

# Qualitative Particle Swarm Optimization (Q-PSO) for Energy-Efficient Building Designs

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**Abstract.** Particle Swarm Optimization (PSO) is a stochastic optimization method, based on the social behavior of bird flocks. The method, known for its high performance in optimization, has been mainly developed for problems involving just quantitative variables. In this paper we propose a new approach called Qualitative Particle Swarm Optimization (Q-PSO) where the variables in the optimization can be both qualitative and quantitative and the updating rule is derived adopting probabilistic measures. We apply this procedure to a complex engineering optimization problem concerning building façade design. More specifically, we address the problem of deriving an energy-efficient building design, i.e. a design that minimizes the energy consumption (and the emission of carbon dioxide) for heating, cooling and lighting. We develop a simulation study to evaluate Q-PSO procedure and we derive comparisons with most conventional approaches. The study shows a very good performance of our approach in achieving the assigned target.

**Keywords:** Qualitative particle swarm optimization · Engineering optimization · Energy-efficient building design

## 1 Introduction

*Particle Swarm Optimization* (PSO) is an optimization method inspired by the social behavior of bird flocks or fish schools. Introduced by Kennedy and Eberhart [1] as a stochastic optimization algorithm, PSO is a population-based search procedure in which the population is conceived as a swarm composed of particles. In this approach each particle moves in the search space with an adaptable velocity, recording the best position it has ever visited in the search space, i.e. the position with the lowest objective function value when we deal with the minimization problems. Each particle has a neighborhood that consists of some pre-defined particles and the best position attained so far by each

member of the neighborhood is communicated to the particle itself and affects its movement. In this way a particle moves in the search space looking for the optimal values with both the information on its best position, called *cognitive component*, and the information on the best position reached by the neighborhood, called *social component*. PSO has been applied to solve a large number of optimization problems [2], mainly to search in continuous domains. There are few variants of the approach that operate in discrete spaces, such as the procedures developed for binary problems [3], or integer and combinatorial problems [4, 6].

In this paper we propose an innovative PSO approach able to deal with problems characterized by both qualitative and quantitative variables. We develop this approach to address an optimization problem for the design of energy-efficient building envelopes. More specifically we consider the problem of minimizing the carbon dioxide emissions due to the heating, cooling and lighting energy consumption in a room regarded as a module of a building.

The paper is structured as follows. In Sect. 2 we describe PSO approach and we show the updating rules by which the particles change their position in the search space; in Sect. 3 we introduce the *Qualitative Particle Swarm Optimization* (Q-PSO) to address problems that involve qualitative variables. In Sect. 4 we perform a simulation study to test the approach and evaluate its performance in deriving optimal values for designing energy-efficient building façades. Finally in Sect. 5, we derive some remarks on the performance of the approach also in comparison with other procedures.

## 2 Particle Swarm Optimization (PSO) Approach

Addressing continuous optimization problems which involve the optimization of an objective function  $h(\mathbf{x})$  of a variable vector  $\mathbf{x}$  defined on a D-dimensional space, PSO algorithm adopts a population (swarm) of particles that adjust their position in time according to their own information and neighborhood particles information. In this procedure the swarm  $S$  is composed by  $P$  *particles*. At each iteration (step  $t$  of the algorithm), the  $i$ -th particle of the swarm is associated with the position in the continuous D-dimensional search space  $\mathbf{x}_i(t) = [x_{i,1}(t), x_{i,2}(t), \dots, x_{i,D}(t)]$ , and with its velocity, describing the last particle position change,  $\mathbf{v}_i(t) = [v_{i,1}(t), v_{i,2}(t), \dots, v_{i,D}(t)]$ . To the  $i$ -th particle it is then associated the objective function  $h(\mathbf{x}_i)$ , with  $i \in \{1, 2, \dots, P\}$ .

At the first step of the procedure, both the position and velocity vectors are randomly initialized within a range of feasible values. Then, both of these parameters are iteratively updated until a stopping criterion is met. The update equation rules are:

$$v_{i,d}(t+1) = f(v_{i,d}(t), x_{i,d}(t), pbest_{i,d}(t), nbest_{i,d}(t)) \quad (2.1)$$

$$x_{i,d}(t+1) = g(x_{i,d}(t), v_{i,d}(t+1)) \quad (2.2)$$

where  $f(\cdot)$  and  $g(\cdot)$  are suitable functions,  $pbest_{i,d}(t)$  is the  $d$ -th element of the vector  $\mathbf{pbest}_i(t)$  representing the historical best position of the particle  $i$ , i.e.

the lowest objective function value (personal best), and  $nbest_{i,d}(t)$  is the  $d$ -th element of vector  $\mathbf{nbest}_i(t)$  related to the best position of the  $i$ -th particle's neighborhood (social best).

According to these rules, the  $i$ -th particle position at step  $(t + 1)$  is determined by the function  $g(\cdot)$  of its previous position and its velocity at time  $t$ . Moreover the velocity of  $i$ -th particle at time  $(t + 1)$  is a function  $f(\cdot)$  depending on the velocity and the position at time  $t$ , and the best positions achieved by the particle and its neighborhood. Based on the paper of Kennedy and Eberhart [1], the functions  $f(\cdot)$  and  $g(\cdot)$  are specified by the following equations:

$$v_{i,d}(t + 1) = v_{i,d}(t) + c_1\rho_1(pbest_{i,d}(t) - x_{i,d}(t)) + c_2\rho_2(nbest_{i,d}(t) - x_{i,d}(t)) \quad (2.3)$$

$$x_{i,d}(t + 1) = x_{i,d}(t) + v_{i,d}(t + 1) \quad (2.4)$$

where  $c_1$  and  $c_2$  are two positive constants representing the cognitive and social acceleration coefficients;  $\rho_1, \rho_2 \sim \mathcal{U}(0, 1)$  are two independent uniformly distributed random values in the range  $[0, 1]$  introduced to weight the velocity toward the particle personal best and the velocity toward the global best solution.

When the neighborhood of each particle is represented by the whole swarm, then  $\mathbf{nbest}_i$  naturally becomes  $\mathbf{gbest}$ , that is the global best solution. This condition is known as the *gbest topology*. Under this condition all the particles are connected among them, achieving a full connected graph: the unique best particle position in the entire population affects all the other particles positions.

One of the most problematic characteristic of PSO is its propensity to converge, prematurely, on early best solutions. In the paper of Shi and Eberhart [7] a *inertia* weight,  $w$ , was introduced to control the velocity of the particles in order to overcome this limitation. In their successive paper [8], a linearly decreasing  $w$  is proposed changing its value according to the number of iterations. Further, in order to improve the performance of the method, Chatterjee and Siarry [9] suggested an updating equation for the *inertia* weight  $w$  that introduces a non linear component in the velocity function. Rather than considering the *inertia* weight, Clerc and Kennedy [5] introduced the constriction coefficient, which alleviates the requirement to clamp the velocity.

Considering discrete optimization problems, several PSO algorithms have been proposed in the literature. Kennedy and Eberhart [3] introduced a procedure able to deal with sequences of bits rather than real numbers. In this approach each element of the velocity vector is bound in the interval  $[0, 1]$  through a sigmoid function, setting a probability threshold to flip the  $d$ -th element of the vector  $\mathbf{x}_i(t)$  from 0 to 1 or vice versa. This procedure can also be adopted when the accessible values are more than two, simply converting decimal to binary numbers (i.e. gray code). Liao et al. [10] extended discrete PSO [3] to solve the flow-shop scheduling problem by redefining the particle and the velocity and incorporating a local search scheme to move a particle to the new sequence. Dealing with integer problems, the approach proposed by Laskari et al. [4] addresses the issue by simply rounding the updated value  $x_{i,d}(t + 1)$  in (2.4)

to the nearest integer  $x_{i,d}^*(t+1)$ . For a review on PSO for discrete optimization problems see [11].

Addressing directly the problem of optimization involving both qualitative and quantitative variables, we propose a novel approach based on the concept of probabilistic attraction.

### 3 Qualitative Particle Swarm Optimization (Q-PSO)

Many real-world optimization problems require the involvement of qualitative variables that are variables whose values are represented by a finite set of labels. In this work, we consider nominal qualitative variables, i.e. variables described by non-ordered labels.

In order to derive an optimization procedure based on the fundamental principles of PSO and suitable to deal with qualitative variables, we propose to introduce in the algorithm an updating probabilistic rule. More specifically we propose to update the value of a qualitative variable using a probability distribution rather than a velocity parameter. For a particle  $i$ , we specify the position vector as follows:

$$\mathbf{x}_i = (\underbrace{x_{i,1}, x_{i,2}, \dots, x_{i,Q}}_{\text{qualitative variables}}, \underbrace{x_{i,Q+1}, x_{i,Q+2}, \dots, x_{i,Q+C}}_{\text{quantitative variables}}), \quad (3.1)$$

$$\mathbf{x}_i \in L_1 \times \dots \times L_Q \times \mathbb{R}^C$$

where the first  $Q$  variables are qualitative while the remaining  $C$  variables are quantitative (continuous). We then denote with  $L_q$  the set of labels of the  $q$ -th qualitative variable, with  $q \in \{1, 2, \dots, Q\}$  and  $n_{L_q}$  the cardinality of the set  $L_q$ .

The position and the velocity of the quantitative variables can be updated using the rules introduced by canonical PSO algorithms. The parameters of the qualitative variables introduced in this algorithm require instead the specification of a new set of equations.

To build a PSO algorithm that allows the consideration of qualitative variables, we introduce the concept of *probabilistic attraction*. The probabilistic attraction consists in sampling from the  $L_q$  possible labels of each qualitative variable with a probability distribution. At generation  $t+1$ , this distribution depends on the contribution of each label in determining the global and local best positions at generation  $t$  of the algorithm. In this way we sample more frequently those labels that contribute most to determine optimal values of the objective function, and less frequently the others. With the probabilistic attraction procedure we also assign a non-zero probability to choose other labels of the variable, avoiding in this way to get stuck in some local optimum.

Considering the  $i$ -th particle, with  $i \in \{1, 2, \dots, P\}$ , and the  $q$ -th qualitative variable, with  $q \in \{1, 2, \dots, Q\}$ , at generation  $t+1$  we define the probabilistic attraction procedure as follows.

Let  $\pi$  be the equal sampling probability of each label of the  $q$ -th variable (equiprobability condition),  ${}_i\pi_q^{pbest}(t)$  the probability to choose the individual best

position of particle  $i$  for the  $q$ -th variable at generation  $t$ ,  $\mathbf{X}(t)$  the position matrix of all the particles in the swarm at time  $t$ ,  $\mathbf{X}(t) = [\mathbf{x}_1(t), \mathbf{x}_2(t), \dots, \mathbf{x}_P(t)]^T$ , and  $\mathbf{\Pi}(t)$  the matrix of the individual best positions of each particle in the swarm at generation  $t$ ,  $\mathbf{\Pi}(t) = [\mathbf{pbest}_1(t), \mathbf{pbest}_2(t), \dots, \mathbf{pbest}_P(t)]^T$ .

Then the sampling probabilities of each label in  $L_q$ , based on the results obtained at the current generation  $t$ , is determined by:

1. the identification of the individual best label  ${}_iL_q^{pbest}(t)$  of the particle  $i$  at the current generation  $t$ , with respect to the objective function to be optimized;
2. the identification of the best global label  $L_q^{gbest}(t)$  of the whole swarm at the current generation  $t$ , in optimizing the objective function;
3. the assessment of the probability  ${}_i\pi_q^{pbest}(t+1)$  to choose the individual best label  ${}_iL_q^{pbest}(t)$  at the next generation  $t+1$

$$\begin{aligned} {}_i\pi_q^{pbest}(t+1) &= \Pr\{x_{i,q}(t+1) = {}_iL_q^{pbest}(t)\} \\ &= f(\pi, {}_i\pi_q^{pbest}(t), \mathbf{X}(t), \mathbf{\Pi}(t)), \end{aligned} \quad (3.2)$$

where  $f(\cdot)$  is a suitable function;

4. the computation of the probability  $\pi_q^{gbest}(t+1)$  to choose the global best label  $L_q^{gbest}(t)$  at the next generation  $t+1$

$$\begin{aligned} \pi_q^{gbest}(t+1) &= \Pr\{x_{i,q}(t+1) = L_q^{gbest}(t)\} \\ &= g(\pi, \pi_q^{gbest}(t), \mathbf{X}(t), \mathbf{\Pi}(t)), \end{aligned} \quad (3.3)$$

where  $g(\cdot)$  is a suitable function and  ${}_i\pi_q^{pbest}(t)$  in 3.2 is substituted by  $\pi_q^{gbest}(t)$ .

5. the computation of the probabilities of the labels that are not the individual or global best as follows:

$$\begin{aligned} {}_i\pi_q^l(t+1) &= h(\pi_q^{gbest}(t+1), {}_i\pi_q^{pbest_i}(t+1), n_{L_q}), \\ \forall l \in L_q, l \notin \{L_q^{gbest}, {}_iL_q^{pbest}\}. \end{aligned} \quad (3.4)$$

where  $h(\cdot)$  is a suitable function.

In this paper the functions in (3.2), (3.3) and (3.4) assume fixed values according to the case study that we will address. In particular we assume that  $h(\cdot)$  depends on  $\pi_q^{gbest}$  and  ${}_i\pi_q^{pbest}$ , as described in the following equation:

$${}_i\pi_q^l(t+1) = \frac{{}_i\pi_q^{res}(t+1)}{(n_{L_q} - 2)}, \quad \forall l \in L_q, l \notin \{L_q^{gbest}, {}_iL_q^{pbest}\}, \quad (3.5)$$

where

$${}_i\pi_q^{res}(t+1) = 1 - [\pi_q^{gbest}(t+1) + {}_i\pi_q^{pbest}(t+1)]. \quad (3.6)$$

For the  $k$ -th particle, which defines the best global label of the whole swarm, we have  $L_q^{gbest} = {}_kL_q^{pbest} = L_q^{best}$ . In this case we define its distribution probability as a monotonically increasing function  $\zeta(\cdot)$  as follows:

$$\Pr\{x_{k,q}(t+1) = L_q^{best}(t)\} = {}_k\pi_q^{best}(t+1) = \zeta(\pi_q^{gbest}, {}_k\pi_q^{pbest}). \quad (3.7)$$

This situation happens at least one time in each generation of the algorithm, because the global best position is selected among the individual best positions. In particular we assume that  ${}_k\pi_q^{best}(t+1) = [\pi_q^{gbest}(t+1) + {}_k\pi_q^{pbest}(t+1)]$ .

According to this assumption, the probability of each label different from  $L_q^{best}$  could be derived as in Eq. (3.5), and normalized to guarantee the constraint  $\sum_{j=1}^{n_L} \pi_q^j(t+1) = 1$ . Then the probability of sampling one of the remaining labels is calculated as follows:

$${}_k\pi_q^l(t+1) = \frac{{}_k\pi_q^l(t+1)}{\sum_{j=1}^{n_L} {}_k\pi_q^j}, \quad l \in \{1, \dots, n_L\}. \quad (3.8)$$

At time  $t+1$ , when all the probability values are determined, we update the  $q$ -th qualitative variable of  $\mathbf{x}_i$  according to the following distribution function:

$$x_{i,q} = \begin{cases} L_q^{gbest}(t) & \pi_q^{gbest}(t+1) \\ {}_iL_q^{pbest}(t) & {}_i\pi_q^{pbest}(t+1) \\ {}_iL_q^l \neq \{L_q^{gbest}(t), {}_iL_q^{pbest}(t)\} & {}_i\pi_q^l(t+1) \quad l \notin \{pbest, gbest\}, \end{cases} \quad (3.9)$$

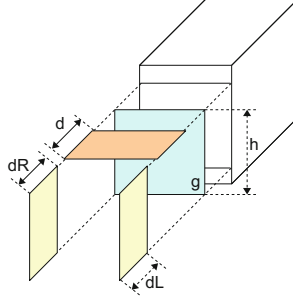
where  $\pi_q^{gbest}(t+1) > {}_i\pi_q^{pbest}(t+1) > {}_i\pi_q^l(t+1)$ .

We denote this approach *Qualitative Particle Swarm Optimization (Q-PSO)*. It represents a generalization of the particle swarm optimization approach, since allows framing the problems considering both quantitative and qualitative variables. In this paper we will derive Q-PSO approach for the problem of designing building envelopes with the objective of reducing the consumption of energy. Some of the variables involved in this optimization problem are in fact qualitative, and a more general PSO approach may lead to a more accurate analysis.

## 4 Q-PSO for Energy-Efficient Building Envelope Designs

The reduction of pollutant emissions is nowadays a fundamental problem related to the climate change issue and environmental sustainability. Major causes of air pollution are the emission of carbon dioxide from energy building consumption, vehicular traffic, and industrial production. In this article, we focus on the objective to reduce building carbon dioxide emissions meanwhile maintaining living and comfort conditions inside buildings. The quantity of energy consumed for heating, cooling and lighting a building is a function of several variables (qualitative and quantitative) and parameters, including the climatic conditions of the construction site, the building orientation, the materials, the type of insulation and the geometry of the interior and exterior. In this field, several studies have addressed the problem by using different optimization procedures, which include the evolutionary optimization approaches [12–14]. In particular, Zemella et al. [13] proposed an Evolutionary Neural Network to optimize a building envelope design where some qualitative variables were involved.

In designing energy-efficient building façades several variables have to be considered and their single and interactive effects on energy consumption should



**Fig. 1.** Graphical representation of variables involved in the optimization problem

be measured under a large set of conditions and constraints. In this research we focus on the following variables, regarded in the literature as the most relevant for the problem: the proportion of glaze in the surface, the depth of different overhangs for shading the windows and the type of glass used. All the considered variables are continuous quantitative variables except for type of glass, which is a nominal qualitative variable. We formulate the optimization problem as the minimization of the energy consumption necessary for maintaining the comfort conditions of a room, regarded as a module of a building. We address this problem by developing a simulation study. We compute the energy loads for heating  $Q_H$ , lighting  $Q_{light}$  and cooling  $Q_{cool}$  adopting the software EnergyPlus<sup>1</sup>.

According to Kragh and Simonella [15], we can estimate the amount of carbon dioxide emissions with the following expression:

$$E_{CO_2} = \frac{f_{gas}}{\eta_H} Q_H + f_{el} \left( Q_{light} + \frac{Q_{cool}}{COP} \right) \quad kgCO_2 \quad (4.1)$$

where  $Q_H, Q_{light}, Q_{cool}$  are measured in kWh,  $f_{gas} = 0.194 kgCO_2/kWh$  and  $f_{el} = 0.422 kgCO_2/kWh$  represent the amount of carbon dioxide for the production of 1kWh using natural gas or electricity respectively,  $\eta_H = 0.89$  is a measure of the efficiency of the heating system and  $COP$  is the performance's coefficient, that is the ratio of useful output to energy input. Here we refer to the London climatic conditions, so we assume  $COP = 3.4$  [13].

The variables involved in the minimization problem are:

- $h$ : height, in meters, of the glazed surface, discretized as follows

$$h \in \{0.75 + 0.05i, i = 0, \dots, N_h\}$$

$$N_h = \frac{h_{max} - h_{min}}{h_{step}}, \quad h_{max} = 2.85, \quad h_{min} = 0.75, \quad h_{step} = 0.05$$

<sup>1</sup> <http://apps1.eere.energy.gov/buildings/energyplus/>

**Table 1.** Parameters of the simulation

Algorithms	Parameters for the quantitative variables	Parameters for the qualitative variable
PSOCC	$\chi = 0.729, c_1 = c_2 = 2.05$	3 bit, Gray code, $V_{max} = 4.0$
PSOIW	$0.4 \leq w \leq 0.9, c_1 = c_2 = 1.49$	
PSOIWNL	$0 \leq w \leq 0.729, c_1 = c_2 = 1.49$	
Q-PSOCC	$\chi = 0.729, c_1 = c_2 = 2.05$	$\pi^{gbest} = 0.35, \pi^{pbest} = 0.25, \pi^{res} = 0.40$
Q-PSOIW	$0.4 \leq w \leq 0.9, c_1 = c_2 = 1.49$	
Q-PSOIWNL	$0 \leq w \leq 0.729, c_1 = c_2 = 1.49$	

- $d$ : depth, in meters, of the horizontal overhang, discretized as follows

$$d \in \{0.10 + 0.10i, i = 0, \dots, N_d\}$$

$$N_d = \frac{d_{max} - d_{min}}{d_{step}}, \quad d_{max} = 1.00, \quad h_{min} = 0.10, \quad h_{step} = 0.10$$

- $dL$ : depth, in meters, of the left vertical overhang, discretized as follows

$$d_L \in \{0.10 + 0.10i, i = 0, \dots, N_{dL}\}$$

$$N_{dL} = \frac{dL_{max} - dL_{min}}{dL_{step}}, \quad dL_{max} = 1.00, \quad dL_{min} = 0.10, \quad dL_{step} = 0.10$$

- $dR$ : depth, in meters, of the right vertical overhang, discretized as follows

$$d_R \in \{0.10 + 0.10i, i = 0, \dots, N_{dL}\}$$

$$N_{dR} = \frac{dR_{max} - dR_{min}}{dR_{step}}, \quad dR_{max} = 1.00, \quad dR_{min} = 0.10, \quad dR_{step} = 0.10$$

- $g$ : type of glass, with the following labels

$$g = \{g_0, g_1, g_2, g_3, g_4\}$$

#### 4.1 The Optimization Problem

Without loss of generality, we restrict the domains of variables  $h$ ,  $d$ ,  $dL$  and  $dR$  to a finite set of possible levels in order to achieve reasonable solutions. The variable  $g$ , coded by labels  $g_i$ ,  $i = 1 \dots 4$ , is considered as a nominal qualitative variable as it describes specific types of glass. A schematic graphical representation of the variables is reported in Fig. 1.

For designing a building façades we derive Q-PSO introduced in Sect. 3, which allows the consideration of both quantitative variables ( $h$ ,  $d$ ,  $dL$  and  $dR$ ) and a qualitative variable ( $g$ ). Since the search space consists of the finite set of points defined by the Cartesian product of the domains of each variable, the number of possible experimental points is equal to 215 000.



This number of candidate points makes extremely difficult to compute the objective function defined in 4.1 for the whole space as the calculation of  $Q_H$ ,  $Q_{light}$  and  $Q_{cool}$  by means of EnergyPlus would involve a very large amount of computational time. We address this problem by considering two different settings, which differ for the number of variables and complexity:

- *Case 1*: we characterize the problem with variables  $h$ ,  $d$  and  $g$ , which requires to test 2 150 experimental points;
- *Case 2*: we characterize the problem with variables  $h$ ,  $d$ ,  $g$ ,  $dL$  and  $dR$ , which requires to test 215 000 experimental points.

The relative small number of experimental points of *case 1* allows the evaluation of all the search space and the identification of the actual minimum value. The structure of *case 1* can then be used as a “test-bed” to evaluate the performance of Q-PSO.

Due to the computational time needed to evaluate the whole search space, for *case 2* we will consider the minimum value that has been found in the simulation study as the problem optimum value.

We address the optimization problem in the following way:

1. Initialization of a random population of  $P$  particles, where each particle represents a specific variable configuration for the façade (i.e. an experimental point);
2. Computation of the energy consumption associated to each particle by means of the simulation software EnergyPlus;
3. Computation of the amount of carbon dioxide produced by each particle using formula (4.1);
4. Update the particle positions according to the achieved result;
5. Repeat steps 2–4  $T$  times, where  $T$  is a parameter (called the number of generations) fixed by the investigator.

In this simulation we set different values of parameters for the experimentation: for *case 1* we use  $P = 30$  and  $T = 10$ , while for *case 2* we assume  $P = 50$  and  $T = 30$ . With respect to the orientation of the façade, we assume in this setting the East orientation.

Due to the stochastic nature of the optimization techniques, we decide to test the algorithm on 5 different runs in order to validate the results and to evaluate the performance of the optimization procedure. The implementation of the algorithm is realized by using the free software R-project<sup>2</sup>.

## 4.2 The Performance of Q-PSO Optimization Approach

In addressing the problem of designing energy-efficient façades we build Q-PSO with sampling probabilities based on the knowledge achieved on the process and some preliminary tests. In particular, we set:

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<sup>2</sup> <http://www.r-project.org>

$$\pi_q^{g^{best}}(t) = 0.35 \quad {}_i\pi_q^{p^{best}}(t) = 0.25 \quad \pi_q^l(t) = \frac{0.40}{3}, \quad (4.2)$$

$$l \notin \{L_q^{g^{best}}, {}_iL_q^{p^{best}}\}, \forall t \in \{1, 2, \dots, T\}, \forall i \in \{1, 2, \dots, P\} \quad (4.3)$$

To evaluate the performance of Q-PSO and develop comparisons with other approaches, we implement *binary PSO* as proposed by [3], currently used to deal with qualitative variables. We derive binary PSO encoding the labels of the variable  $g$  with a 3 bit Gray code. The comparison between binary PSO and Q-PSO is realized under different hypotheses on the *inertia* weight.

In particular, we consider:

- PSOCC and Q-PSOCC, a constriction coefficient  $\chi$  is introduced in according with [5];
- PSOIW and Q-PSOIW, linearly decreasing *inertia* weight is considered [8];
- PSOIWNL and Q-PSOIWNL, non-linearly decreasing *inertia* weight is considered [9].

In Table 1 we summarize the parameters,  $\chi$ ,  $w$ ,  $c_1$  and  $c_2$ , of the implemented algorithms for different weight structures. First we consider *case 1*, in order to evaluate the performance of the algorithms when the optimal solution is known, and later we will consider *case 2*, where the complexity of the problem imposes to regard the optimum as 175.60 (the best minimum value obtained in the simulations).

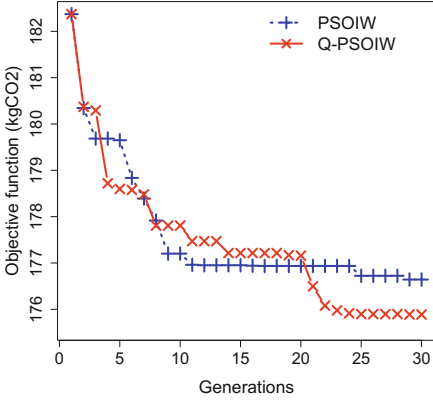
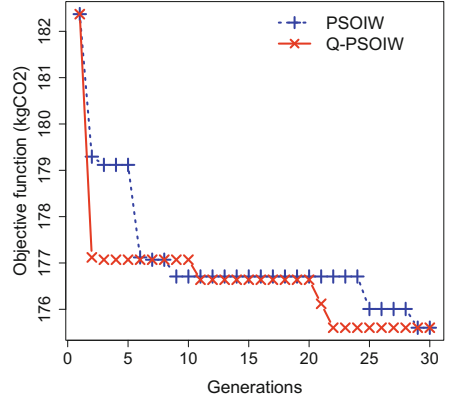
For case 1 we evaluate by simulation all the 2150 possible different façade configurations and achieve the minimum value 179.55, regarded as the target of the optimization procedures. The comparison of the algorithms performance is then derived by computing the average of the minimum values and the minimum value of the objective function determined on the best positions  $g^{best}(t)$  at each generation  $t \in \{1, 2, \dots, T\}$  by each method over 5 runs. The main results are presented in Table 2. We observe that Q-PSO finds better averages of the minimum values with respect to standard PSO. Both the approaches find the minimum value identified as target, but Q-PSO improves the average minimum solution in 5 runs reducing its variability. Q-PSO always reaches the target of the minimization problem, exhibiting a particular stability of the approach.

**Table 2.** Performance of the algorithms for case 1

	PSOCC	Q-PSOCC	PSOIW	Q-PSOIW	PSOIWNL	Q-PSOIWNL
Target	179.55	179.55	179.55	179.55	179.55	179.55
Average of the minimum value in 5 runs	180.04	<b>179.57</b>	180.04	<b>180.03</b>	180.48	<b>179.55</b>
(sd)	(1.05)	(0.04)	(1.05)	(1.03)	(1.27)	(0.00)
Num. of runs achieving the minimum	3	<b>4</b>	3	3	3	<b>5</b>

**Table 3.** Performance of the algorithms for case 2

	PSOCC	Q-PSOCC	PSOIW	Q-PSOIW	PSOIWNL	Q-PSOIWNL
Minimum value	175.60	175.60	175.60	175.60	175.60	175.60
Average of the minimum value in 5 runs	176.05	<b>175.90</b>	176.64	<b>175.89</b>	175.90	<b>175.60</b>
(sd)	(0.45)	(0.66)	(0.61)	(0.64)	(0.66)	(0.00)
Num. of runs achieving the minimum	1	<b>4</b>	1	<b>4</b>	4	<b>5</b>

(a) Average minimum value in 5 runs  
PSOIW vs Q-PSOIW(b) Minimum value in 5 runs  
PSOIW vs Q-PSOIW**Fig. 2.** Case 2: comparison of PSO and Q-PSO.

Similarly for *case 2* we should evaluate by simulation the 215000 possible different façade configurations to discover the minimum value of the objective function and regard this value as the target of the optimization procedures. Given the computational time needed to evaluate the objective function we proceed by comparing the behavior of the two approaches in 30 generations and different *inertia* weights. Results are summarized in Table 3. In Fig. 2 we can observe that also for complex problems, as the one proposed in *case 2*, Q-PSO outperforms PSO both in the behavior of the average of the minimum values and in the minimum values, for all the *inertia* weight structures here considered. Moreover, Q-PSO achieves the minimum value in fewer generations than PSO.

## 5 Concluding Remarks

In this paper we addressed an optimization problem which involves both quantitative and qualitative variables adopting the successful and well-known procedure of PSO. Given the difficulties of this procedure to deal with qualitative

variables we introduced a generalization of the procedure: Qualitative Particle Swarm Optimization (Q-PSO), which is a stochastic optimization approach able to deal both with qualitative and quantitative variables. Q-PSO approach is based on the idea of sampling labels of qualitative variables with a probability distribution depending on the global and individual best labels achieved in each generation of the algorithm.

We implemented Q-PSO to the case study of deriving the optimal design of a building envelope able to minimize the carbon emissions due to heating, cooling and lighting, and compared this approach to standard PSO (binary PSO). The study, which involves different structures of the problem, shows that Q-PSO outperforms PSO in almost all the parameter configurations that we considered. We observe in fact that Q-PSO achieves very good results in finding minimum values and averages of minimum values over a set of runs and in few generations of the algorithm. In conclusion, in this study we introduced a new, easy to implement and effective approach that allows considering qualitative variables in complex optimization problems.

As future works we intend to test Q-PSO to different case studies in order to generalize its properties and to study the robustness increasing the number of runs.

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