

Preface

“Topological” is an unlikely adjective to be associated with signal processing, which is dominated by results flowing from geometric rigidity. However, heterogeneous sensors and sensors in heterogeneous environments present a unique challenge; there is too much uncertainty for rigidity to be of much use. This book takes the perspective that signal processing has much to gain by taking a more *local* approach; consistency between nearby sensors or measurements is expected, but is not expected between sensors that are far apart. But how does one measure *distance* without explicitly invoking geometry, which is potentially very uncertain? This is the purview of *topology*; the lesson is that nearness can be studied implicitly and local signals can be studied through the theory of *sheaves*.

Sheaves have an unduly bad reputation, even among mathematicians, so it was unexpected that they could unify a number of signal processing concepts. Therefore, this book places sheaves at the conceptual and computational center by focusing exclusively on sheaves over cellular spaces. In this context (which is somewhat more limiting than the usual definition), many of the venerable results about sheaves still hold, but the proofs are substantially easier. Focusing on sheaves means that the exposition treats *cohomology* almost exclusively, which is more natural than singular or cellular homology for signals. For these reasons, it is usually easy to connect a local signal-processing concept to the correct sheaf theoretic tool.

Because the author is both a practitioner as well as a theoretician, due consideration is given to implementation concerns. Each chapter contains at least one *case study* in which the theoretical tools are used to address a relevant engineering problem. In most cases, the case study outlines a prototype implementation developed by the author that uses simulated or experimental data.

This book is intended for first-year graduate students and advanced undergraduates in mathematics and engineering. As such, a background including both linear algebra and multivariable calculus is expected and is used to motivate the necessary topological concepts. Algebraic topology relies on point-set topology, at least theoretically. There is an appendix on point-set topology, though intuition is usually a good guide to the necessary concepts. In a few places, a passing familiarity with abstract algebra and the structure of the ring \mathbb{Z} is helpful but never necessary, since another appendix reviews the necessary background.

Because the book brings together many dissimilar concepts, it is meant to be read in a linear fashion. Those readers comfortable with the material can skim ahead, but should be wary. Of necessity, the treatment differs from the traditional one both from a mathematical and an engineering perspective. That said, proofs, remarks, and exercises are called out carefully so that they may be omitted on first reading. No essential details are “buried” inside proofs, and some of the more technical or less insightful proofs are merely referenced or sketched. Because of this format, each topic is introduced informally, defined precisely, and then explained through examples. Reading the examples is essential for mastery of the subject. Engineering is built through a combination of theory and practice; the examples highlight both.

Finally, I welcome comments, suggestions, and corrections from you, the reader. Feel free to send me a message! I maintain a list of emendations on my website <http://www.dr-michael-robinson.net/> which you may find useful.

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