Preface

Nonholonomic systems are control systems which depend linearly on the control. Their underlying geometry is the sub-Riemannian geometry, which plays for these systems the same role as Euclidean geometry does for linear systems. In particular the usual notions of approximations at the first order, that are essential for control purposes, have to be defined in terms of this geometry. The aim of these notes is to present these notions of approximation and their application to the motion planning problem for nonholonomic systems.

The notes are divided into three chapters and two appendices. In Chap. 1, we introduce the basic definitions on nonholonomic systems and sub-Riemannian geometry, and give the main result on controllability, namely the Chow–Rashevsky Theorem. Chapter 2 provides a detailed exposition of the notions of first-order approximation, including nonholonomic orders, privileged coordinates, nilpotent approximations, and distance estimates such as the Ball-Box Theorem. As an application, we show how these notions allow us to describe the tangent structure to a Carnot-Carathéodory space (the metric space defined by a sub-Riemannian distance). The chapter ends with the presentation of desingularization procedures that are necessary to recover uniformity in approximations and distance estimates. Chapter 3 is devoted to the motion planning problem for nonholonomic systems. We show in particular how to apply the tools from sub-Riemannian geometry in order to give solutions to this problem, first in the case where the system is nilpotent, and then in the general case. An overview of the existing methods for nonholonomic motion planning conclude this chapter. Finally, we present some results on composition of flows in connection with the Campbell-Hausdorff formula in Appendix A, and some complements on the different systems of privileged coordinates in Appendix B.

From the point of view of the sub-Riemannian geometry, this book is intended to be complementary to that of Ludovic Rifford in the same collection [1]. As a consequence, the subjects that are extensively talked about in the latter (for instance sub-Riemannian geodesics) are not discussed here.

Notice finally that the main theoretical part about controllability and first-order theory is self-contained, all the results being proved. However for some applications, we took the liberty of stating some results without demonstration. They concern either developments beyond the scope of these notes (Theorem 2.4 called the Uniform Ball-Box Theorem, Theorem 2.5 on the metric tangent cone) or technical results on algorithmic procedures (Theorem 2.9 on the desingularization procedure, the fact that the formula (3.17) on sinusoidal controls may be inverted).

These notes grew out of a series of lectures given at the *Trimester on Dynamical and Control Systems* in Trieste in 2003, and more recently at the CIMPA Schools *Géométrie sous-riemannienne* in Beirut, Lebanon, in 2012, and *Contrôle géométrique, stochastique et des équations aux dérivées partielles* in Tlemcen, Algeria, in 2014. I am most grateful to the organizers of these events, Andrei Agrachev and Ugo Boscain for the first one, Fernand Pelletier, Ali Fardoun, and Mohamad Mehdi for the second one, and Sidi Mohammed Bouguima, Benmiloud Mebkhout, and Yacine Chitour for the third one. The materials of the third chapter mostly come from a collaboration with Yacine Chitour and Ruixing Long, during the Ph.D. thesis of the latter. This book is also thanks to them.

Paris, April 2014 Frédéric Jean

Reference

1. Rifford, L.: *Sub-Riemannian geometry and optimal transport*. SpringerBriefs in Mathematics. Springer International Publishing, New york (2014)

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