Preface

This textbook is an introduction to the representation theory of finite-dimensional algebras with a strong emphasis on quivers and their representations. The book is intended to be used as a graduate textbook for a one-semester course. The first three chapters are completely self-contained, assuming only familiarity with basic notions of linear algebra, and could be used also for an advanced undergraduate topics course or as a quick introduction to Auslander–Reiten quivers for mathematicians who are mostly interested in applying the theory to other fields of research without necessarily becoming an expert in representation theory. In Chaps. 4–7, prior experience with rings is beneficial, but the main concepts are recalled in Chap. 4.

The use of quivers in the representation theory of finite-dimensional algebras gives us the possibility to visualize the modules of a given algebra very concretely as a collection of matrices, each of which is associated to an arrow in a certain diagram—the quiver. To every quiver one can associate the path algebra, whose elements are finite sums of paths in the quiver and whose multiplication is given as concatenation of paths. The modules of the path algebra correspond precisely to the representations of the quiver. Thus the quiver does give not only an example of an algebra but also a very concrete model for the representation theory of the algebra. The beauty of the theory is that the quiver approach can be used to study the representation theory of an *arbitrary* finite-dimensional algebra!

The main tool for describing the representation theory of a finite-dimensional algebra is the Auslander–Reiten quiver, which gives explicit information about the modules as well as the morphisms between them in a most convenient way. When making the choices on how to develop the material in this book, my main goal was to get to the construction of Auslander–Reiten quivers as soon as possible. This is why, in the first three chapters, I only use the language of quiver representations, postponing the viewpoint of algebras and modules to Chaps. 4–7. For the student, this approach has the advantage of having the wealth of examples of the first three chapters at hand, when studying the somewhat abstract notion of a module.

Chapter 1 starts with the definition of quivers and their representations and then develops the basic tools such as morphisms, direct sums, exact sequences, etc. The concepts of projective and injective representations as well as the Auslander–Reiten translation are introduced in Chap. 2. Chapter 3 contains various methods for the construction of Auslander–Reiten quivers and describes explicitly how to use them to compute morphisms and extensions between representations. Chapter 4 introduces algebras in general and path algebras in particular, while Chap. 5 is devoted to bound quiver algebras, which are quotients of path algebras by admissible ideals. The proof of the equivalence of the notions of modules over the bound quiver algebra and representations of the bound quiver is given in Chap. 5. In Chap. 6, we present several popular constructions of algebras. The Auslander–Reiten formulas are proved in Chap. 7 and Gabriel's Theorem in Chap. 8. Chapter 8 does not use the results of Chaps. 4–7 and could be read right after Chap. 3.

Representation theory is an ideal context to introduce the student to the basic concepts of category theory, and the language of categories is developed along the way as needed.

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The following diagram shows how the different chapters depend on each other:

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