationary distribution of the system at use VP scheme has the following multiplicative form:

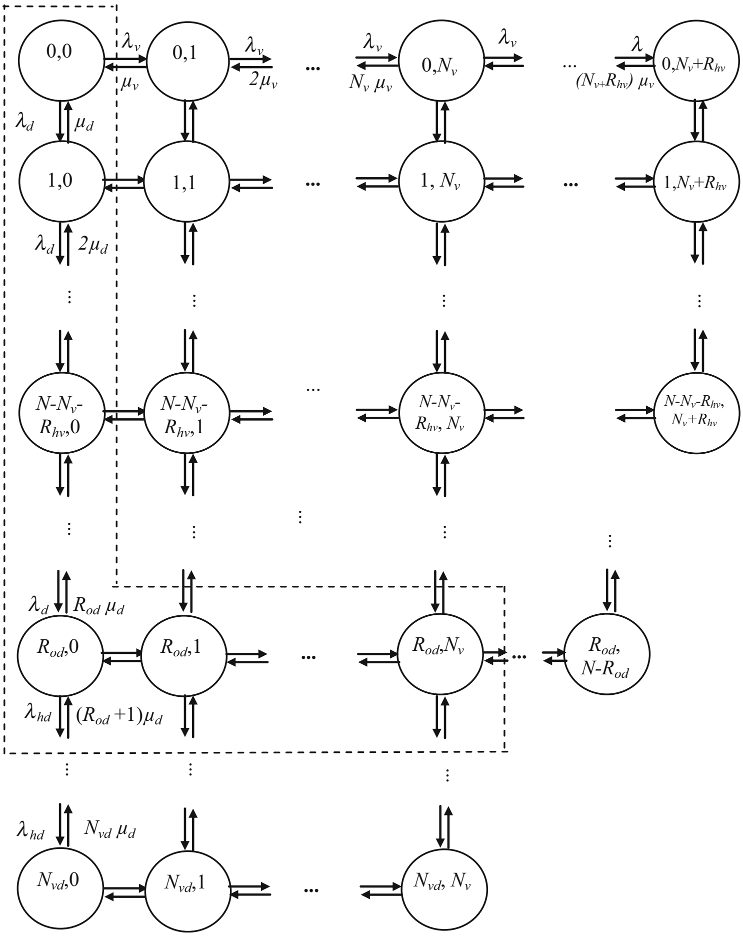


Fig. 2.5 State transition diagram of the model with VP scheme for partition of channels

Case $R_{od} \le N_{vd} - R_{hv}$:

$$p(i,j) = \begin{cases} \dfrac{v_d^i}{i!}\dfrac{v_v^j}{j!}p(0,0) & \text{if}\, 0 \le i \le R_{od}, 0 \le j \le N_v, \\ \left(\frac{v_d}{v_{hd}}\right)^{R_{od}} \dfrac{v_{hd}^i}{i!}\dfrac{v_v^j}{j!}p(0,0) & \text{if}\, R_{od}+1 \le i \le N_{vd}, 0 \le j \le N_v, \\ \left(\frac{v_v}{v_{hv}}\right)^{N_v} \dfrac{v_d^i}{i!}\dfrac{v_{hv}^j}{j!}p(0,0) & \text{if}\, 0 \le i \le R_{od}, N_v+1 \le j \le N_v+R_{hv}, \\ \left(\frac{v_d}{v_{hd}}\right)^{R_{od}} \left(\frac{v_v}{v_{hv}}\right)^{N_v} \dfrac{v_{hd}^i}{i!}\dfrac{v_{hv}^j}{j!}p(0,0) & \text{if}\, R_{od}+1 \le i \le N_{vd}-1, N_v \\ & +1 \le j \le \min(N_v+R_{hv}, N-i); \end{cases} \quad (2.50)$$

Case $R_{\text{od}} > N_{\text{vd}} - R_{\text{hv}}$:

$$p(i,j) = \begin{cases} \dfrac{v_{\text{d}}^i}{i!}\dfrac{v_{\text{v}}^j}{j!}p(0,0) & \text{if } 0 \le i \le R_{\text{od}}, 0 \le j \le N_{\text{v}}, \\ \left(\dfrac{v_{\text{d}}}{v_{\text{hd}}}\right)^{R_{\text{od}}} \dfrac{v_{\text{hd}}^i}{i!}\dfrac{v_{\text{v}}^j}{j!}p(0,0) & \text{if } R_{\text{od}} + 1 \le i \le N_{\text{vd}}, 0 \le j \le N_{\text{v}}, \\ \left(\dfrac{v_{\text{v}}}{v_{\text{hv}}}\right)^{N_{\text{v}}} \dfrac{v_{\text{d}}^i}{i!}\dfrac{v_{\text{hv}}^j}{j!}p(0,0) & \text{if } 0 \le i \le R_{\text{od}}, N_{\text{v}} + 1 \le j \le \min(N_{\text{v}} + R_{\text{hv}}, N - i), \\ \left(\dfrac{v_{\text{d}}}{v_{\text{hd}}}\right)^{R_{\text{od}}} \left(\dfrac{v_{\text{v}}}{v_{\text{hv}}}\right)^{N_{\text{v}}} \dfrac{v_{\text{hd}}^i}{i!}\dfrac{v_{\text{hv}}^j}{j!}p(0,0) & \text{if } R_{\text{od}} + 1 \le i \le N_{\text{vd}} - 1, N_{\text{v}} \\ & +1 \le j \le N - i. \end{cases} \quad (2.51)$$

In both formulas (2.50) and (2.51), $p(0,0)$ is determined from the normalizing condition.

Finally we obtain the following explicit formulas to calculate the QoS metrics at use VP scheme for partition of channels' pool:

$$\text{PB}_{\text{ov}} = \sum_{i=0}^{N_{\text{vd}}-R_{\text{hv}}} \sum_{j=N_{\text{v}}}^{N_{\text{v}}+R_{\text{hv}}} p(i,j) + \sum_{i=N_{\text{vd}}-R_{\text{hv}}+1}^{N_{\text{vd}}} \sum_{j=N_{\text{v}}}^{N-i} p(i,j), \quad (2.52)$$

$$\text{PB}_{\text{hv}} = \sum_{i=0}^{N_{\text{vd}}-R_{\text{hv}}} p(i, N_{\text{v}} + R_{\text{hv}}) + \sum_{i=N_{\text{vd}}-R_{\text{hv}}+1}^{N_{\text{vd}}} p(i, N - i), \quad (2.53)$$

$$\text{PB}_{\text{od}} = \sum_{i=R_{\text{od}}}^{N_{\text{vd}}} \sum_{j=0}^{\min(N_{\text{v}}+R_{\text{hv}}, N-i)} p(i,j), \quad (2.54)$$

$$\text{PB}_{\text{hd}} = \sum_{i=0}^{N_{\text{v}}-1} p(N_{\text{vd}}, i) + \sum_{i=N_{\text{vd}}-R_{\text{hv}}}^{N_{\text{vd}}} p(i, N - i). \quad (2.55)$$

2.2.3 Numerical Results

The developed above explicit formulas allow to investigate the behavior of QoS metrics of the both partition schemes over any range of change of values of loading parameters of heterogeneous calls and number of channels. First of all, here it is assumed that allocation of entire pool of channels between zones is fixed, and only regulated parameters are R_{hv} and R_{od}. It is clear that the behavior of QoS metrics with respect to the indicated parameters is identical in both partition schemes. In other words, the increase in value of one of the parameters R_{hv} and R_{od} (in an admissible area) favorably influences the QoS metric of calls of the corresponding type only.

The initial data for total number of channels and loading parameters of heterogeneous calls are as in [4], i.e., $N = 30$, $\lambda_{\mathrm{ov}} + \lambda_{\mathrm{hv}} = 0.15$ call/s, $\lambda_{\mathrm{od}} + \lambda_{\mathrm{hd}} = 0.3$ call/s, $\mu_{\mathrm{v}}^{-1} = 2$ s, and $\mu_{\mathrm{d}}^{-1} = 120$ s. Below, assume that $N_{\mathrm{v}} = 12$, $N_{\mathrm{vd}} = 18$ and 30 % of the total intensity of voice calls are handover voice calls and 80 % of the total intensity of data calls are new data calls.

First consider the results of numerical experiments for the model with CP scheme for partition of channels. Some results for behavior of QoS metrics versus R_{hv} are shown in Fig. 2.6. Since loss probability of ov-calls is determined by Erlang's B-formula (i.e., it is independent on R_{hv}), then function $\mathrm{PB}_{\mathrm{ov}}$ is constant

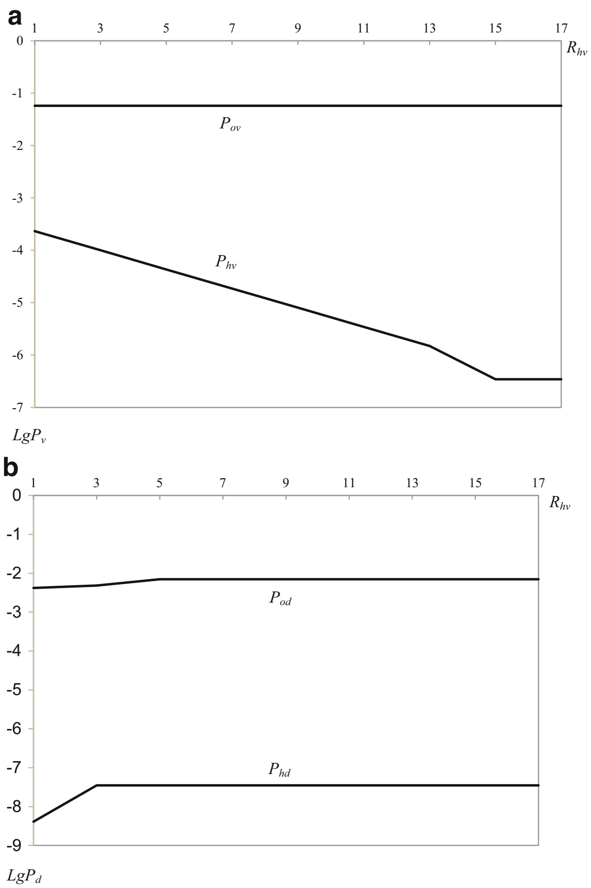
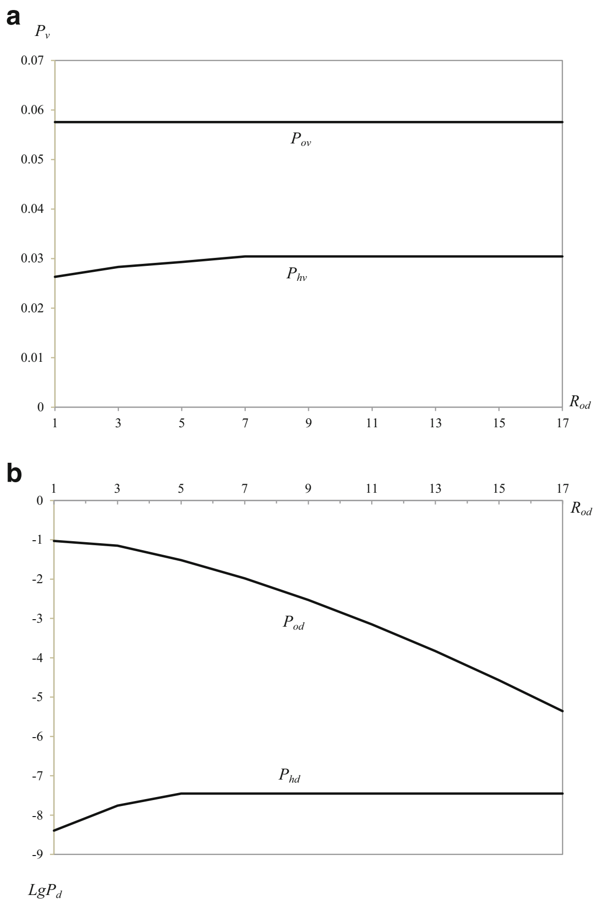


Fig. 2.6 QoS metrics versus R_{hv} under CP scheme of partition

one, while loss probability of hv-calls (PB_{hv}) is decreasing function versus R_{hv} (see Fig. 2.6a). Note that function PB_{hv} is almost piecewise linear one and has high rate of decreasing especially for small values of R_{hv}. Both QoS metrics for data calls are non-decreasing function with respect to R_{hv}, and they become almost constant for the large values of indicated parameter (see Fig. 2.6b). The last facts are explained by the following arguments: for given initial data, intensity of handover voice calls essentially is less than intensity of data calls, and at the same time handle rate of data calls essentially is more than appropriate parameter for voice calls.

Results for behavior of QoS metrics versus R_{od} are shown in Fig. 2.7. As above, function PB_{ov} is a constant one, but here PB_{hv} is a non-decreasing function, since increase in value of parameter R_{od} leads to decreasing the chances of hv-calls for access to the channels of vd-zone (see Fig. 2.7a). At that rate of change of function, PB_{hv} is inconsiderable for large values of parameter R_{od}. In this case, function PB_{od} is decreased with high speed in small values of parameter R_{od}, while function PB_{hd} has a small increasing rate in large values of indicated parameter.

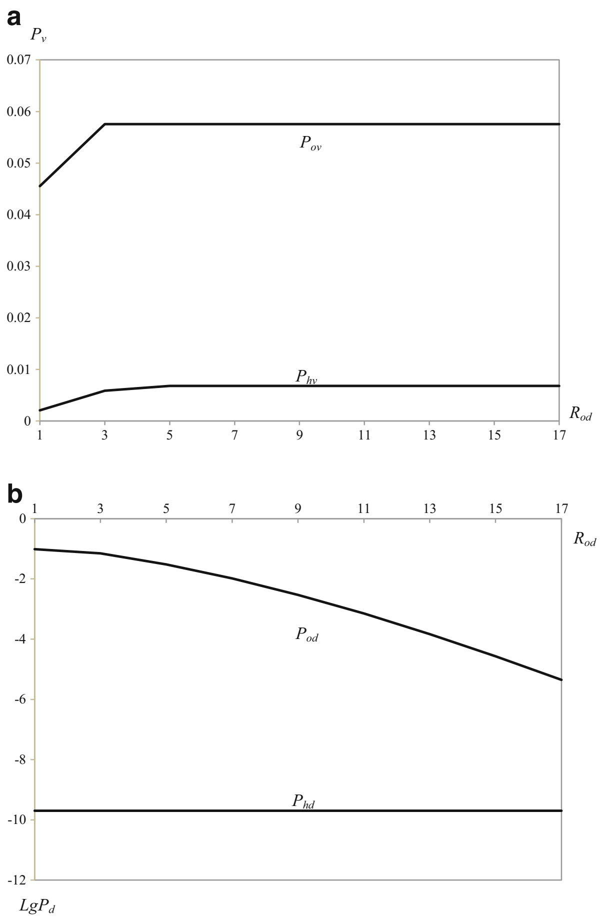
Fig. 2.7 QoS metrics versus R_{od} under CP scheme of partition



Now consider the results of numerical experiments for the model with VP scheme for partition of channels (see Figs. 2.8 and 2.9). In Fig. 2.8, the dependency of QoS metrics on the parameter R_{hv} is shown. It is seen from Fig. 2.8a that function PB_{hv} decreases in small values of parameter R_{hv} with high speed; thereafter, it becomes almost constant; function PB_{ov} increases with insignificant speed in small values of indicated parameter; thereafter, it becomes almost constant also. Almost constants are both functions PB_{od} and PB_{hd} versus R_{hv} (see Fig. 2.8b). Such behavior of functions PB_{od} and PB_{hd} is explained via small intensity of handover voice calls.

Dependency of QoS metrics on the parameter R_{od} is shown in Fig. 2.9. Here both functions PB_{od} and PB_{hv} increase with insignificant speed in small values of indicated parameter; thereafter, it becomes almost constant (see Fig. 2.9a). However, function PB_{od} decreases with significant speed versus PB_{od} and PB_{od}, while function PB_{od} and PB_{hd} is almost constant one (see Fig. 2.9b).

Fig. 2.8 QoS metrics versus R_{od} under VP scheme of partition



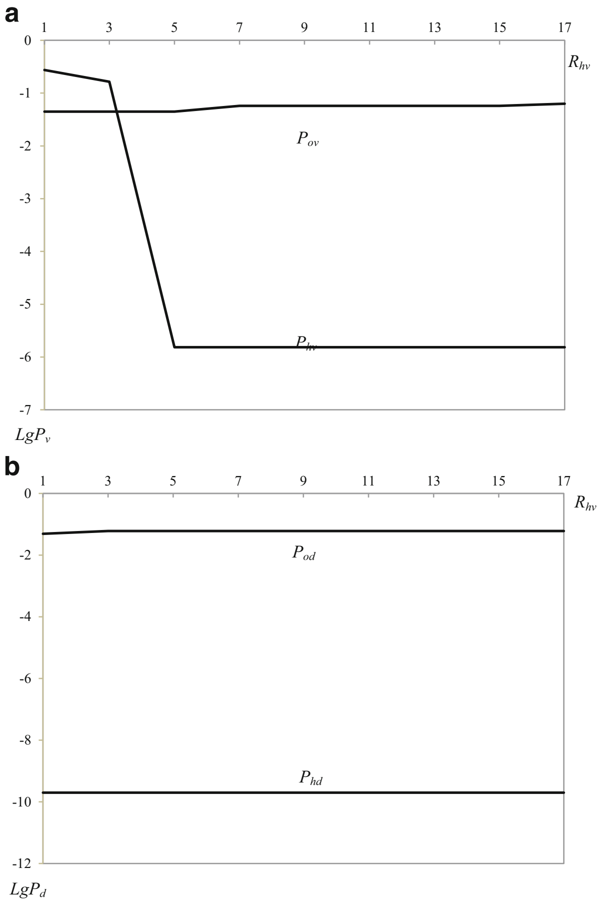


Fig. 2.9 QoS metrics versus R_{hv} under VP scheme of partition

Now briefly consider comparative analysis of the QoS metrics of two partition schemes at fixed values of structural and loading parameters of the model. Controllable parameters are $\mathrm{PB}_{\mathrm{hv}}$ and $\mathrm{PB}_{\mathrm{od}}$. As shown above (see Figs. 2.6, 2.7, 2.8, and 2.9), the behavior of QoS metrics versus these controllable parameters in different partition schemes is identical.

Some results of the comparison are shown in Figs. 2.10, 2.11, 2.12, and 2.13 where labels 1 and 2 denote QoS metrics for CP scheme and VP scheme, respectively. The input data are the same as for Figs. 2.6, 2.7, 2.8, and 2.9.

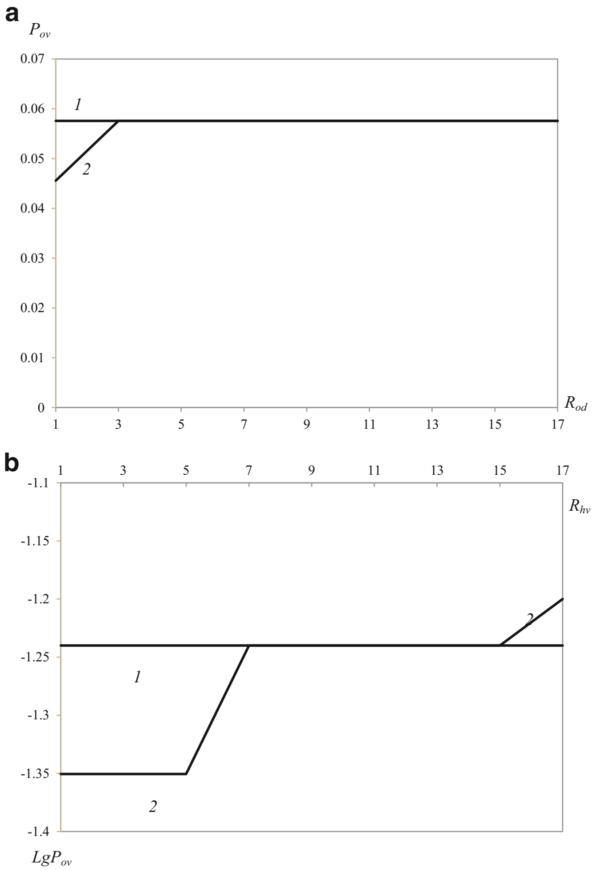


Fig. 2.10 Comparison for P_{ov} under different partition schemes; (**a**) $R_{hv} = 9$; (**b**) $R_{od} = 9$

It is interesting to note that character of change of QoS metrics versus change of parameters PB_{hv} and PB_{od} is almost identical at both partition schemes of channels (except for the QoS metric PB_{hv} versus R_{hv}, see Fig. 2.11b). However, in some cases, their absolute values are in different quantitative ranges.

From Fig. 2.10a, we conclude that for the chosen initial data, QoS metric PB_{ov} is better under VP scheme of partition for values $R_{od} \leq 2$ and for $R_{od} \geq 3$ both partition schemes have the same performance. However, from Fig. 2.10b it is seen that this QoS metric is better under VP scheme of partition for values $R_{od} \leq 8$ and in cases $R_{od} > 9$ favorably scheme for QoS metric PB_{ov} is CP scheme of partition.

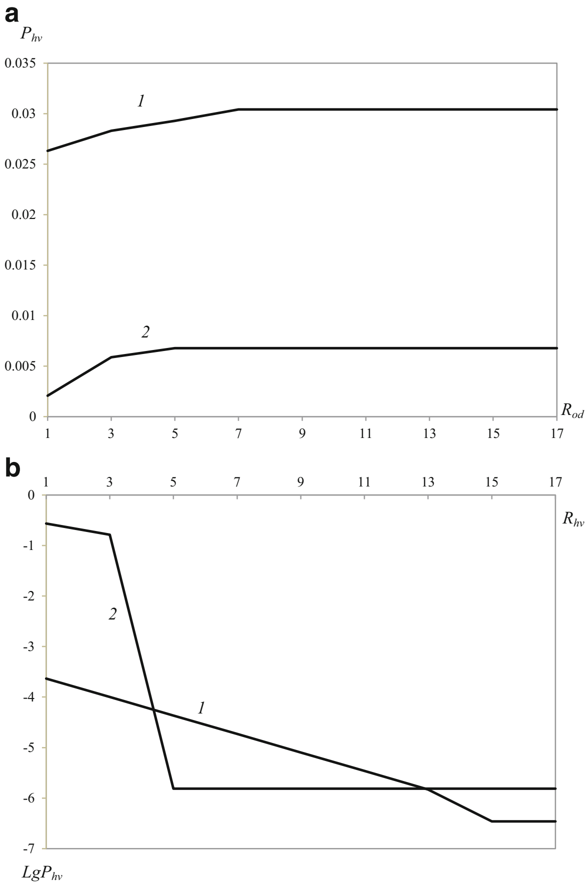
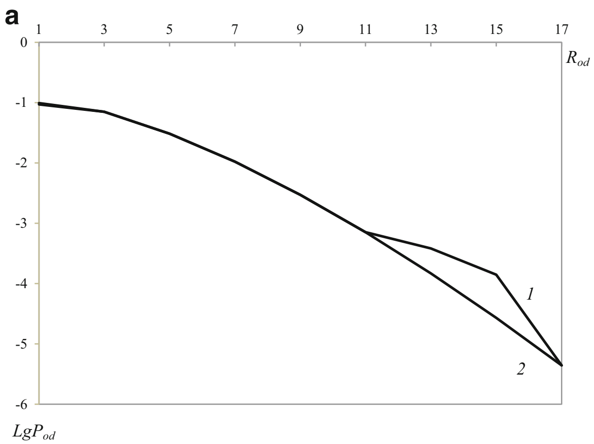


Fig. 2.11 Comparison for P_{hv} under different partition schemes; (**a**) $R_{\text{hv}} = 9$; (**b**) $R_{\text{od}} = 9$

In Fig. 2.11 comparative results are shown for QoS metric PB_{hv}. It is seen from Fig. 2.11a that this QoS metric is essentially better under VP scheme of partition for all values of R_{od}. However, this QoS metric is better under VP scheme for values of $R_{\text{hv}} \in [3, 7]$, and in other values this metric favorably is CP scheme (see Fig. 2.12b).

It is seen from Fig. 2.12a that for QoS metric PB_{od} at $R_{\text{od}} < 12$ both partition schemes have the same performance, but at $R_{\text{od}} \geq 12$ one of schemes, i.e., VP scheme, has good performance for this metric. Note that this QoS metric is essentially better under CP scheme of partition for all values of R_{hv} (see Fig. 2.12b).



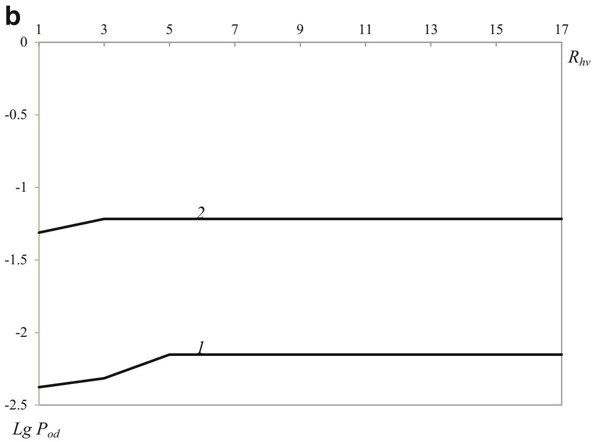


Fig. 2.12 Comparison for P_{od} under different partition schemes; (**a**) $R_{\text{hv}} = 9$; (**b**) $R_{\text{od}} = 9$

From Fig. 2.13 we conclude that QoS metric PB_{hd} is essentially better under VP scheme of partition for all values of both parameters R_{od} and R_{hv}.

The numerical results show that all QoS metrics in both partition schemes have monotony property. These facts allow to develop the algorithms to finding the set of effective values (SEVs) in order to satisfy the given QoS level. Such kind of problems has been considered in Sect. 2.1.1; thus they are not considered here. We left them to reader.

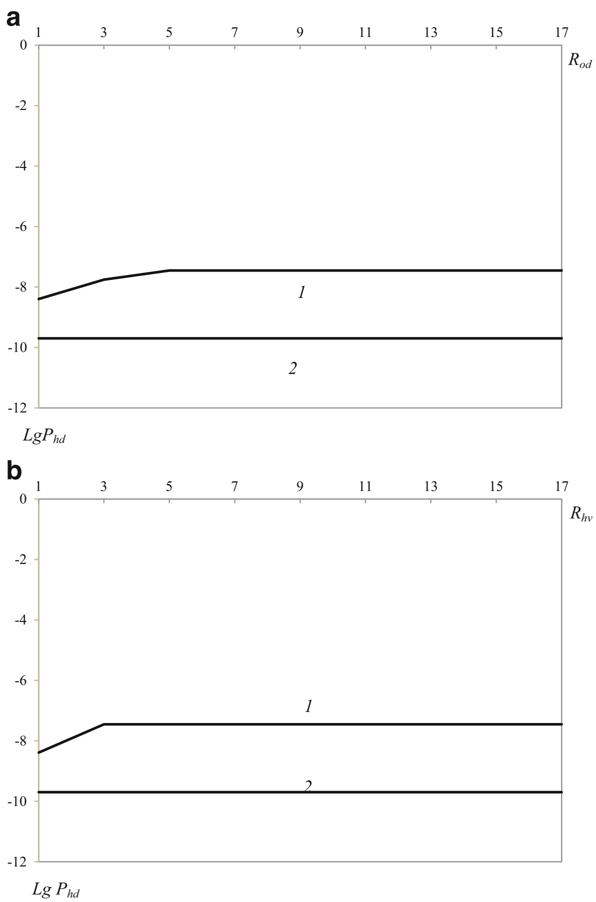


Fig. 2.13 Comparison for P_{hd} under different partition schemes; (**a**) $R_{\mathrm{hv}} = 9$; (**b**) $R_{\mathrm{od}} = 9$

2.3 Conclusion

In the last decades are published many books [1, 2, 5, 6, 10–12, 19, 27, 30, 31, 33] and reviews [3, 14, 15, 24] which deal with applications of queueing theory in telecommunication networks. In this direction, multi-rate Erlang's models are subjects of many researches. Rather detailed description of the MRQ results can be found in appropriate chapters of books [11, 19, 27, 30] and in papers [14, 15, 23, 26]. Here we only note that in the theory of MRQ models, the main results are based on the following fact [13, 29]: the stationary distribution of unbuffered Markov model of MRQ in the full availability of channels (i.e., in CS scheme) has a multiplicative form. Note that as in case of single-rate multidimensional Erlang's models, a multiplicative solution exists also in the cases when the distribution function of the service time of multi-rate traffic is arbitrary with the fixed mean value.

In this chapter an analytical approach to the analysis of a multi-rate Erlang's model with the state-dependent randomized access strategy is proposed [21, 23]. In contrast to the known numerical methods, the proposed approach does not require generation of the large state space of the model, and, therefore, finding the desired QoS metrics is carried out by explicit formulas. It is shown that from the results of this chapter, the results for the MRQ models with the known access schemes can be easily obtained. It is proved that the results, obtained in previous studies using heuristic considerations, which have been regarded as approximate, are in fact accurate ones in some cases. The proposed method also allows one to develop simple one-dimensional recurrence formulas for calculating QoS metrics of integral wireless networks of voice calls and data calls. The simplicity of the obtained formulas allows one to formulate and solve important problems of optimization of the studied models.

Note that Kaufman–Roberts algorithm together with its various modifications is the main tool to study the characteristics of multi-rate queues. For instance, in [8] the formula to calculate the occupancy distribution in the CS scheme for the model MRQ with mixture of flows of Erlang, Engset, and Pascal type is proposed. Here the considered models of MRQ systems with randomized access strategy have been studied in [31], Chap. 7, where they are called state-dependent systems. In the indicated book, the equations (2.18) are obtained also. However, in [31] these equations are obtained subject to the following conditions: $(\alpha_i(n))/(\alpha_i(n+b_j)) = (\alpha_j(n))/(\alpha_j(n+b_i))$ for any $i, j = 1, 2, \ldots, K$. The last conditions are necessary to providing the reversibility of appropriate K-dimensional Markov chain which are results from Kolmogorov's theorem [18]. At the same time, as authors noted, in practice the fulfillment of these conditions is extremely difficult problem. The equations (2.18) without any proofs are resulted in the book [19], Chap. 11 also. Authors simply verbally assume that these equations are true. Alternative and effective approach to calculate the QoS metrics of multi-rate models is using convolution algorithms [11].

Attempt to the solution of similar problems which appear in communication networks has been made in work [32]. Unfortunately, this attempt has appeared unsuccessful [22].

Here two partition schemes for distribution of entire pool of channels among voice and data calls in integral wireless networks are proposed also. One of them uses isolated (rigid) distribution of channels, while in another scheme, virtual distribution procedure is applied. In both schemes, a voice call seizes the free channel in own zone, and if there is no free channel in this zone, only handover voice calls might search free channel in another zone. Moreover the state-dependent limit to the both number of handover voice calls and new data calls in zone of channels for data calls are defined. It is shown that in both partition schemes stationary distribution of appropriate 2-D MC has multiplicative form. By using this fact the explicit formulas to calculate the QoS metrics of the integral networks under given partition schemes are developed. The proposed formulas allow to perform comparative analysis of QoS metrics in various partition schemes.

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