

Preface

Leon Henkin (1921–2006) was an extraordinary logician and an excellent teacher. His writings became influential from the very start of his career with his doctoral thesis, *The completeness of formal systems*, defended in 1947 under the direction of Alonzo Church. He then published two papers in the *Journal of Symbolic Logic*, the first, on completeness for first order logic, in 1949, and the second devoted to completeness in type theory, in 1950. He applied the same basic idea to a generalization of ω -consistency and ω -completeness; by similar methods he proved a completeness theorem using Craig's interpolation lemma. Not only Henkin extended his method to other logics, it has become a standard procedure thereafter in all kinds of logics, not just the classical ones.

His main interests were logic and algebra. In particular, Henkin devoted a large portion of his research to algebraic logic. The theory of *Cylindric Algebras* was studied by Leon Henkin, Alfred Tarski, and Donald Monk during an extended period, and their discoveries provided a class of models which are to first-order logic what Boolean algebras are to sentential logic. Further research was taken up by Istvan Németi and Hajnal Andréka.

In some of his publications Henkin went into the philosophy of mathematics, taking a nominalistic position. Another issue with an ample historical precedent is the search for a system that takes the *identity* relation as the sole primitive constant. He also constructed a special theory of propositional types having lambda and equality as the only primitive signs and proved the completeness of that logic.

Several of his publications, dealing with elementary concepts, fall under the category of mathematical education; we believe that his work *On mathematical induction* was the result of his devotion to mathematical education. Between 1957 and 1972, Henkin divided his time between his mathematical research work and enquiries into the teaching of mathematics.

Answering an invitation from Alfred Tarski, Henkin joined the Mathematics Department in Berkeley (University of California) in 1953. When Tarski and Henkin were able to assemble a number of logicians from the Department of Mathematics and Philosophy, they created an interdepartmental agency, the very famous *Group in Logic and the Methodology of Science*. He stayed with the Department until 1991, when he retired and became an Emeritus Professor.

Henkin was often described as a social activist, he labored much of his career to boost the number of women and underrepresented minorities in the upper echelons of mathematics. He was also very aware that we are beings immersed in the crucible of history from which we find it hard to escape.

The editors felt that it was about time for a comprehensive book on the *The Life and Work of Leon Henkin* and we sent a proposal to Springer Basel to be published in *Studies on Universal Logic*. We are grateful to the Series Editor Jean-Yves Béziau for accepting our proposal. It seems that the Logic community shared a similar feeling and the response to our call was incredible. We wish to thank all the contributors for their interesting papers, including personal and historical material. We are sure their contributions provide a foundational logic perspective on Henkin's work.

This book is dedicated to Leon Henkin, a creative and influential logician who has changed the landscape of logic, while also being a highly principled, generous and humble human being. He was and continues to be our mentor.

Henkin's Investigations and Beyond

Two parts of this book are devoted to Henkin's work, where we include chapters on:

Algebraic Logic by Donald Monk, *Leon Henkin and Cylindric Algebras*. Cylindric algebras are abstract algebras which stand in the same relationship to first-order logic as Boolean algebras do to sentential logic. Monk analyzes Henkin's contributions on the subject and also includes a section on *Publications of Henkin Concerning Cylindric Algebras*. Henkin devoted a large portion of his works to this theory, many of them having Tarski and/or Monk as coauthors.

Completeness Two chapters are directly devoted to this issue, *Henkin on Completeness* and *Henkin's Theorem in Textbooks*, by María Manzano and Enrique Alonso. In the first one, four of his completeness proofs are analyzed—namely, for first-order logic, type theory, propositional type theory and first-order logic using interpolation—as well as the proofs Henkin used to explain in class; the paper ends with two expository works that Henkin devoted to the same issue (*Truth and Provability* and *Completeness*). The aim in the second paper is to examine the incorporation and acceptance of Henkin's completeness proof in some textbooks on classical logic. All of these textbooks come from the Anglo-Saxon tradition and were published before the beginning of the 1980s.

General Models *Changing a Semantics: Opportunism or Courage?* by Hajnal Andréka, Johan van Benthem, Nick Bezhaishvili, and Istvan Németi. This paper gives a systematic view of generalized model techniques, discusses what they mean in mathematical and philosophical terms, and presents a few technical themes and results about their role in algebraic representation, calibrating provability, lowering complexity, understanding fixed-point logics, and achieving set-theoretic absoluteness. It is also shown how thinking about Henkin's approach to semantics of logical systems in this generality can yield new results, dispelling the impression of ad-hocness.

Propositional Type Theory Peter Andrews wrote *A Bit of History Related to Equality as a Logical Primitive*. This historical note illuminates how Leon Henkin's work influenced that of the author. It focuses on Henkin's development of a formulation of type theory based on equality, *A Theory of Propositional Types*, and the significance of this contribution.

A Problem Concerning Provability is the title of a problem posed by Henkin in 1952—in the Problem Section of the *JSL*. Volker Halbach and Albert Visser contributed to this volume with *The Henkin Sentence*. They discuss Henkin’s question concerning a formula that has been described as expressing its own provability. They analyze Henkin’s formulation of the question and the early responses by Kreisel and Löb, and sketch how this discussion led to the development of Provability Logic.

Beyond Henkin’s Method The chapter by Robert Goldblatt, *The Countable Henkin Principle*, presents a general result about the existence of finitely consistent theories which encapsulates a key aspect of the “Henkin method”. The countable version of the principle is applied here to derive a variety of theorems, including omitting-types theorems, and to strong completeness proofs for first-order logic, omega-logic, countable fragments of languages with infinite conjunctions, and a propositional logic with probabilistic modalities. The paper concludes with a topological approach to the countable principle.

Henkin’s Method The chapter by Franco Parlamento, *Henkin’s Completeness Proof and Glivenko’s Theorem*, also deals with Henkin’s method. The author observes that Henkin’s argument for the completeness theorem yields also a classical semantic proof of Glivenko’s theorem and leads in a straightforward way to the weakest intermediate logic for which that theorem still holds. He also outlines and comments on its application to the logic of partial terms, when “existence” is formulated as equality with a (bound) variable.

Finite Type Theory William Gunther and Richard Statman contributed with *Reflections on a Theorem of Henkin*. In a full type structure with a finite ground domain, say \mathcal{M}^n , it is folklore that a member of \mathcal{M}^n is symmetric if and only if it is definable in type theory. A proof follows immediately from an observation due to Leon Henkin in *A Theory of Propositional Types*. In this note the authors generalize the folklore theorem to other calculi and also they provide a straightforward proof of that theorem itself using only simple facts about the symmetric group and its action on equivalence relations.

Extending Completeness in the Theory of Types to Fuzzy Type Theory Vilém Novák contributed with *From Classical to Fuzzy Type Theory*. Mathematical fuzzy logic is a special many-valued logic whose goal is to provide tools for capturing the vagueness phenomenon via degrees. Both propositional and first-order fuzzy logic were proved to be complete, but only in 2005. Also, higher-order fuzzy logic (called the Fuzzy Type Theory, FTT) was developed and its completeness with respect to general models was proved. The proof is based on the ideas of the Henkin’s completeness proof for TT.

Many-Sorted Logic In the chapter *April the 19th*, María Manzano credits most of the ideas involved in her translation of a variety of logics into a many-sorted framework to Henkin’s paper of 1950 *Completeness in the Theory of Types* and to his paper of 1953, *Banishing the Rule of Substitution for Functional Variables*. April the 19th, 1996, was Henkin’s 75 birthdate, but also the date when María was going to give a talk in Berkeley to explain how the ideas of Henkin were applied in her book *Extensions of First-Order Logic*.

Applying Henkin's Method to Hybrid Logic In *Henkin and Hybrid Logic*, by Patrick Blackburn, Antonia Huertas, María Manzano and Klaus Frovin Jørgensen, the authors explain why Henkin's techniques are so important in hybrid logic, an extension of orthodox modal logic in which special proposition symbols are used to name worlds. These symbols, called nominals, allow Henkin's witnessing technique to be applied in modal logic. As a result, in higher-order settings the use of general interpretations and the construction of type hierarchies can be (almost) pure Henkin. The paper ends with the words: '*Hybrid logic? It's a suggestive name. But it could without exaggeration be called: Henkin-style modal logic*'.

Reviews for The Journal of Symbolic Logic Concepción Martínez Vidal and José Pedro Úbeda Rives contribute with *Leon Henkin The Reviewer*. In this chapter they look at the minor, though abundant, works by Henkin; namely his reviews—a total of forty six—and other minor papers. They sort the reviews into four categories and provide a brief summary as well as an analysis of each of them. This analysis reveals Henkin's personal views on some of the most important results and influential books in his time. Finally, they relate these reviews and minor works to Henkin's major contributions.

Mathematics Education Nitsa Movshovitz-Hadar contributes with *Pairing Logical and Pedagogical Foundations for the Theory of Positive Rational Numbers*. Five different ways of "founding" the mathematical theory of positive rational numbers for further logical development are presented. This is done as Leon Henkin outlined in 1979 in the form of notes for a future paper, suggesting that pairing them up with five representation models could possibly lead to further pedagogical development. His wish was to explore how varying modes of deductive development can be mirrored in varying classroom treatments rooted in children's experience and activities. This dream of Henkin never came to full fruition.

Leon Henkin Up Close and Personally

Some of the contributions to this volume fall under the category of very personal writings, describing the relationship the authors have maintained with Leon Henkin. We also include a biographical chapter on Henkin's life and work with some suggestions on future research.

Arithmetization of Metamathematics Solomon Feferman contributes with a very personal paper, *A Fortuitous Year with Leon Henkin*, in which he tells us how his doctoral thesis changed when his advisor Tarski went on sabbatical leave and Henkin acted as supervisor. '*What if Henkin had not been in Berkeley to act in his place? In fact, none of the "what ifs" held and I am eternally grateful to Leon Henkin for his being there for me at the right place at the right time*'.

Henkin and the Suit Albert Visser tell us a funny story concerning a young obedient respectful student (himself) during a short visit Leon and Ginette paid to Amsterdam. The story ends with these words: '*This is all well and good, but what does the story teach us about the Henkins? The story makes clear that the Henkins were very open and friendly*'.

people. They easily gained someone's trust. The fact that I told them the story during the dinner is the best possible testimony of that.'

Lessons from Leon Diane Resek, one of his doctoral students, describes Leon as a human being. In her words, '*Leon was the least intimidating of the professors I met at Berkeley*'. She highlights his love for good writing and his strength as a lecturer, and she concludes saying: '*In general, Leon's recognition of values in life beyond pushing for the best and strongest theorems set him apart from many mathematicians at Berkeley.*'

Tracing Back 'Logic in Wonderland' to My Work with Leon Henkin Nitsa Movshovitz-Hadar describes in her personal relationship with Leon, who was her thesis advisor, defended in 1975. She ends the contribution with these words: '*I feel very fortunate to have been his graduate student as I learned from him much more than logic. It is his humanity that conquered my heart.*'

Leon Henkin and a Life of Service The contribution by Benjamin Wells is entirely of biographical character. The author includes several letters from Henkin and some personal communications held between them. He analyzes some historical events surrounding some classical discoveries, including the controversy around some of the Mal'tsev's contributions.

Biographical Note *Leon Henkin*, by Maria Manzano and Enrique Alonso, provides some key biographical notes to Henkin's life. It guides readers through Henkin's formative period and his most outstanding logical contributions, but also discusses his other interests, most notably his dedication to mathematical pedagogy and minority-group education.

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