Preface

Stochastic Analysis is the branch of mathematics that deals with the analysis of dynamical systems affected by noise. It emerged as a core area of mathematics in the late twentieth century and has subsequently developed into an important theory with a wide range of powerful and novel tools, and with impressive diverse applications within and beyond mathematics. As so many systems are profoundly affected by stochastic fluctuations, it is not surprising that the array of applications of Stochastic Analysis is vast and touches many aspects of life.

This volume includes articles from some of the main contributors to the recent progress in stochastic analysis, and provides a snapshot of the current state of the area and its ongoing developments. It constitutes the proceedings of the conference on “Stochastic Analysis and Applications” held at the University of Oxford and the Oxford-Man Institute during 23–27 September 2013. The conference honoured the 60th birthday of Professor Terry Lyons FLSW FRSE FRS, Wallis Professor of Mathematics, University of Oxford. Terry Lyons is one of the leaders of the field of stochastic analysis. His introduction of the notion of rough path has revolutionized the field in both theory and applications.

A Biographical Account of Terry Lyons

Terence John Lyons was born in May 1953 in London. He went to Trinity College, Cambridge as an undergraduate from 1972–1976. His doctoral studies were in Oxford under the supervision of Richard Hayden. During this period he became interested in stochastic analysis and in particular some of the work of Paul Malliavin. In his early development he was influenced by Ted Gamelin and Henry McKean.

After completing his D.Phil. he held a Junior Research Fellowship at Jesus College, Oxford before taking up a Hedrick visiting professorship at UCLA for 1981–1982. He returned to the UK to take up a lectureship at Imperial College, London which he held until 1985. At that time he was appointed to the Colin

MacLaurin chair at the University of Edinburgh, where he took a turn as Head of Department. He moved back to a chair at Imperial College in 1993 but still maintained links with Edinburgh to encourage the group that he had started there. In 1993 he was awarded an EPSRC Senior Fellowship and used this to develop the theory of rough paths. In 2000 he moved to the Wallis Professorship of Mathematics at the University of Oxford and a Professorial Fellowship at St Anne's College.

Over the course of his career he has been awarded many prizes and honours. He was awarded the Rollo Davidson prize in 1985, a London Mathematical Society Junior Whitehead prize in 1986. He was awarded the Polya prize of the LMS in 2000. He was elected a fellow of the Royal Society of Edinburgh in 1987, the Royal Society in 2002 and the Learned Society of Wales in 2011. He has an Honorary degree from the University Paul Sabatier, Toulouse as well as Honorary fellowships at the Universities of Aberystwyth and Cardiff. He became the first director of the Wales Institute of Mathematical and Computational Sciences in 2008, which he held until 2011 when he became the director of the Oxford-Man Institute of Quantitative Finance. In 2013 he became president of the London Mathematical Society. He has been an invited speaker at the International Congress of Mathematicians, the Schramm lecturer for the Institute of Mathematical Statistics as well as delivering expository lecture courses such as the Summer School in Probability at St Flour.

Terry has been very much concerned with providing for the next generation of mathematicians. From 1988–2000 he managed a series of three EU grants bringing together the leading European institutions in the field, allowing the development of a generation of stochastic analysts. He has also been a great proponent of mathematics in bringing new ideas and techniques from academia to industry and, in particular, its transformative power in tackling the challenges thrown up by the growth and complexity of many aspects of modern society. He has written ground breaking papers on potential theory, Dirichlet forms, Markov chains, numerical analysis, filtering and mathematical finance. However, it is his pathwise view of integration, developed as the theory of rough paths, that has had a profound effect both within and outside the field of stochastic analysis. It has led to deep and powerful results in stochastic differential equations and stochastic partial differential equations. In particular it provided the initial tools that were built on by Martin Hairer in the work that led to the award of a Fields Medal in 2014. Terry Lyons' introduction of the signature of a path as a tool to provide an effective description of a path and the way that it acts has already provided new insights into efficient extraction of information from data.

In his personal life Terry married Barbara in 1975 and has two children, Barnaby, born 1981, and Josephine, born 1983, and currently one grandchild, born 2014.

Throughout his career, Terry Lyons supervised a large number of Ph.D. students. The following is the list of students that have completed or are currently working on their Ph.D. under his supervision:

Babbar, Katia	Boutaib, Youness	Caruana, Michael
Chang, C.Y.	Chevyrev, Ilya	Crisan, Dan
Dickinson, Andrew S.	Fawcett, Thomas	Flint, Guy
Gaines, Jessica	Gyurko, Greg	Hitchcock, David
Hoff, Ben	Janssen, Arend	Liang, Gechun
Litterer, Christian	Lunt, John	Ni, Hao
Pan, Wei	Penrose, Mathew	Potter, Chris
Rapoo (Sipilainen), Eeva	Skipper, Max	Smith, Adam
Tchernychova, Maria	Victoir, Nicolas	Williams, David
Xu, Weijun	Yam, Phillip	Yang, Danyu

The Contents of This Volume

The first chapter starts with the contribution of Shigeki Aida. In it, the author continues his previous work on the study of the strong convergence of Wong-Zakai approximations of the solution to the reflecting stochastic differential equations. In this chapter, he proves the strong convergence under weaker assumptions on the domain. The first main theorem shows the convergence when the domain is convex. The estimate of the order of the convergence is the same as that given in the previous work. The second main theorem establishes the convergence when the domain is not convex, but satisfies certain additional conditions.

The contribution of Dominique Bakry contains a description of symmetric diffusion operators where the spectral decomposition is given through a family of orthogonal polynomials. In dimension one, this reduces to the case of Hermite, Laguerre and Jacobi polynomials. In higher dimension, some basic examples arise from compact Lie groups. The author gives a complete description of the bounded sets on which such operators may live and a classification of those sets when the polynomials are ordered according to their usual degrees.

The contribution of Erich Baur and Jean Bertoin discusses old and new results related to the destruction of a random recursive tree (RRT), in which its edges are cut one after the other in a uniform random order. In particular, the authors study the number of steps needed to isolate or disconnect certain distinguished vertices when the size of the tree tends to infinity. New probabilistic explanations are given in terms of the so-called cut-tree and the tree of component sizes, which both encode different aspects of the destruction process. Finally, the authors establish the connection to Bernoulli bond percolation on large RRTs and present recent results on the cluster sizes in the supercritical regime.

The contribution of René Carmona and Francois Delarue discusses the Master Equation for large population equilibriums. The authors use a simple N-player stochastic game with idiosyncratic and common noises to introduce the concept of the Master Equation, originally proposed by Lions in his lectures at the Collège de France. They highlight the stochastic nature of the limit distributions of the states of the players due to the fact that the random environment does not average out in the limit, and recast the Mean Field Game (MFG) paradigm in a set of coupled Stochastic Partial Differential Equations. The first one is a forward stochastic Kolmogorov equation giving the evolution of the conditional distributions of the states of the players given the common noise. The second equation has the form of a stochastic Hamilton Jacobi Bellman (HJB) equation providing the solution of the optimization problem when the flow of conditional distributions is given. Being highly coupled, the system reads as an infinite dimensional Forward Backward Stochastic Differential Equation (FBSDE). Uniqueness of a solution and its Markov property lead to the representation of the solution of the backward equation (i.e. the value function of the stochastic HJB equation) as a deterministic function of the solution of the forward Kolmogorov equation, function which is usually called the decoupling field of the FBSDE. The (infinite dimensional) PDE satisfied by this decoupling field is identified with the master equation. Finally the authors show that this equation can be derived for other large populations equilibriums like those given by the optimal control of McKean-Vlasov stochastic differential equations.

The contribution of Thomas Cass, Martin Clark and Dan Crisan revisits the filtering equations. The problem of nonlinear filtering has engendered a surprising number of mathematical techniques for its treatment. A notable example is the change-of-probability-measure method introduced by Kallianpur and Striebel to derive the filtering equations and the Bayes-like formula that bears their names. More recent work, however, has generally preferred other methods. In this chapter, the authors reconsider the change-of-measure approach to the derivation of the filtering equations and show that many of the technical conditions present in previous work can be relaxed. The filtering equations are established for general Markov signal processes that can be described by a martingale problem formulation. Two specific applications are treated.

The contribution of Ana Bela Cruzeiro and Remi Lassalle discusses the stochastic least action principle for the Navier-Stokes equation. The authors extend the class of stochastic processes allowed to represent solutions of the Navier-Stokes equation on the two-dimensional torus to certain non-Markovian processes, which they call admissible. More precisely, they provide a criterion for the associated mean velocity field to solve this equation. Due to the fluctuations of the shift, a new term of pressure appears which is of purely stochastic origin. The authors also provide an alternative formulation of this least action principle by means of transformations of measure. Within this approach, the action is a function of the law of the processes, while the variations are induced by some translations on the space of the divergence-free vector fields. Due to the renormalization in the definition of the cylindrical Brownian motion, this action is only related to the relative entropy by an inequality. However it is shown that, if the high frequency modes are cut, this new approach provides a least action principle for the Navier-Stokes equation based on the relative entropy.

The contribution of Sandy Davie studies the dyadic method of Komlós, Major and Tusnády (KMT), which is a powerful way of constructing simultaneous normal approximations to a sequence of partial sums of i.i.d. random variables. The author uses a version of this KMT method to obtain a first-order approximation in a Vaserstein metric to solutions of vector SDEs under a mild nondegeneracy condition using an easily implemented numerical scheme.

The contribution of Joscha Diehl, Peter Friz and Harald Oberhauser studies partial differential equations driven by rough paths. This is a continuation of the authors' earlier work on the subject motivated by the Lions-Souganidis theory of viscosity solutions for SPDEs. The authors continue and complement the previous (uniqueness) results with general existence and regularity statements. Much of this is transformed to questions for deterministic parabolic partial differential equations in viscosity sense. On a technical level, the authors establish a refined parabolic theorem of sums which may be useful in its own right.

The contribution of Yidong Dong and Ronnie Sircar discusses time-inconsistent portfolio investment problems. The explicit results for the classical Merton optimal investment/consumption problem rely on the use of constant risk aversion parameters and exponential discounting. However, many studies have suggested that individual investors can have different risk aversions over time, and they discount future rewards less rapidly than exponentially. While state-dependent risk aversion and nonexponential type (e.g. hyperbolic) discounting align more with real life behaviour and household consumption data, they have tractability issues and make the problem time-inconsistent. In their contribution, Dong and Sircar analyse the cases where these problems can be closely approximated by time-consistent ones. Using asymptotic approximations, they are able to characterize the equilibrium strategies explicitly in terms of corrections to solutions for the base problems with constant risk aversion and exponential discounting. The authors also explore the effects of hyperbolic discounting under proportional transaction costs.

The contribution of David Elworthy discusses decompositions of diffusion operators and related couplings. Results by Cranston, Greven and Feng-Yu Wang on relationships between coupling and shift coupling, and harmonic functions and space time harmonic functions are reviewed. These lead to extensions of a result by Freire on the separate harmonicity of bounded harmonic functions on certain product manifolds. The extensions are to situations where a diffusion operator is decomposed into the sum of two other commuting diffusion operators. This is shown to arise for a class of foliated Riemannian manifolds with totally geodesic leaves. A form of skew product decomposition of Brownian motions on these foliated manifolds is obtained, as are gradient estimates in leaf directions. Relationships between stochastic completeness of the manifold itself and stochastic completeness of its leaves are established. Baudoin and Garafola's "sub-Riemannian manifolds with transverse symmetries" are shown to be examples.

The contribution of Hans Föllmer and Claudia Klüppelberg studies a mathematical consistency problem motivated by the interplay between local and global risk assessment in a large financial network. In analogy to the theory of Gibbs measures in Statistical Mechanics, they focus on the structure of global convex risk measures which are consistent with a given family of local conditional risk measures. Going beyond the locally law-invariant (and hence entropic) case, the authors show that a global risk measure can be characterized by its behaviour on a suitable boundary field. In particular, a global risk measure may not be uniquely determined by its local specification, and this can be seen as a source of "systemic risk" in analogy to the appearance of phase transitions in the theory of Gibbs measures. The proof combines the spatial version of Dynkin's method for constructing the entrance boundary of a Markov process with a certain nonlinear extension of backwards martingale convergence.

In their contribution, Masatoshi Fukushima and Hiroshi Kaneko discuss the Villat's kernels and BMD Schwarz kernels in Komatu-Loewner equations. The classical Loewner differential equation for simply connected domains is attracting new attention since Schramm launched in 2000 the stochastic Loewner evolution (SLE) based on it. The Loewner equation itself has been extended to various canonical domains of multiple connectivity after the works by Komatu in 1943 and 1950, but the Komatu-Loewner (K-L) equations have been derived rigorously only in the left derivative sense. In a recent work, Chen, Fukushima and Rhode prove that the K-L equation for the standard slit domain is a genuine ODE by using a probabilistic method together with an SDE method, and that the right-hand side of the equation admits an expression in terms of the complex Poisson kernel of the Brownian motion with darning (BMD). In the present paper, K-L equations for the annulus and circularly slit annili are investigated. For the annulus, they establish a K-L equation as a genuine ODE possessing a normalized Villat's kernel on its right-hand side by using a variant of the Carathéodory convergence theorem for annuli indicated by Komatu. This method is also used to obtain the same K-L equation in the right derivative sense on annulus for a more general family of growing hulls that satisfies a specific right continuity condition usually adopted in the SLE theory. Villat's kernel is then identified with a BMD Schwarz kernel for the annulus. Finally, the authors derive K-L equations for circularly slit annuli in terms of their normalized BMD Schwarz kernels, but only in the left derivative sense when at least one circular slit is present.

Tomoyuki Ichiba and Ioannis Karatzas study the unfolding of the Skorokhod reflection of a continuous semimartingale, in a possibly skewed manner, into another continuous semimartingale on an enlarged probability space according to the excursion-theoretic methodology of Prokaj. This is done in terms of a skew version of the Tanaka equation, whose properties are studied in some detail. The result is used to construct a system of two diffusive particles with rank-based characteristics and skew-elastic collisions. Unfoldings of conventional reflections are also discussed, as are examples involving skew Brownian Motions and skew Bessel processes.

David Nualart contributes with a survey of some recent developments in the applications of Malliavin calculus combined with Stein's method to derive central limit theorems for random variables on a finite sum of Wiener chaos. Starting from the fourth moment theorem by Nualart and Peccati, the author discusses several related topics such as conditions for the convergence in total variation, absolute continuity of probability laws and uniform convergence of densities under suitable nondegeneracy assumptions. The fact that the random variables belong to a fixed

Wiener chaos (or to a finite sum of Wiener chaos) will play a fundamental role in the results. Normal approximation on a finite Wiener chaos.

Zhenjie Ren, Nizar Touzi and Jianfeng Zhang provide an overview of the recently developed notion of viscosity solutions of path-dependent partial differential equations. The authors review the Crandall-Ishii notion of viscosity solutions, so as to motivate the relevance of the definition in the path-dependent case. The authors focus on the well-posedness theory of such equations. In particular, they provide a simple presentation of the current existence and uniqueness arguments in the semilinear case and review the stability property of this notion of solutions, including the adaptation of the Barles-Souganidis monotonic scheme approximation method. The results rely crucially on the theory of optimal stopping under nonlinear expectation. In the dominated case, we provide a self-contained presentation of all required results. The fully nonlinear case is more involved and is addressed elsewhere.

Marta Sanz-Sole and Andre Suess study logarithmic asymptotics of the densities of SPDEs driven by spatially correlated noise. The authors consider the family of stochastic partial differential equations indexed by a parameter $\varepsilon \in (0,1]$,

$$Lu^{\varepsilon}(t,x) = \varepsilon\sigma(u^{\varepsilon}(t,x))\dot{F}(t,x) + b(u^{\varepsilon}(t,x)),$$

$(t,x) \in (0,T] \times \mathbb{R}^d$ with suitable initial conditions. In this equation, L is a second-order partial differential operator with constant coefficients, σ and b are smooth functions and $\dot{F}$ is a Gaussian noise, white in time and with a stationary correlation in space. Let $p^{\varepsilon}_{t,x}$ denote the density of the law of $u^{\varepsilon}(t,x)$ at a fixed point $(t,x) \in (0,T] \times \mathbb{R}^d$. The authors study the existence of $\lim_{\varepsilon\downarrow 0} \varepsilon^2 \log p^{\varepsilon}_{t,x}(y)$ for a fixed $y \in \mathbb{R}$. The results apply to classes of stochastic wave equations with $d \in \{1,2,3\}$ and stochastic heat equations with $d \geq 1$.

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