Chapter 2
Asymmetric Continuum: Basic Motions and Equations

2.1 Introduction

For axial, shear and rotational strains, we present the basic motion equations, which follow directly from the derivatives of the classic Newton formula; these motion equations can be quite independent; their reference displacements can be quite different. However, for the waves emitted from a common source (e.g., an earthquake focus) some of these motions might be correlated or phase shifted; these interaction fields explain the wave propagation due the rotation strain and shear strain mutual release-rebound processes. For a solid continuum we introduce the molecular transport motion as nuclei of a possible real transport inside a fracture domain. We consider also the important experimental data related to the appearances of the $E_{\varphi\varphi}$ components (strain measurement by Gomberg and Agnew 1996). Our theory explains these important discoveries. A similar system of the independent motions is introduced for fluids; besides the transport motion we consider the molecular shear strains and also the rotation molecular strains, both as the time derivatives of these strains.

2.2 Solids: Strain Fields and Molecular Transport

First we recall that the seismometer recordings do not prove the existence of displacements (cf., Eqs. 1.1a, b), just the existence of displacement derivatives; the seismometers record the integrated effect of the displacements derivatives. The strain waves could have even a length of some hundreds kilometers for a very big wave period. We assume that in our Asymmetric Continuuam the real displacements, even very small, do not exist; the real displacements appear only in the fracture processes as an integrated recording effect (cf., Chap. 1).

On the other hand, recent seismological observations of rotation waves by the very precise instruments, based on the Sagnac effect, were able to reveal and measure a very small spin, up to 10^{-9} rad/s. These recorded rotation motions are

R. Teisseyre and M. Teisseyre-Jeleńska, *Asymmetric Continuum*,
GeoPlanet: Earth and Planetary Sciences, DOI: 10.1007/978-3-642-31860-3_2,

not the simple point rotations, but relate to the rotation strains. These experimental observations give grounds for a new, relatively simple theoretical approach in which the strain moments used in a classic theory are replaced by the antisymmetric part of strains. In the Asymmetric Continuum Theory, we use displacements only as the reference motions.

Thus, in solids we can relay on the axial, shear and rotation motions, while the displacements and point rotations are neglected. However, we can add the molecular transport, as possible nuclei for the fracture processes. In this way we arrive at a kind of similarity with the motions in fluids (the transport velocities and the molecular strains). Solids include the strain motions and molecular displacements, while in fluids there will appear the molecular strains and transport motions.

We present the basic strains (axial, shear and rotational) as being related to the independent reference displacements (cf., Teisseyre 2009, 2011); thus all these strains, $E_{ik} = E_{(ik)} + E_{[ik]}$ (or deformations $D_{ik} = E_{ik} = \partial u_k / \partial x_i$) could be released quite independently in a seismic source, or may be mutually related by phase shifted reference displacements, e.g., $\bar{u}_s, \hat{u}_s, \breve{u}_s$:

$$\bar{E}(\bar{u}) = \frac{1}{3}\sum_s E_{ss} = \frac{1}{3}\sum_s \frac{\partial \bar{u}_s}{\partial x_s} \quad - \text{ axial strain} \tag{2.1a}$$

$$\hat{E}_{(ik)}(\hat{u}) = E_{(ik)} - \delta_{ik}\bar{E} = \frac{1}{2}\left(\frac{\partial \hat{u}_k}{\partial x_i} + \frac{\partial \hat{u}_i}{\partial x_k}\right) - \delta_{ik}\frac{1}{3}\sum_{s=1}^{3}\frac{\partial \bar{u}_s}{\partial x_s} \quad - \text{shear strains} \tag{2.1b}$$

$$\breve{E}_{[ik]}\left(\breve{u}\right) = \frac{1}{2}\left(\frac{\partial \breve{u}_k}{\partial x_i} - \frac{\partial \breve{u}_i}{\partial x_k}\right) \quad - \text{rotation strain} \tag{2.1c}$$

where the reference displacements may be mutually related, with possible phase shifts, due to the joint source processes; e.g., as follows:

$$\bar{u}_s = \xi^0 u_s^{\%}, \hat{u}_s = e^0 u_s^{\%}, \quad \breve{u}_s = \chi^0 u_s^{\%}; \quad \left\{\xi^0, e^0, \chi^0\right\} = \{0, \pm 1, \pm i\} \tag{2.2a}$$

In some situations, the reference displacement may be very useful, e.g., when considering the reflection and refraction rules in the processes related to wave propagation.

We shall note that the presented approach with the independent strain relations (2.1a), or even using the reference displacements, differs essentially from the classic approach in which the solutions might be obtained with the help of a unique displacement field, u (which moreover might be replaced by the potentials, $u = u^P + u^S; \quad u^P = \text{grad } \varphi, \quad u^S = \text{rot } \psi$). Of course, we should remember that a strain rotation, $\breve{E}_{[ik]}$, has a quite different meaning than a simple rotation motion.

The compatibility conditions assure, in a mathematical sense, that the deformations and strains could be expressed by some displacement derivatives:

$$
\begin{aligned}
I_{ij} &= \varepsilon_{ikm}\varepsilon_{jtn}\frac{\partial^2 D_{mn}}{\partial x_k \partial x_t} = 0 \rightarrow D_{mn} = \frac{\partial u_n}{\partial x_m} \\
I_{(ij)} &= \varepsilon_{ikm}\varepsilon_{jtn}\frac{\partial^2 E_{(mn)}}{\partial x_k \partial x_t} = 0 \rightarrow E_{(mn)} = \frac{1}{2}\left(\frac{\partial u_n}{\partial x_m} + \frac{\partial u_m}{\partial x_n}\right) \\
I_{[ij]} &= \varepsilon_{ikm}\varepsilon_{jtn}\frac{\partial^2 E_{(mn)}}{\partial x_k \partial x_t} = 0 \rightarrow E_{[mn]} = \frac{1}{2}\left(\frac{\partial u_n}{\partial x_m} + \frac{\partial u_m}{\partial x_n}\right)
\end{aligned}
\tag{2.2b}
$$

The independent motion equations (cf., Teiseyre and Górski 2009; Teisseyre 2009, 2011) should follow directly from the derivatives of the classic Newton formula:

$$
\begin{aligned}
&\mu \sum_s \frac{\partial^2 u_i}{\partial x_s \partial x_s} - \rho \frac{\partial^2 u_i}{\partial t^2} + (\lambda + \mu) \sum_s \frac{\partial^2 u_s}{\partial x_i \partial x_s} = 0 \text{ and for } D_{ni} = \frac{\partial u_i}{\partial x_n}: \\
&\mu \sum_s \frac{\partial^2 D_{ni}}{\partial x_s \partial x_s} - \rho \frac{\partial^2}{\partial t^2} D_{ni} + (\lambda + \mu) \frac{\partial^2}{\partial x_n \partial x_i} \sum_s D_{ss} = 0
\end{aligned}
\tag{2.3}
$$

We obtain (the external forces omitted):

$$
(\lambda + 2\mu) \sum_s \frac{\partial^2 \bar{E}}{\partial x_s \partial x_s} - \rho \frac{\partial^2 \bar{E}}{\partial t^2} = 0; \bar{E} = \frac{1}{3} \sum_s \frac{\partial \bar{u}_s}{\partial x_s} = \frac{1}{3} \sum_s E_{ss}
\tag{2.4a}
$$

$$
\begin{aligned}
&\mu \sum_s \frac{\partial^2 \hat{E}_{ni}}{\partial x_s \partial x_s} - \rho \frac{\partial^2}{\partial t^2} \hat{E}_{ni} + (\lambda + \mu) \left(\frac{3\partial^2}{\partial x_n \partial x_i} \bar{E} - \delta_{ni} \sum_s \frac{\partial^2}{\partial x_s \partial x_s} \bar{E} \right) = 0 \text{ or} \\
&\mu \sum_s \frac{\partial^2 \hat{E}_{ni}}{\partial x_s \partial x_s} - \rho \frac{\partial^2}{\partial t^2} \hat{E}_{ni} + (\lambda + \mu) \left(\frac{\partial^2}{\partial x_n \partial x_i} \sum_k E_{kk} - \frac{\delta_{ni}}{3} \sum_{s,k} \frac{\partial^2 E_{kk}}{\partial x_s \partial x_s} \right) = 0
\end{aligned}
\tag{2.4b}
$$

$$
\mu \sum_s \frac{\partial^2}{\partial x_s \partial x_s} \breve{E}_{[ni]} - \rho \frac{\partial^2}{\partial t^2} \breve{E}_{[ni]} = 0
\tag{2.4c}
$$

$$
\begin{aligned}
&E_{kl} = E_{(kl)} + E_{[kl]} = \delta_{kl}\bar{E} + \hat{E}_{kl} + \breve{E}_{[kl]}; \; \bar{E} = \frac{1}{3}\sum_{s=1}^{3} E_{(ss)}, \\
&\hat{E}_{(ik)} = E_{(ik)} - \delta_{ik}\frac{1}{3}\sum_{s=1}^{3} E_{(ss)}
\end{aligned}
\tag{2.4d}
$$

Thus, all these strains (axial, deviatoric, and rotational—related to a load angular moment) could be released, in a common process, with these phase shifts; in that case, these strains will be interrelated through the reference motion, u, as a basic reference field introduced in a mathematical sense. In such a way, we can present the relations for the independent fields with the help of the phase factors, and give their connections to the common reference displacements. In these formulae we have used the different reference displacements and the constitutional relations, joining stresses and strains.

To obtain a relation between the antisymmetric part of stresses, $S_{[kl]}$, and related rotation strains, $E_{[kl]}$, we must introduce the proper constitutive laws, e,g., the Shimbo law (1975, 1995) based on the friction processes and rotation of grains.

Another equivalent constitutive law has been introduced in the Kröner approach to the continuum theory with the self-fields and related internal nuclei (Kröner 1981).

In our approach we join the shear and rotation stresses and strains with common constants μ:

$$S_{(ik)} = 2\mu E_{(ik)} + \lambda\delta_{ik}E_{(ss)} \, and \, \breve{S}_{[ik]} = 2\mu\breve{E}_{[ik]}; \bar{S} = (2\mu + 3\lambda)\bar{E}, \hat{S}_{(ik)} = 2\mu\hat{E}_{(ik)} \tag{2.5a}$$

and

$$S_{kl} = S_{(kl)} + S_{[kl]} = \delta_{kl}\bar{S} + \hat{S}_{(kl)} + \breve{S}_{[kl]}; \;\; \bar{S} = \frac{1}{3}\sum_{s=1}^{3} S_{(ss)}, \;\; \hat{S}_{(ik)} = S_{(ik)} - \delta_{ik}\frac{1}{3}\sum_{s=1}^{3} S_{(ss)}$$

$$E_{kl} = E_{(kl)} + E_{[kl]} = \delta_{kl}\bar{E} + \hat{E}_{(kl)} + \breve{E}_{[kl]}; \;\; \bar{E} = \frac{1}{3}\sum_{s=1}^{3} E_{(ss)}, \;\; \hat{E}_{(ik)} = E_{(ik)} - \delta_{ik}\frac{1}{3}\sum_{s=1}^{3} E_{(ss)}$$

$$S_{(ik)} = 2\mu E_{(ik)} + \lambda\delta_{ik}E_{(ss)} \, and \, \breve{S}_{[ik]} = 2\mu\breve{E}_{[ik]}; \;\; \bar{S} = (2\mu + 3\lambda)\bar{E}, \;\; \hat{S}_{(ik)} = 2\mu\hat{E}_{(ik)} \tag{2.5b}$$

The fracture processes in a source can occur due to the release-rebound processes; such interactive processes explain a propagation pattern with the consecutive rotation and shear strains (Teisseyre 1985, 2009, 2011).

When an axial strain is constant, then the release-rebound system may be described by the linear relations between the time and space derivatives; the related equations remind the Maxwell equations. However, to this end we must choose the coordinate system oriented in a special way which presents the deviatoric strains as the off-diagonal tensor:

$$\hat{E}_{(ik)} = E_{(ik)} - \delta_{ik}\frac{1}{3}\sum_{s=1}^{3} E_{(ss)} \to \hat{E}_{(ik)} = \begin{bmatrix} 0 & \hat{E}_{(12)} & \hat{E}_{(13)} \\ \hat{E}_{(12)} & 0 & \hat{E}_{(23)} \\ \hat{E}_{(13)} & \hat{E}_{(23)} & 0 \end{bmatrix} \tag{2.6}$$

In this system we may define the shear vector as $\hat{E}_i = \left\{\hat{E}_{(23)}, \hat{E}_{(31)}, \hat{E}_{(12)}\right\}$, and the rotation vector $\breve{E}_i$ as $\breve{E}_i = \left\{\breve{E}_{[23]}, \breve{E}_{[31]}, \breve{E}_{[12]}\right\}$; note that when using the 4D approach, the vector $\hat{E}_i$can be defined invariantly (cf., Teisseyre 2009; see Chap. 12 on the 4D Maxwell-like invariant relations).

The release-rebound process means that a break of molecular bonds releases rotation field, $\partial\breve{E}/\partial t$, and then in a rebound motion there appears rot$\hat{E}$; reversely, the release of shears, $\partial\hat{E}/\partial t$, leads to rot$\breve{E}$. Finally, we arrive at relations for the release-rebound processes adequately described by the Maxwell-like relations (see: Chap. 12 and Teisseyre 2009, 2011):

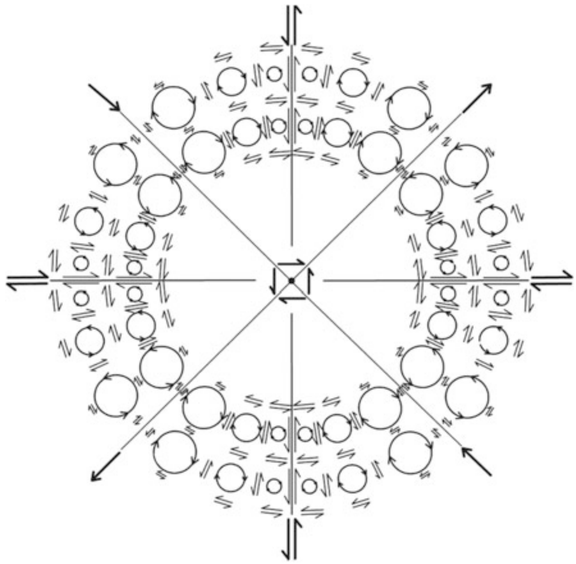


Fig. 2.1 Wave interaction pattern: the shears and rotation strains; their interaction enables propagation of these waves

$$\mathrm{rot}\,\breve{E} - \frac{\chi \partial \hat{E}}{c\partial t} = 0,\ \mathrm{rot}\,\hat{E} + \frac{\chi \partial \breve{E}}{c\partial t} = 0;\ \frac{c}{\chi} = V^S = \sqrt{\frac{\mu}{\rho}} \tag{2.7a}$$

where c is the light velocity and χ is the material constant.

From these relations we obtain that shear and rotation strain must propagate with the same velocity and that the wave equations which coincide with the previously derived formulae (2.4a), when the axial strains $\sum_s \frac{\partial \bar{u}_s}{\partial x_s}$ remain to be constant, will be given as:

$$\Delta \breve{E} - \frac{\chi^2}{c^2}\frac{\partial^2 \breve{E}}{\partial t^2} = 0 \quad \text{and} \quad \Delta \hat{E} - \frac{\chi^2}{c^2}\frac{\partial^2 \hat{E}}{\partial t^2} = 0 \tag{2.7b}$$

The related wave mosaic (Teisseyre and Gorski 2011) explains the interrelated propagation pattern of the shear and rotation motions as presented in Fig. 2.1 (the rotation strain motions being represented as the double opposite-sense arrows).

The interaction of fields explains the propagation pattern (some critical remarks against rotation motions have been based on the supposition that a separate propagation of rotation motions will be immediately attenuated and therefore could not exist): in our interactive propagation pattern the rotations and shears exist and these motions visualize the wave interactions corresponding to the release-rebound processes at a fracture source.

As mentioned at the beginning of this chapter we should supplement the solid continuum by the molecular transport motion as nuclei of the possible real transport inside a fracture domain, that is outside the considered Asymmetric Continuum. The fracture slip-displacements at a seismic source could develop due to the existence of the assumed molecular displacement velocities, which can relate to the real displacement transport (fracture) forming a fracture zone (formally outside the considered continuum). However, we shall underline that these molecular displacement transports should depend and follow the actual solutions for the strains; in this way the molecular displacements become formed with the help of the real strains and their molecular reference direct displacement transport, or that with the possible phase shifts:

$$d\tilde{\upsilon} \quad \text{or} \quad \left(\xi^0 + e^0 + \chi^0\right) d\tilde{\upsilon} \tag{2.8a}$$

with $\left\{\xi^0, e^0, \chi^0\right\} = \{0, \pm 1, \pm \mathrm{i}\}$ (Eq. 2.2a), as related to the fields $\delta_{kl}\bar{E}, \hat{E}_{(kl)}, \breve{E}_{[kl]}$ (Eqs. 2.1a–c).

Now, we may define the molecular transport field, w_s, in the following way:

$$w_s = \frac{\partial \upsilon_s}{\partial x_s} \to \left\{ w_1 = \frac{\partial \upsilon_1}{\partial x_1}, w_2 = \frac{\partial \upsilon_2}{\partial x_2}, w_3 = \frac{\partial \upsilon_3}{\partial x_3} \right\} \tag{2.9a}$$

$$\upsilon = 0, \text{ but } d\upsilon \text{ exists: } d\upsilon = d\frac{\partial u}{\partial t} \tag{2.9b}$$

Here, the derivatives $d\upsilon_s$ mean the derivatives of real transport velocities and the field $w_s = \frac{\partial \upsilon_s}{\partial x_s}$ might become related, after an integration process, to the possible real fracture transport velocities, υ_s, but it appears outside a frame of the continuum.

This molecular transport term should fulfil the Navier–Stokes-like equation:

$$\frac{\mathrm{D}(\rho w_s)}{Dt} = w_s \frac{\partial \rho}{\partial t} + \rho \frac{\partial w_s}{\partial t} + \rho l \sum_k w_k \frac{\partial w_s}{\partial x_k} = \breve{\eta} \sum_k \frac{\partial^2 w_s}{\partial x_k \partial x_k} \tag{2.10a}$$

where a dynamic micro-viscosity $\breve{\eta}$ (the new material constants) and an additional constant l, are introduced.

Thus, we obtain the governing relation for these molecular transports including possible forces:

$$\rho \frac{\partial w_i}{\partial t} + w_i \frac{\partial \rho}{\partial t} + \rho l \sum_k w_k \frac{\partial w_i}{\partial x_k} - \eta \sum_k \frac{\partial^2 w_i}{\partial x_k \partial x_k} = \tilde{F}_i$$

at ρ = const.:

$$\rho \frac{\partial w_i}{\partial t} + \rho l \sum_k w_k \frac{\partial w_i}{\partial x_k} - \eta \sum_k \frac{\partial^2 w_i}{\partial x_k \partial x_k} = \tilde{F}_i \tag{2.10b}$$

Due to this molecular transport and related fracture mechanism we may expect a possible intensity increase of the wave motions in an active seismic region; we will return to these problems in the next chapter.

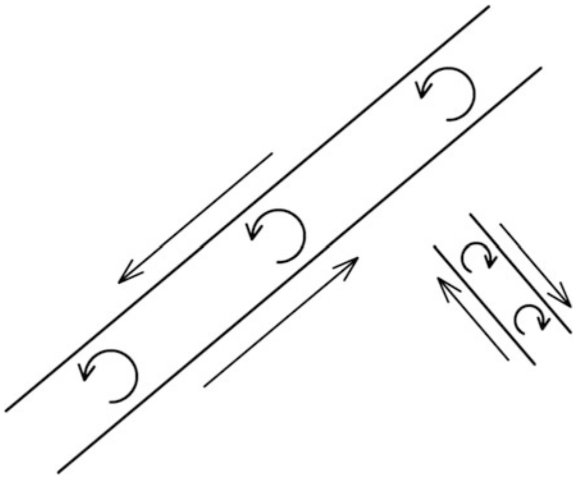


Fig. 2.2 Shear load: sketch of slip elements and the opposite rotations as appear along the main slip and a double couple partner

We may remark that in a similar way to (2.9a), we could define the molecular displacements:

$$w_s^u = \frac{\partial u_s}{\partial x_s} \to \left\{ w_1^u = \frac{\partial u_1}{\partial x_1}, w_2^u = \frac{\partial u_2}{\partial x_2}, w_3^u = \frac{\partial u_3}{\partial x_3} \right\} \text{ at } \sum_n w_n^u = 0 \tag{2.11a}$$

and this relation leads us to the equation similar to eqs. (2.4a):

$$\rho \frac{\partial w_n^u}{\partial t} - \overleftrightarrow{\mu} \sum_k \frac{\partial^2 w_n^u}{\partial x_k \partial x_k} = \overleftrightarrow{F}_n^u \tag{2.11b}$$

These molecular fields may lead to some fracture process, see, e.g., Fig. 2.2.

Also the molecular transport could lead to an appearance of an interaction chain, the string chain; e.g., such phenomena might be expected near some structure inside the Earth which effectively reflect the seismic waves and also modify the micro-seismic waves. These reflected micro-seismic waves become modified due to the local molecular fields. In this way, the observed reflected micro-seismic parts might bring some earthquake precursory signals as discovered by Sobolev and Lyubushin.

In our final statement we underline that the possible independent fields (displacements, transports, strains and molecular strains) and the different possible float transport motions may be treated as quite independent fields; this is our main postulate. However, it is to be remembered that in some circumstances these fields might be mutually related and shifted in phases; e.g., due to the common fracture processes in a seismic source.

As an example of the powerful abilities of the Asymmetric Theory let us consider the single and double couples. First, let us note the following properties of single and double couples:

- A double couple does not contain any rotational part
- A single couple contains shears and rotation
- Series of the perpendicular couple pairs form rotations with mutually compensated shears.

A double couple field is usually assumed to model a fault slip mechanism (Knopoff and Gilbert 1960); however, when a fault zone has a finite thickness there appears an additional term, the single couple (Knopoff and Chen 2009); the appearing torque imbalance is compensated by a rotation component at the crack tip and therefore the symmetry of Classic Theory is maintained. Nevertheless, the obtained results indicate that the radiation from an advancing crack tip and related strength weakening zone may explain a number of observed rotational phenomena; an additional constitutive law for friction properties should be included.

However, note that in the Asymmetric Continuum Theory a single couple can be defined directly as a sum of the symmetric deviatoric strains and the antisymmetric rotations, $D_{ks} = E^D_{(ks)} + \omega_{ks}$; the related rotation effects remain incorporated into the theory's structure and the theoretical results could be directly related to the observed rotation processes; for the related fracture process we should again include the friction constitutive law.

In classic theory we may consider the solutions for displacements (or displacement potentials) and then estimate a deformation $D_{ks} = \frac{\partial u_s}{\partial x_k}$; otherwise, we can find this solution from the wave equations, and for rotations and deviatoric strains, when assuming a constant confining strain, and putting $F_{nl} = F_{(nl)} + F_{[nl]}$, we will obtain:

$$D_{nl} = E^D_{(nl)} + \omega_{nl},\ \omega_{nl} \equiv E_{[nl]};\ \mu \frac{\partial^2}{\partial x_k \partial x_k} D_{nl} - \rho \frac{\partial^2}{\partial t^2} D_{nl} = F_{nl} \tag{2.12a}$$

The same relation can be obtained in the asymmetric theory, but in that case the fields $E^D_{(nl)}$ and ω_{nl} may be independent or mutually related:

$$D_{nl} = E^D_{(nl)} + \omega_{nl} = \frac{1}{2}\left(\frac{\partial u_l}{\partial x_n} + \frac{\partial u_n}{\partial x_l}\right) - \frac{\delta_{nl}}{3}\frac{\partial u_s}{\partial x_s} + \chi \frac{1}{2}\left(\frac{\partial u_l}{\partial x_n} + \frac{\partial u_n}{\partial x_l}\right);\ \chi = \pm i \tag{2.12b}$$

where this definition of deformation may differ from that given in relation (2.11a).

We may consider the solutions for displacements (or displacement potentials) and then derive the rotation wave and the deviatoric strains; this will be the classic approach. Otherwise, using the asymmetric approach we can solve directly the independent wave equations for the rotation and deviatoric strain fields (with a certain assumption related the axial strain).

These two possible procedures will lead us to quite different results. This is a similar difference between the classic and asymmetric approaches as discussed already in Chap. 1, when considering the problem how to derive the $E_{\varphi\varphi}$ component.

We arrive at some conclusions: confrontation between the Classic Elasticity and Asymmetric Continuum Theory is related to the basic point motions involved; in the former we use exclusively the displacement field, while the asymmetry includes the four point motions: molecular displacements, rotations, deviatoric shears, and axial scalar field. Our consideration indicates an essential difference between the Classic and Asymmetric approaches. Underlining such differences we have pointed out their advantages and deficiencies, but an important fact is that these two theoretical approaches refer, in fact, to the very different technical and measurement systems.

A basic difference between the considered theories follows from the assumption that in the Asymmetric Theory we may admit a simultaneous appearance of a number of fields independently released in a source. This is due to the fact that we admit an independent release of some physical fields, e.g., strains and rotations, in an earthquake source. Formally these released fields might be expressed again by some displacements and, therefore, the resulting rotation and strain fields become related to a number of independent fields. However, the special analysis may permit to separate some observed wave groups originated due to the interaction processes between the rotation and strain fields.

Finally, we shall add that the measurement devices register of course a sum of all possible contributions to a given field, e.g., for displacement we measure a sum of all its contributions from the different source-release processes jointly. Only additional information may discriminate an origin of some component contribution, e.g., for displacement it might be a separately found time period when a wave group of displacement correlates with rotational oscillations (Teisseyre 2007), while for the strain component $E_{\varphi\varphi}$ it is a fact that the displacement-related contribution to that field appears negligible in comparison to the contributions caused by a direct strain effect, as discussed in Chap. 1.

At the end of these considerations let us analyse one special case related to the natural earthquake events in rocks that are under a vertical gradient of pressure; here, it is convenient to use the cylindrical system (r,ϕ,z), with the z-axis oriented in vertical direction. In the considered case we assume the angular bond deformations described by the angular squeeze, $E_{\varphi\varphi}$, in the horizontal plane. Under a significant compression load there may appear some centers with micro-breaks and induced opposite-sense shears (Teisseyre et al. 2006). Such processes can be understood in the following way: the defects become activated under compression load and produce centers with shears of the opposite sense. In effect we observe a rock fragmentation and rotation release followed by the rebound slips. The centers of the opposite sense shears may create dynamic angular deformations leading to the bond breaks and slip propagation followed by the rebound rotations retarded in phase. We can assume that the induced shears and fragmentation depend on the applied load and defect content (cf., Chap. 7). Such induced angular squeeze, $E_{\varphi\varphi}$, can be related to the applied axial stresses due to the defect co-action:

$$\sum S_{ss} = \varepsilon E_{\varphi\varphi} \tag{2.13a}$$

where we insert a new constant, ε, which may relate to the unknown angular squeeze material properties and to a specific angular squeeze structure.

Note a difference between the $E_{\varphi\varphi}$ component and the E_{rr}, E_{zz} ones; the latter components represent typical compression strains, the $E_{\varphi\varphi}$ component represents a circular squeeze which seems to fit to the local fragmentation processes under a high confining load. Such fragmentations in a circular shape are typical in some material crushing under compression. For processes under constant applied load, $E_{rr} + E_{zz} = \text{const}$, we write after eqs. (2.4a) the wave in the cylindrical coordinates, for the field $E_{\varphi\varphi}$:

$$\mu\left(\frac{\partial}{r\partial r}\left(\frac{r\partial}{\partial r}\right) + \frac{\partial^2}{r^2\partial\psi^2} + \frac{\partial^2}{\partial z^2}\right)E_{\varphi\varphi} - \rho\frac{\partial^2}{\partial t^2}E_{\varphi\varphi} = \left(\frac{\mu}{\lambda+2\mu}\right)^{1/2}\frac{\partial F_{\varphi}}{r\partial\psi} \tag{2.13b}$$

where $\psi = \varphi\left(\frac{\mu}{\lambda+2\mu}\right)^{1/2}$

Considering the angular squeeze deformations related to the independent axial strains, we get a completely new view on the appearance of the $E_{\varphi\varphi}$ component discovered in strain measurement related to earthquake events (Gomberg and Agnew 1996). In these authors' original opinion, related to the Classical Elasticity, the squeeze strain angular components, $E_{\varphi\varphi}$, can be estimated from the solution for the displacement potentials. A pressure gradient (z-direction) may have an angular symmetry of a squeeze appearing in a horizontal plane (parallel to the Earth surface). These authors, using the scalar and vector potentials, Φ and Ψ, considered the asymptotic solution for the cylindrical waves excited by an earthquake source:

$$(\Phi, \Psi) \to f(r, \varphi, t) = AJ_m(kr)\exp[\mathrm{i}(m\varphi - \varpi t)], \tag{2.14a}$$

where for m = 0 we obtain:

$$f(r, t) \approx A\sqrt{\frac{2}{\pi k_r r}}\exp[\mathrm{i}(k_r r - \varpi t - \pi/4)] \tag{2.14b}$$

The exact solutions would be given by an expansion of the Bessel functions with cylindrical harmonics (cf., Udias 2002).

An asymptotic expression for displacements, and further for the strain angular squeeze $E_{\varphi\varphi}$, becomes as follows (Gomberg and Agnew 1996):

$$E_{\varphi\varphi} \approx \frac{u_r}{r} + i\frac{u_{\varphi}}{r} \tag{2.14c}$$

The approximated solution decays rapidly with distance; the cylindrical wave front may be estimated as close to zero. These authors wanted to explain the experimentally discovered angular squeeze strains discovered by the systematic squeeze measurements. The obtained theoretical result (2.14c) forced them to assume that the experimentally discovered angular squeeze strains appear only as the local distortions of the strain field, in spite of the fact that for all analysed events the obtained $E_{\varphi\varphi}$ values have been estimated as significant.

However, when we return to the relations valid in the Asymmetric Continuum Theory, an interpretation of the above-mentioned results look quite differently. There appears a big difference between the Classical approach of an angular squeeze, derived from the wave equation for displacement potentials, and the Asymmetric Theory considering directly the theoretical solution for the independent squeeze waves. In this case instead of the solution (2.14c) we arrive at the Bessel wave solution for this angular term:

$$E_{\varphi\varphi} = AJ_m(kr)\exp[\mathrm{i}(k_z z + m\psi - \varpi t)]; \quad \psi = \varphi\left(\frac{\mu}{\lambda + 2\mu}\right)^{1/2} \tag{2.15a}$$

In first approximation the asymptotic expression for $E_{\varphi\varphi}(r, \psi, z, t)$ can be given by an expansion of the Bessel functions:

$$\begin{aligned} E_{\varphi\varphi} \propto \sqrt{\frac{2}{\pi k_r r}}\exp[\mathrm{i}(k_r r + k_z z + m\psi - \varpi t - \pi/4)] \approx \\ \sqrt{\frac{2}{\pi k_r r}}\exp\left[\mathrm{i}\left(k_r r + k_z z + \frac{m\varphi}{\sqrt{3}} - \varpi t - \pi/4\right)\right] \end{aligned} \tag{2.15b}$$

In this solution, the angular squeeze decays with r quite differently than in the asymptotic expression (2.14c) derived from the Classic Theory: this difference follows from the fact that we rely on an independent wave equation for all strains, including the squeeze ones. The related wave solution represents the propagation of the circular squeeze deformation $E_{\varphi\varphi}(r, z)$; such an independent physical field can propagate due the asymmetric continuum structure exciting much greater effects than those previously estimated by Gomberg and Agnew (1996).

Concluding, we can treat the observed deformations, $E_{\varphi\varphi}$, as real experimental facts explained by the Asymmetric Theory; the obtained result leads to an agreement between observations and theory. We repeat that the angular squeeze strains considered by Gomberg and Agnew (1996) were recorded as significant for all the events they analysed.

Further, let us assume a micro-fracture at $r = 0$ under the condition $E_{rr} + E_{\varphi\varphi} = \text{constant}\ (E_{rr} = -E_{\varphi\varphi})$, then the wave oscillations of these components become given by the equations (cf. Eq. 2.4a):

$$\mu\Delta E_{rr} - \rho\frac{\partial^2}{\partial t^2}E_{rr} = \frac{\partial F_r}{\partial r}, \mu\Delta E_{\varphi\varphi} - \rho\frac{\partial^2}{\partial t^2}E_{\varphi\varphi} = \frac{\partial F_\varphi}{r\partial\varphi} \tag{2.16a}$$

This will lead to the wave solution for a radial component, E_{rr}, and the Bessel solution for squeeze, $E_{\varphi\varphi}$, strains (Fig. 2.3.); in approximation:

$$E_{\varphi\varphi} \propto \sqrt{\frac{2}{\pi k_r r}}\exp[\mathrm{i}(k_r r + m\varphi - \varpi t - \pi/4)] \tag{2.16b}$$

The next figure, Fig. 2.4, presents a combined propagation of these deformations, $E_{rr}\, E_{rr}$ and $E_{\varphi\varphi}$, with the P-wave velocity: $V^P = \varpi/k_r$.

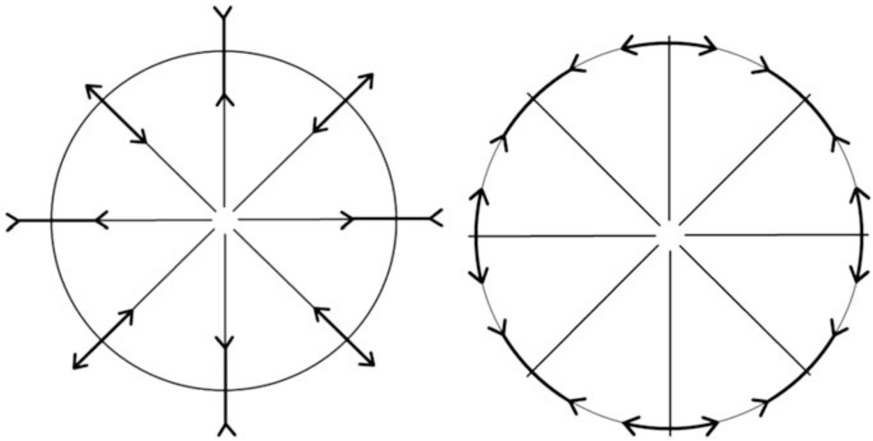
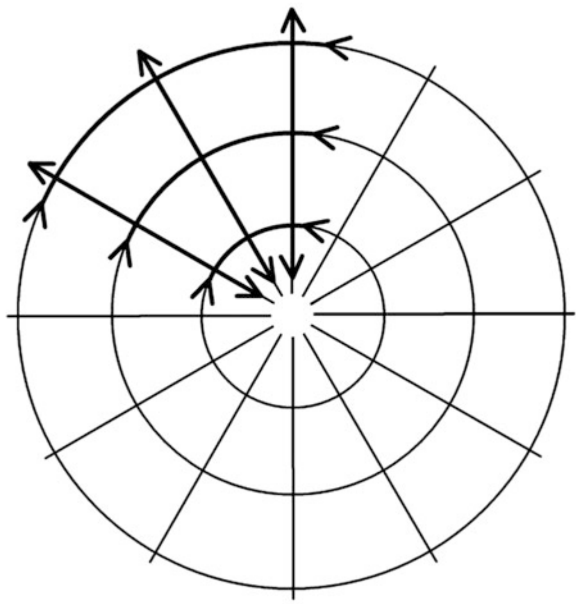


Fig. 2.3 Sketch of the E_{rr} deformations (*left part*) and $E_{\varphi\varphi}$ deformations (*right part*) related to the solutions for m = 2

Fig. 2.4 Propagation related to the interaction of the E_{rr} and $E_{\varphi\varphi}$ strains



2.3 Fluids: Molecular Strains and Transport

A similar asymmetric approach can be introduced for fluids; we also postulate that the different motion processes in fluids may either remain as the quite independent fields, or be mutually interdependent.

A transport motion in fluids is described by the Navier–Stokes transport equations. To consider the transport processes we recall the Euler equation

$$\frac{\partial \upsilon}{\partial t} + (\upsilon \nabla)\upsilon = -\frac{1}{\rho}\,\mathrm{grad}\,p + g \tag{2.17}$$

The transport processes at a constant density can be presented symbolically by the following transition:

$$\frac{\partial}{\partial t} \to \frac{\mathrm{d}}{\mathrm{d}t} = \frac{\partial}{\partial t} + \sum_s \upsilon_s \frac{\partial}{\partial x_S} \tag{2.18}$$

leading to the Navier–Stokes equation at a constant density:

$$\rho \frac{\mathrm{d}\upsilon_i}{\mathrm{d}t} = \rho \frac{\partial \upsilon_i}{\partial t} + \rho \sum_s \upsilon_s \frac{\partial \upsilon_i}{\partial x_s} = \eta \sum_s \frac{\partial^3 \upsilon_i}{\partial x_s^2} + F_i \tag{2.19}$$

where F are the body forces, υ is the displacement velocity, η is the dynamic viscosity, and p is the pressure.

Now we note that these transport motions, υ_i, should influence directly the molecular strain fields, $\tilde{E}_{(ki)}$ and $\tilde{E}_{[ki]}$, as a kind of their reference field. The molecular stresses and strains present the symmetric and anti-symmetric molecular deformations (Teisseyre 2009):

$$\tilde{S}_{kl} = \tilde{S}_{(kl)} + \tilde{S}_{[kl]}, \tilde{E}_{kl} = \tilde{E}_{(kl)} + \tilde{E}_{[kl]} \tag{2.20a}$$

Here, we refer to some of our former papers (Teisseyre 2007, 2008, 2009); these fields can be inter-related by means of the following constitutive relations

$$\begin{gathered}\frac{1}{3}\tilde{S}_{ss} = -p = \frac{1}{3}k\sum_s \tilde{E}_{ss}, \quad \bar{\tilde{S}} = \frac{1}{3}k\sum_s \bar{\tilde{E}}_{ss} = k\bar{\tilde{E}}; \\ \hat{\tilde{S}}_{(kl)} = \tilde{\mu}\hat{\tilde{E}}_{(kl)}, \breve{\tilde{S}}_{[kl]} = \tilde{\mu}\breve{\tilde{E}}_{[kl]}; \hat{\tilde{S}}_{(kl)} = \tilde{S}_{(kl)} - \frac{1}{3}\delta_{kl}\tilde{S}_{ss}, \hat{\tilde{E}}_{(kl)} = \tilde{E}_{(kl)} - \frac{1}{3}\delta_{kl}\tilde{E}_{ss}\end{gathered} \tag{2.20b}$$

where $\hat{\tilde{S}}_{(kl)}$ and $\hat{\tilde{E}}_{(kl)}$ mean the deviatoric parts of the symmetric tensors and $\breve{\tilde{S}}_{[kl]}$, $\breve{\tilde{E}}_{[kl]}$ relate to antisymmetric molecular tensors; for both constitutive relations we have assumed the same rigidity constant $\tilde{\mu}$ (this assumption is supported by the similar arrivals of the shear and rotation motions).

The molecular shear strain rate and molecular shear rotation rate (molecular spin) can be related to some reference displacement velocities in the following way:

$$\tilde{E}_{(kl)} = \frac{1}{2}\left(\frac{\partial\bar{\bar{\upsilon}}_l}{\partial x_k} + \frac{\partial\bar{\bar{\upsilon}}_k}{\partial x_l}\right), \ \breve{\tilde{E}}_{[kl]} = \frac{1}{2}\left[\frac{\partial\breve{\tilde{\upsilon}}_l}{\partial x_k} - \frac{\partial\breve{\tilde{\upsilon}}_k}{\partial x_l}\right], \ \bar{\bar{E}} = \frac{1}{3}\sum_s \frac{\partial\bar{\bar{\upsilon}}_S}{\partial X_S}$$
$$\hat{\tilde{E}}_{(ik)} = \tilde{E}_{(ik)} - \delta_{ik}\bar{E} = \frac{1}{2}\left(\frac{\partial\hat{\tilde{\upsilon}}_k}{\partial x_i} + \frac{\partial\hat{\tilde{\upsilon}}_i}{\partial x_k}\right) - \delta_{ik}\frac{1}{3}\sum_{s=1}^{3}\frac{\partial\bar{\bar{\upsilon}}_S}{\partial x_S} \quad (2.21)$$

where the relation for antisymmetric molecular strains follows from the fact that any rotation field can be expressed as a rotation of some velocity field; the reference velocities $\bar{\bar{\upsilon}}_k, \hat{\tilde{\upsilon}}_k, \breve{\tilde{\upsilon}}_k$ are introduced as mathematical reference fields only.

The relations for the molecular strain follow automatically from those presented for the real strains. We may assume an independence between the introduced molecular strains (strain rates), or these fields could be mutually related due to some internal processes in fluid, e.g., vortex motion.

It shall be noted, also, that for the equal reference motions, $\bar{\bar{\upsilon}}_l, \breve{\tilde{\upsilon}}_l, \tilde{\upsilon}_l$, and for $k = \tilde{\mu}$, we obtain the relation between the molecular stresses and displacement velocity as a sum of molecular strain rate and spin:

$$\tilde{S}_{kl} = \tilde{S}_{(kl)} + \tilde{S}_{[kl]} = \tilde{\mu}\frac{\partial\tilde{\upsilon}_l}{\partial x_k} = \tilde{\mu}\left(\tilde{E}_{(kl)} + \tilde{E}_{[kl]}\right), \ \tilde{S}_{kl} = \tilde{\mu}\tilde{E}_{kl} \quad (2.22)$$

We may consider the balance equation for spin as follows (Teisseyre 2007, 2008, 2009):

$$\tilde{M}_{lk} = l^2\varepsilon_{lki}\sum_n \frac{\partial\tilde{S}_{[ni]}}{\partial x_n} = \tilde{\mu}l^2\varepsilon_{lki}\sum_n \frac{\partial\tilde{E}_{[ni]}}{\partial x_n} \quad (2.23a)$$

$$\frac{\partial\tilde{M}_{pk}}{\partial x_k} = l^2\sum_n \varepsilon_{pki}\frac{\partial^2\tilde{S}_{[ni]}}{\partial x_k\partial x_n} = l^2\sum_n \varepsilon_{pki}\frac{\partial^2\tilde{S}_{[ki]}}{\partial x_n\partial x_n} = l^2\rho\varepsilon_{pki}\frac{\partial\tilde{E}_{[ki]}}{\partial t} + \varepsilon_{pki}\tilde{K}_{[ki]} \quad (2.23b)$$

where l is the Cosserat length enabling us to formulate the action of molecular stresses and moments on the fluid elements, and $\tilde{K}_p = \varepsilon_{pki}\tilde{K}_{[ki]}$ means the external moment.

We noticed also an important equivalence of these different summations:

$$\sum_n \varepsilon_{pki}\frac{\partial^2\tilde{S}_{[ni]}}{\partial x_k\partial x_n} = \sum_n \varepsilon_{pki}\frac{\partial^2\tilde{S}_{[ki]}}{\partial x_n\partial x_n} \quad (2.24)$$

Due to the further transformations we can write

$$\frac{\partial\tilde{M}_{pk}}{\partial x_k} = \tilde{\mu}l^2\varepsilon_{pks}\sum_n \frac{\partial^2\tilde{E}_{[ks]}}{\partial x_n\partial x_n} = \rho l^2\varepsilon_{pks}\frac{\partial\tilde{E}_{[ks]}}{\partial t} + \varepsilon_{pks}\tilde{K}_{[ks]}, \quad (2.25)$$

We may define now an average rotation over some group of the neighbouring rotations considered in an element related to the characteristic length element; we put:

$$\tilde{\mu}\sum_k \frac{\partial^2 \tilde{E}_{[ns]}}{\partial x_k \partial x_k} = \rho \frac{\partial^2 \tilde{E}_{[ns]}}{\partial t^2} \rightarrow \tilde{\mu}\sum_k \frac{\partial^2 \Omega_p}{\partial x_k \partial x_k} = \rho \frac{\partial^2 \Omega_p}{\partial t^2} \tag{2.26}$$

where η, ρ are constants and $\omega_p = \varepsilon_{pks}\tilde{E}_{[ks]}$ and $\Omega_p = \frac{1}{\Delta\omega}\int \omega_p \mathrm{d}\omega$.

There may be some reasons to average the pure spin (molecular rotation strains) into a molecular rotation related to some Cosserat characteristic molecular length, we will return to these relations in the next chapter, however, for a constant density the considered structure as a whole can be attributed to the field Ω (Eq. 2.26), while rotation motions at each of points, of course, may be described by the field ω.

At the end of this chapter we recall the structure of the Kröner theory (Kröner 1981) which allows to introduce additional fields, e.g., the rotation ones. Comparing our assumptions to that theory we note that the Kröner elastic fields correspond to the symmetric fields and the self-fields to the asymmetric fields, while the total fields correspond to the physical fields, which in the Kröner theory are represented by the elastic fields.

We should also remember that instead of stress moments we use an equivalent system representing antisymmetric stresses $S_{[ik]}$; the related balance law expresses the rotation force moment acting on a body element as the antisymmetric stresses (Teisseyre and Boratyński 2003, 2006).

Stress moment M_i can be formed from the stress moment tensor M_{is}. On the basis of the antisymmetric stresses

$$M_i = \frac{\partial}{\partial x_s} M_{is} = \in_{iks} \frac{\partial^2}{\partial x_n \partial x_s} S_{[kn]} \text{ and } M_{is} = \in_{iks} \frac{\partial}{\partial x_n} S_{[kn]} \tag{2.27a}$$

we arrive at

$$\varepsilon_{iks} \frac{\partial^2}{\partial x_n \partial x_s} S_{[kn]} = 2\mu\varepsilon_{iks} \frac{\partial^2}{\partial x_n \partial x_s} E_{[kn]} = \frac{1}{2}\varepsilon_{iks}\rho \frac{\partial^2}{\partial t^2} E_{[kn]}, \tag{2.27b}$$

where rotation might be represented as: $\omega_i = \frac{1}{2}\varepsilon_{iks}E_{[ks]}$.

We may note that according to Shimbo (1975) we use the same modulus for the symmetric and antisymmetric constitutive law joining the stresses and strains (see: Teisseyre et al. 2006, Chaps. 4–6).

In our approach, instead of stress moments we use an equivalent system representing antisymmetric stresses S[ik]; the related balance law expresses, on the one hand, the rotation of force acting on a body element due to the antisymmetric stresses (stress moment divided by an infinitesimal arm length), and, on the other hand, the balancing term, i.e., the acceleration related to angular momentum (Teisseyre and Boratyński 2003, 2006).

We may note that in the micromorphic continuum (Eringen 1999) the microstrain can contain its antisymmetric part while the gyration tensor can contain its symmetric part; these peculiarities extend, moreover, into the stresses and stress moments and on the inertia spin tensor (in the latter, some additional asymmetric properties may result from the micro-inertia tensor).

References

Eringen AC (1999) Microcontinuum field theories I, foundations and solids. Springer, Berlin, p 325

Gomberg J, Agnew DC (1996) The accuracy of seismic estimates of dynamic strains from Pinyon Flat Observatory, California, strainmeter and seismograph data. Bull Seismol Soc Am 86:212–220

Knopoff L, Chen YT (2009) The single couple equivalent force component of dynamical fractures and intrafault rotations. Bull Seismol Soc Amer 50:117–133

Knopoff L, Gilbert F (1960) First motions from seismic sources. Bull Seismol Soc Amer 50:117–133

Kröner E (1981) Continuum theory of defect. In: Les houches, session

Shimbo M (1975) A geometrical formulation of asymmetric features in plasticity, Hokkaido University. Bull Fac Eng 77:155–159

Shimbo M (1995) Non-Riemannian geometrical approach to deformation and friction. In: Teisseyre R (ed) Theory of earthquake premonitory and fracture processes. PWN (Polish Scientific Publishers), Warszawa, pp 520–528

Teisseyre R (1985) New earthquake rebound theory. Phys Earth Planet Inter 39:1–4

Teisseyre KP (2007) Analysis of a group of seismic events using rotational omponents. Acta Geophys 55:535–553

Teisseyre R (2008) Asymmetric continuum: standard theory. In: Teisseyre R, Nagahama H, Majewski E (eds) Physics of asymmetric continua : extreme and fracture processes. Springer, Berlin, p 95–109

Teisseyre R (2009) Tutorial on new development in physics of rotation motions. Bull Seismol Soc Am 99(2B):1028–1039

Teisseyre R (2011) Why rotation seismology: confrontation between classic and asymmetric theories. Bull Seismol Soc Am 101(4):1683–1691

Teisseyre R, Boratyński W (2003) Continua with self-rotation nuclei: evolution of asymmetric fields. Mech Res Commun 30:235–240

Teisseyre R and Górski M (2009) Fundamental deformations in asymmetric continuum: motions and fracturing, Bull Seismol Soc Am 99, 2B:1132–1136

Teisseyre R, Białecki M, Górski M (2006) Degenerated asymmetric continuum theory. In: Teisseyre R, Takeo M, Majewski E (eds) Earthquake source asymmetry, structural media and rotation effects. Springer, Berlin, p 43–56

http://www.springer.com/978-3-642-31859-7

Asymmetric Continuum
Extreme Processes in Solids and Fluids
Teisseyre, R.; Teisseyre-Jeleńska, M.
2014, XIV, 180 p. 40 illus., 1 illus. in color., Hardcover
ISBN: 978-3-642-31859-7