

# Preface

This booklet is devoted to the study of controllability of systems with persistent memory, which are important in engineering applications, for example, to viscoelasticity, non-Fickian diffusion and thermal processes with memory. Earlier studies of controllability are due to Leugering, see [63–65], and by now controllability under the action of boundary deformation has been studied with several different methods while controllability under boundary traction has been less considered. We give an overview of some of the ideas used in this kind of study for systems of “hyperbolic” type, i.e. finite velocity of signal propagation (the “parabolic” case is far less studied, see [8, 37, 41–43]): operator methods (introduced by Belleni-Morante in [10] and used for control problems in [74]) are in Chap. 2; a moment method approach to controllability is in Chap. 5; a circle of ideas introduced by Kim in [52] and which relays on the observation inequality is in Chap. 6.

To familiarize the readers with the subject, Chap. 1 treats a very simple example; Chap. 4 recalls known properties of the controllability of the (memoryless) wave equation and Chap. 3 presents the results we shall need on Riesz sequences, stressing the applications to the solution of the moment problems encountered in “hyperbolic” type systems.

We note that “controllability” as studied here is not state controllability as defined in [50] since the “state” of a system with persistent memory takes into account the entire past evolution of the system, see [18, 19, 25], and so state controllability is clearly impossible: we study the possibility of hitting a target at a given time. Section 5.4 shows an application of this notion of controllability to a source identification problem.

Finally, we mention problems that we are not going to study. First, we note that new possibilities arise when studying systems with persistent memory: the flux of heat and the temperature or, analogously, the stress and the deformation or the velocity of deformation, are only weakly related and it makes sense to understand how strong the relation is. This problem can be recast in the form of a control problem and leads to the study of the controllability of the pairs (temperature/flux), (deformation/stress) and (velocity of deformation/stress), which is a novelty of memory systems, see [4, 3, 80]. We shall always assume that the memory kernels

are smooth. Approximate controllability for kernels with (mild) singularities has been studied for example in [31, 40, 49]. Finally we cite [29, 97], where controllability is studied using Carleman estimates.

This book is addressed to people interested in control theory, with basic knowledge of functional analysis. In order to state clearly the results that are needed, the final section of Chap. 1 recalls general notions while more special properties are explicitly recalled when needed.

The solutions to the problems proposed in every chapter can be downloaded from the author's WEB page at the address <http://calvino.polito.it/~lucipan/ricerca.html>.

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