

Preface

Nonlinear analysis is a remarkable mixture of topology (of several different types), analysis (both “hard” and “soft”), and applied mathematics. Mathematicians with a correspondingly wide variety of interests should become acquainted with this important, rapidly developing subject. But it’s a BIG subject. You can feel it: just weigh in your hand Eberhard Zeidler’s *Nonlinear Functional Analysis and Its Applications* (Zeidler, 1986). It’s heavy, as a 900 page book must be. Yet this is no encyclopedia; the preface accurately describes its “. . . very careful selection of material . . .” And what you are holding in your hand is Part I of a five-part work.

So how do you get started learning nonlinear analysis? Zeidler’s book has a first page, and some people are quite content to begin right there. For an alternative, the bibliography in Zeidler (1986), which is 42 pages long, contains exposition as well as research results: monographs that explain greater or lesser portions of the subject to a variety of audiences. In particular Deimling (1985) covers much of the material of Zeidler’s book. Then what’s different about the exposition in this book? My answer is in three parts: this book is (i) topological, (ii) goal-oriented, and (iii) a model of its subject. The next three little paragraphs explain what each of these means.

(i) As the title states, this is a *topological* book (though it’s not a book of topology). I’m a topologist and, as I’ve studied nonlinear analysis, I’ve become impressed by the extent to which the subject rests, in a strikingly simple and natural way, on basic topological ideas. These ideas come from general (point-set) topology, from metric space topology and, in the form of classical homology theory, from algebraic topology as well. It’s possible to disguise, or even to replace to some extent, the substantial topological content of this subject, but that won’t happen in this book. On the contrary, we’ll make sure our analysis rests on a secure base of carefully expounded topology.

(ii) The goal of this book has a name: the Krasnoselski–Rabinowitz bifurcation theorem. By the time you finish this book you will know what this beautiful result says, understand why it is true, and, through a single but very striking instance, get some idea of how it is applied. You can come to this book with little specific preparation beyond the undergraduate real analysis level. Yet by the end of its

relatively few pages you will see how, in the late twentieth century (ca. 1970), we gained a new understanding of an eighteenth-century model of a column collapsing under excessive weight.

(iii) Beyond its power and elegance, the Krasnoselski–Rabinowitz theorem has another virtue that made it irresistible as a topic for this book: the structure of its proof and this application is itself a model of the interplay of topological and analytic ideas that is characteristic of much of nonlinear analysis. The topological ingredients for the proof come from all the branches I mentioned: a separation theorem for compact topological spaces from general topology, Ascoli–Arzela theory from metric space topology, and the Leray–Schauder degree from algebraic topology. A key step in the proof is a calculation formula for the Leray–Schauder degree which, in turn, depends on a substantial topic in functional (“soft”) analysis: the spectral theory of compact linear operators on Banach spaces. The classical “hard” analysis comes into play once we have the relatively abstract bifurcation theorem and want to use it to study the ordinary differential equation problem that models the column buckling.

As a curtain raiser to the relatively extensive discussions that lead us to the Krasnoselski–Rabinowitz theorem, I’ll show you a simpler and more classical tool from the nonlinear analyst’s toolbox: the Schauder fixed point theorem, along with a rather easily understood application of it. This is also a model of nonlinear analysis: the topological topics of the Ascoli–Arzela theorem and fixed point theory are applied, with the help of some elementary but clever calculus, to investigate the equilibrium distribution of heat in a rod.

Jean Mawhin’s eloquent argument in Mawhin (1988) that much of nonlinear analysis could be illustrated in the context of the forced pendulum suggested some quite direct applications of the two main tools of this book, the Schauder fixed point theorem and the Leray–Schauder degree. In particular, the reader can see a demonstration of the usefulness of the degree before being introduced to bifurcation theory.

This book was born at a conference at the University of Montreal organized by Andrzej Granas in 1983 where the talks, especially those of Ronald Guenther, Roger Nussbaum, and Paul Rabinowitz, made nonlinear analysis accessible. UCLA gave me the opportunity to communicate what I was learning about this subject, and to refine these notes, through specialized courses I taught in 1984, 1987, and 1992. The students and colleagues who attended these courses or talked to me about my plans helped me in many ways. I thank especially Joseph Bennish, Jerzy Dydak, Massimo Furi, Reiner Martin, and PierLuigi Zezza. The first time I taught about topology and nonlinear analysis, my late colleague Earl Coddington faithfully attended my lectures and didn’t seem to think it was ridiculous for a topologist to try to present analysis from his own point of view. The fact that this book was written is a consequence of Earl’s encouragement.

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