Chapter 1
Mass and Gravity

Besides length and volume, the mass, earlier designated as weight, is an important parameter characterising a body. It was recognised that there is proportionality between the ponderosity of a body, the resistance against moving it when it is at rest and stopping it when it is in motion. Nevertheless, mass as a physical quantity is a relatively new concept, defined mathematically first by Isaac Newton in 1697 [1, 2]. Since that time we distinguish between mass as an extensive property of a body and weight as a force exerted by the gravitational field of the earth on that body.

Xenia 165. Die Möglichkeit	**The possibility**
Liegt der Irrtum nur erst,	If a mistake is made at the beginning
wie ein Grundstein unten im Boden,	And used in subsequent work
Immer baut man darauf,	It will never be revealed later
nimmermehr kommt er an Tag.	Like building over a foundation stone.
Goethe/Schiller: Xenien [3]	Translation: S.A.A. Jayaweera

This xenia (host gift) most probably was written by Goethe against Newton. Goethe detested Newton's method of physical investigation. In his opinion Newton, therefore, made many mistakes.

In the International System of Units (SI) mass is one of the basic quantities. Today, due to the Theory of Relativity and Quantum Mechanics the concept mass becomes again fundamentally unclear, but not in conventional practice.

1.1 The Concept of Mass

Today, the aim of weighing is clearly the determination of the mass of a body (Table 1.12) or of a material sample; but earlier it was determination of its value [4]. In earlier times there were weight units with identical designation but different masses for different materials [5]. We still have some identical designations of weights and coins. Of course, the difference between weight and value of body was recognised in

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earlier times. In the Old Kingdom the Egyptians had already distinguished between concrete 'weight' from abstract 'value', using different vocabulary for each. The unit of weight was Deben whereas of value was Sna (vocalised as *shena*), perhaps written during Dynasty 19 as *sniw*. We have no feeling of sense for mass but have for weight which is the force of a body in the Earth's gravitational field.

The word mass is derived from the Latin *massa* which means lumps of a homogeneous material without specific form, but also a conglomerate of bodies [6]. The idea of the 'quantitas materiae' and its conservation goes back to Greek philosophers. Anaxagoras (499–428 BC) argued "nothing comes into being or ceases to exist" [7]. Democritus (460–371 BC) said: "from nothing comes nothing and can only become nothing".

In the middle ages some alchemists doubted this concept and hoped to create noble metals from earthy matter, accompanied by weight increase. Furthermore, on religious reports like the creation of a woman from one rib of the man [8] or of the transubstantiation of bread and wine into Jesus' flesh and blood, difficulties arose in understanding mass as an invariable quantity [9].

Antoine Lavoisier (1743–1794) deduced a general conservation of mass from the weight ratios in chemical reactions [10]. John Dalton (1766–1844) established the atomistic theory in chemistry and made a first table of relative atomic masses [11]. The principle of conservation of mass was formulated finally by Isaac Newton (1643–1717) [1]. In classical physics mass is defined as an inertial property of matter. Nevertheless, what mass really is had not yet been explained reliantly. On the contrary, the concept mass is even today subject to continual changes. Today mass is a basic unit of our international system of units.

In the Newtonian mechanics, two types of mass were distinguished: inert mass and gravitational mass. Bondi [12] distinguishes three types of mass: inert mass, active gravitational mass and passive gravitational mass. A universal proportionality between those different types of mass was assumed and confirmed by many so-called Eötvös experiments (see Chap. 4). For Albert Einstein (1879–1955) that 'Weak Equivalence Principle' gave the basis for his theory of relativity. Furthermore, mass can be regarded as a special type of energy [13].

It is assumed that in the Universe most of mass is connected with invisible 'dark matter' and furthermore not obvious sources of 'dark energy' exist. On Earth we have to do in practice with the visible matter. This includes also transparent substances like gases, glass or water which can be made visible by electromagnetic radiation of certain frequencies.

1.2 Mass and Weight

The hypothesis of Leucippus (500–450 BC) and Democritus (460–371 BC) held everything to be composed of atoms, which are physically, but not geometrically, indivisible; that between atoms lays empty space; that atoms are indestructible; have always been, and always will be, in motion; that there are an infinite number of

Fig. 1.1 Freely adapted from Drew. LHC = Large Hadron Collider, CERN, Genève, Switzerland, FAIR = Facility for Antiproton Research in Europe, Darmstadt, Germany

atoms, and kinds of atoms, which differ in shape, size, and temperature. Of the weight of atoms, Democritus said "The more any indivisible exceeds, the heavier it is." But their exact position on weight of atoms is disputed [14, 15].

In daily life, in trade and commerce as well as in many physical and chemical processes mass is an important parameter [16, 17]. For more than 5000 years balances have been used and for centuries we find balances in every factory and in households. Nevertheless the parameter "mass" is relatively new and still under discussion. When weighing we measure not the mass but the force exerted on the sample by the gravitational field (Fig. 1.1). That force is directly proportional to the mass of the sample. On Earth, however, gravitational acceleration depends on the geographical location of the balance. On account of the elliptic shape of Earth, in equatorial regions its radius is larger. Indeed the shape of the Earth differs significantly from a sphere or ellipsoid. On mountains the distance to the centre is greater and in addition the mass distribution within the Earth is not uniform. In order to formulate the laws of motion it is desirable to have mass as an invariant parameter of a material.

Mass is a measure of the amount of material in an object, being directly related to the number and type of atoms present in the object. We have no sense for the quantity mass; however we do for the force of an accelerated body or of a body in rest within a gravitational field. Newton's second law [1] states: A body of mass m subject to a net force F undergoes an acceleration a that has the same direction as the force and a magnitude that is directly proportional to the force and inversely proportional to the mass. "Mutationem motus proportionalem esse vi motrici impressae, et fieri secundum lineam rectam qua vis illa imprimitur."

$$\vec{F} = m\vec{a} \tag{1.1}$$

The corresponding SI units are Newton: $\mathrm{N} = \mathrm{kg\,m/s^2}$, kg, and $\mathrm{m/s^2}$, respectively.

Weight, W, is defined as the force acting on a body with mass m under gravitational acceleration g

$$W = mg \tag{1.2}$$

On Earth the acceleration is given by the gravitational field due to the large mass of the Earth. Correspondingly the mass will be determined only indirectly by measuring the weight which is a force W due to the gravitational field:

$$m = \frac{W}{g} \tag{1.3}$$

where m denotes the mass to be determined. g is the acceleration due to gravity which varies, depending on the geographical location, between about 9.77–9.83 $\mathrm{m\,s^{-2}}$. The standard acceleration of gravity (standard acceleration of free fall) g_n is defined to be 9.806 65 $\mathrm{m\,s^{-2}}$. Thus whilst mass is an unchanging quantity in classical mechanics, weight changes with gravitational acceleration. On the moon and the other planets that value is quite different depending on the mass of that celestial body (Table 1.2).

1.3 Gravity

Equation (1.3) is a special case of Newton's universal law of gravity: Two bodies of masses m_1, m_2 attract each other and the force between F_g is given by:

$$F_g = G\frac{m_1 m_2}{d^2} \tag{1.4}$$

where d is the distance between the centres of gravity of the masses and G is the gravitational constant. Obviously

$$g_n = \frac{Gm_1}{d^2} \tag{1.5}$$

Newton started his investigation of the laws of gravity when observing an apple falling from a tree. William Stukeley [18] reported that in 1726 Newton told him that story which happened in his garden in the year 1660. Newton was fascinated from the fact that the apples fall always in direction to the Earth centre.

In comparison to other known forces of nature gravity is a weak force. Only the large mass of the Earth gives us the impression of its essential effect. Therefore, it is difficult to determine the gravitational constant [19]. For its measurement extremely well defined and homogeneous test samples are required and screened from disturbing strong forces (electric, magnetic) and influences from the environment. It's no wonder that the value of the gravitational constant is less accurate than all others; its uncertainty is 1×10^{-4}. That means that the accuracy of all formula including G is limited to the per mille range. Many calculations in geological, meteorological, astronomical as well as of space operations are burdened with a basic uncertainty of per mille. With regard to that uncertainty until now it was not possible to check whether G is really a universal constant and to detect violations of the Equivalence Principle.

Methods to determine the value of G include free fall experiments from a tower or along an inclined plane. Better results are obtained by the measurement of the attraction of highly homogeneous test samples of several kilograms by means of a torsion pendulum (Chap. 4, Sect. 4.5.2). Similar experiments can be carried out likewise with a conventional beam balance. New techniques to measure G using atom interferometry are currently under development. This method is also being developed to measure the local acceleration due to gravity g. Recent experiments combine two vertically separated atomic clouds forming an atom-interferometer-gravity-gradiometer that measures the change in the gravity gradient when a well characterised source mass is displaced [20]. In Table 1.1 recent determinations of the gravitational constant and the CODATA recommendation for its value are listed [21].

1.4 Gravity and Motion

Since the time of the Greek philosopher Aristotle [31] in the 4th century BC, there have been many attempts to understand and explain attraction of material objects. Aristotle hypothesised that there was no effect without a cause and that each motion was caused by a force. He believed that everything tries to move towards its proper place in the crystalline spheres of the heavens, and that physical bodies fell towards the centre of the Earth in proportion to their mass [32].

The universal validity of free fall [33] was stated by the Byzantine Johannes Philoponos (485–555) [6] and in 1553 by Giambattista Bendetti. Galileo Galilei (1564–1642) rejected Aristotle's assumption that the velocity of bodies in free fall is proportional to its mass and quantified the theorem that all falling bodies descend with equal velocity "if one could totally remove the resistance of the medium, all substances would fall at equal speeds". He demonstrated this by dropping balls from the Leaning Tower of Pisa. Suitable measurements in this way however had not been possible because no clocks existed at his time with the required precision. He made measurements with balls of different mass rolling down inclined planes and pendulum experiments. In this way he reduced the velocity so that he was able to make reasonable measurements of time even by means of the inaccurate clocks. He determined the mathematical law for acceleration: The total distance covered, starting from rest, is proportional to the square of the time [34]. This law holds correctly only in vacuum because under atmospheric conditions air resistance obstructs the motion and this depends on the body's volume. That effect he discussed in mental

Table 1.1 Measurements 1996–2006 of the Newtonian constant of gravitation and 2006 CODATA recommend value

Year	Investigator	Method	G $\mathrm{m^3\,kg^{-1}\,s^{-2}}$	u_r	Ref.
1996	Karagioz, Izmailov	Fibre torsion balance, dynamic mode	6.6729(5)	7.5×10^{-5}	[22, 23]
1997	Bagley, Luther	Fibre torsion balance, dynamic mode	6.6740(7)	1.0×10^{-7}	[24]
2000 2002	Gundlach, Merkowitz	Fibre torsion balance, dynamic compensation	6.674255(92)	1.4×10^{-5}	[25]
2001	Quinn, Speake, Richman, Davis, Picard	Strip torsion balance, compensation mode, static deflection	6.67559(27)	4.0×10^{-5}	[26]
2002	Kleinevoß	Suspended body, displacement	6.67422(98)	1.5×10^{-4}	[27]
2003	Armstrong, Fitzgerald	Strip torsion balance, compensation mode	6.67387(27)	4.0×10^{-5}	[28]
2005	Hu, Guo, Luo	Fibre torsion balance, dynamic mode	6.6723(9)	1.3×10^{-4}	[29]
2006	Schlamminger, Holzschuh, Kündig, Nolting, Pixley, Schurr, Staumann	Stationary body, weight change	6.67425(12)	1.9×10^{-5}	[30]
2006	CODATA	CODATA-06 adjustment	6.67428(67)	1.0×10^{-4}	[20]

Newtonian constant of gravitation $G = 6.674\,28 \times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}$, Standard uncertainty $0.000\,67 \times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}$, Relative standard uncertainty $u_r 1.0 \times 10^{-4}$, Concise form $6.674\,28(67) \times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}$

experiments of bodies falling from a tower [35]. Dropping of two different bodies in a gravitational field, the “weak equivalence principle” states that the bodies fall with the same acceleration; this is usually termed as the universality of free fall.

In a document “Theoremata circa centrum gravitatis solidorum” [36] Galileo was the first to discuss in 1585–1586 first results on the mass of solid bodies. In his manuscript “La Bilancetta” [37] he improved Archimedes’s method of density determination and designed a hydrostatic balance. Galileo also put forward the basic principle of relativity, that the laws of physics are the same in any system that is moving at a constant speed in a straight line, regardless of its particular speed or direction. Hence, there is no absolute motion or absolute rest. This principle provided the basic framework [33] for Newton’s (1643–1727) laws of motion [1] and is the infinite speed of light approximation to Einstein’s special theory of relativity.

Galileo’s Principle of Inertia stated: “A body moving on a level surface will continue in the same direction at constant speed unless disturbed.” This principle was incorporated into Newton’s 1st law of motion. Lex I: Corpus omne perseverare in statu suo quiescendi vel movendi uniformiter in directum, nisi quatenus a viribus impressis cogitur statum illum mutare: An object at rest or travelling in uniform motion will remain at rest or travelling in uniform motion unless acted upon by a net force.

Additionally measurements within the gravitational field force of a moving body demonstrate Newton's 2nd law of motion or law of inertia: Lex II: Mutationem motus proportionalem esse vi motrici impressae, et fieri secundum lineam rectam qua vis illa imprimitur: The rate of change of momentum of a body is equal to the resultant force acting on the body and is in the same direction.

Newton's second law as originally stated in terms of momentum p is: An applied force F_I is equal to the rate of change of momentum.

$$F_I = \frac{\mathrm{d}p}{\mathrm{d}t} \tag{1.6}$$

where t stands for time. If the mass m of the object is constant the differentiation of the momentum becomes

$$F_I = ma = m\frac{\mathrm{d}v}{\mathrm{d}t} \tag{1.7}$$

where a is the acceleration and v the velocity of the object.

According to Newton's classical mechanics the gravitational force F_g results from a field between two bodies and allows the determination of the mass of a sample in the gravitational field of a body which is large in comparison with the sample, for example, the Earth. This mass is called 'inert mass'. The force F_I according to Newton's law results from the motion of a single sample. The gravitational constant has been defined to a value that the mass of a sample is the same if determined with both methods. The 'weak equivalence principle' states that the property of a body called 'mass' is proportional to the force 'weight'. "This quantity that I mean hereafter under the name of … mass … is known by the weight … for it is proportional to the weight as I have found by experiments on pendulums, very accurately made …"

Within a closed room with opaque walls we cannot decide without additional information whether a gravitational field is due to mass attraction or due to acceleration. So as the basis of the General Theory of Relativity Einstein's Equivalence principle expresses the identity of the gravitational field with that generated by motion [38]: the gravitational 'force' as experienced locally while standing on a massive body (such as the Earth) is actually the same as the pseudo-force experienced by an observer in a non-inertial (accelerated) frame of reference [39].

Günter Sauerbrey demonstrated that 'General Equivalence Principle' experimentally [40, 41]. With two quartz resonators he measured the mass of a deposited thin film using the harmonic inertial field created on the surface of these resonators. With a torsion balance he measured the mass of an equivalent deposited thin film using the gravitational field. In the first case he used a time related parameter (frequency) to evaluate the mass of the deposition. In the second case he used a space related parameter to evaluate the mass. Mass, space and time are closely related as revealed by Einstein's General Theory of Relativity.

Newtonian mechanics turned out to be a special case within the General Theory of Relativity, Newton's law of universal gravitation provides an excellent approximation in many cases and most non-relativistic gravitational calculations are based on it. Up to now there are no hints from experimental investigations for a 'Fifth forth' or of weakening of the gravitational force when passing through matter [42] which could necessitate corrections of Newton's laws or of the Theory of Relativity.

1.5 Gravitational Field

1.5.1 Earth Gravity

The magnitude of the gravitational field near the surface of the Earth is about 9.81 $\mathrm{m\,s^{-2}}$. That means that ignoring air resistance, the speed of an object falling freely near the Earth's surface increases every second by about 9.81 metres per second. The standard value is defined as $g_n = 9.80665\ \mathrm{m\,s^{-2}}$ or 9.80665 $\mathrm{N\,kg^{-1}}$. The vector, gravity, is the force that acts on a body at rest near the surface towards the centre of the Earth. However, the strength of Earth's gravity varies with latitude, altitude, local topography and geology.

Due to its rotation, the Earth is shaped as an ellipsoid with a smaller distance from surface to the centre at the poles. The mean diameter of the equator is about 42 km larger than the rotational axis through the pole. Furthermore, landmass and oceans are distributed non- uniformly and mountains and valleys of the sea bed are irregular. In addition, the density of the material varies. So the Earth is described more accurately as an irregular geoid rather than as a sphere. According to C.F. Gauss, the geoid is considered as the mathematical surface of the Earth, as opposed to the visible topographical surface. Since 2009 the European research satellite Goce measured gravity variations of Earth's surface by means of a gradiometer. This satellite orbits at a height of 265 km. It contains six weights each of 300 g of a platinum/rhodium alloy suspended in electric fields. The acceleration by gravity is measured. In this way the shape of the Earth can be determined to an accuracy of 2 cm. The deepest dent of the geod with -110 m was detected in the Pacific Ocean and the highest bump with $+80$ m below Iceland, the North Atlantic and at New Guinea.

Gravity provides centripetal force which is weakened by centrifugal force due to Earth's rotation. The maximum of the centrifugal force is at the equator. At the poles the centrifugal force is zero. That small centrifugal force is inseparably superimposed on the attraction and this effect is usually included in the local value of the gravitational force. In combination, the equatorial bulge and the effects of centrifugal force mean that sea-level gravitational acceleration increases from about 9.780 $\mathrm{m\,s^{-2}}$ at the equator to about 9.832 $\mathrm{m\,s^{-2}}$ at the poles, so an object will weigh about 0.5 % more at the poles than at the equator.

The net force exerted on an object by the Earth is called apparent or effective gravity and may be influenced by other forces. For most purposes, the Earth's gravitational field may be considered invariable in time. However, attraction of the Sun and moon superimpose periodically the values because of their changing distance from the Earth. These attractions act also indirectly by slightly deforming the Earth and shifting the waters of the oceans, so that the attracting terrestrial masses themselves are modified. The effects are well within the measuring accuracy of modern gravimeters but negligible for practical purposes.

Gravity decreases with altitude, since greater altitude means greater distance from the Earth's centre. Approximately we can use:

$$g_h = g_n \left(\frac{r_e}{r_e + h} \right)^2 \tag{1.8}$$

where g_h is the gravity at height h above sea level, r_e is the Earth's mean radius and g_n the standard gravity. An increase from sea level to the top of Mount Everest (8.8 km) causes a decrease of weight of only about 0.28 %. At an altitude of 400 kilometres, equivalent to a typical orbit of the Space Shuttle, gravity is still nearly 90 % as strong as at the Earth's surface [43]. Nevertheless, when calibrating sensitive balances, this effect of the altitude must be considered (see Sect. 3.4.3 in Chap. 3).

Satellite methods have enormously improved knowledge of the gravity field. The Gravity Information System of the Physikalisch-Technische Bundesanstalt, Braunschweig, Germany provides gravity information for nearly every location above sea level without claiming highest scientific accuracy [44]. The information portal is designed for interested users from physics and metrology whose mechanical applications are affected by the gravitational field of the Earth. The data are of importance for calibration of balances.

1.5.2 Planet Gravity

The magnitude of the gravitational field near the surface of the Sun and the planets depends of the composition and diameter of the bodies. A survey of relative weights and of gravity on the Earth, other planets and the moon is given in Table 1.2.

1.6 Planetary Motion

Johannes Kepler (1571–1630) working with data collected by Tycho de Brahe (1546–1601) [45] without the aid of a telescope developed three laws which described the motion of the planets across the sky [46, 47]. Kepler's laws were derived for orbits around the Sun, but they apply to satellite orbits as well.

- Kepler's First Law (law of orbits): All planets move in elliptical orbits, with the Sun at one focus.
- Kepler's Second Law (law of areas): A line that connects the planet to the Sun sweeps out equal areas in equal times.

Table 1.2 Relative weights and of gravity on the Earth, other planets and the moon

Body	Multiple of Earth gravity	Gravity m/s^2	Body	Multiple of Earth gravity	Gravity m/s^2
Sun	27.90	274.1	Jupiter	2.640	25.93
Mercury	0.3770	3.703	Saturn	1.139	11.19
Venus	0.9032	8.872	Uranus	0.917	9.01
Earth	1 (by definition)	9.8226[a]	Neptune	1.148	11.28
Moon	0.1655	1.625	Pluto	0.0621	0.610
Mars	0.3895	3.728			

[a]This value excludes the adjustment for centrifugal force due to Earth's rotation and is therefore greater than the 9.80665 m/s^2 value of standard gravity

- Kepler's Third Law (law of periods) [48]: The square of the period t_p of any planet is proportional to the cube of the semi-major axis d of its orbit. The third Kepler's law is valid exactly for the period of two bodies.

$$\left(\frac{t_{p1}}{t_{p2}}\right)^2 = \left(\frac{d_1}{d_2}\right)^3 \tag{1.9}$$

Kepler nominated a as mean distance of Earth to the Sun i.e. mean of perigee and apogee. Combined with the law of gravity Kepler's Third Law for two masses m_1, m_2 becomes:

$$t_p^2 = \frac{4\pi^2 d^3}{G(m_1 + m_2)} \tag{1.10a}$$

$$t_p^2 \approx \frac{4\pi^2 d^3}{G m_1} \quad \text{if } m_1 \gg m_2 \tag{1.10b}$$

These equations allow the determination of the total mass of a binary star system from distance d and period t_p. The approximation can be applied e.g. in our Solar System. Although Kepler's laws are valid exactly only for the two-body problem they are a good approximation for describing planet movements. Deviations from that law are due to gravitational influences of other planets, fluctuations of the Sun movement, imperfect spherical shape of the revolving planets and relativistic effects. Such effects on the elliptical orbit are summarised as irregularities of the orbit. Indeed the computational orbit can be regarded as an attractor only for the irregular orbital movement of planets.

Taking into account the different masses m_{p1} and m_{p2} of two planets as a three-body problem the Third Kepler Law writes:

$$\left(\frac{t_{p_1}}{t_{p_2}}\right)^2 = \left(\frac{d_1}{d_2}\right)^3 \frac{m_S + m_{P2}}{m_S + m_{P1}} \tag{1.11}$$

This is of importance only in case the masses m_{P1}, m_{P2} of both planets differ widely and the mass of the central star m_S is similar to the mass of one planet.

Using Eq. (1.10b), for the rotation of the Earth around the Sun, the sidereal period is given by

$$t_u \approx \sqrt{\frac{(1.5 \cdot 10^{11})^3 4\pi^2}{6.67 \cdot 10^{-11} \cdot 2 \cdot 10^{30}}} \text{ [s]} = 31602834 \text{ [s]} = 365 \text{ [d]} \tag{1.12}$$

Mass determination of astronomical objects is made by observation of the relative motion under the influence of a neighbouring object and application of Kepler's Laws.

Newton's theory enjoyed its greatest success when it was used to predict the existence of Neptune based on motions of Uranus that could not be accounted by the actions of the other planets. By the end of the 19th century, it was stated that the orbit of Mercury could not be accounted for entirely under Newton's theory, and this was a first proof for the Theory of Relativity. Nevertheless, in most cases Kepler's Laws can be applied as a good approximation.

1.7 Expansion of the Universe

Gravity is responsible also for movement of galaxies, which—on account of large distances—can be regarded as point shaped objects. However, these motions cannot be explained quantitatively if no additional sources of gravity are introduced. Because the gravitational laws are regarded as valid within the whole Universe it was necessary to assume an invisible matter. Recent observations confirm such 'dark matter'.

According to Einstein's prediction the Universe expands. To explain this phenomenon the concept of 'dark energy' was introduced which works opposite to gravity.

1.8 Mass and Theory of Relativity

In 1881 Michelson and Morley [49] detected the independence of the speed of light from any motion of the light source or of its receiver. That was the basis for Einstein's Theory of Relativity [38]. As a consequence of that theory mass lost its feature as an independent parameter of a body on account of its dependence on its velocity v [13]:

$$m = m_{rest} \frac{1}{\sqrt{1 - v^2/c_0^2}} \tag{1.13}$$

where m_{rest} stands for the mass of the body in rest, c_0 is the velocity of electromagnetic radiation in a vacuum. At ordinary velocities of terrestrial objects, v is very small compared with c, and $m \approx m_0$. However, in an electron beam (for example in an electron microscope), where v is much higher, the difference is significant, and the above relativistic correction is applicable: The mass of a particle increases when moving with a velocity near the speed of light.

According to the Theory of Relativity the velocity of electromagnetic radiation in a vacuum is a maximum and a universal constant. Also any matter cannot travel faster. A recent observation of neutrino's moving at a velocity greater than that of light has been found to be a measuring error and all the physicists are happy that Einstein is correct.

A fundamental principle is conversation of mass and energy which is believed to hold for the whole Universe. In 1905 Einstein formulated his famous equation [38, 50]:

$$E = m_{rest} c_0^2 \tag{1.14}$$

where E denotes the energy, m the mass and c_0 the velocity of light in a vacuum. So mass can be regarded as a special type of energy. When calculating the energy loss of a body emitting electromagnetic radiation, its decrease of mass should be considered. It should be noted that already much earlier the correlation between mass, energy and velocity of light had been stated and that Henri Ponicaré in 1900 had already formulated that equation. However Einstein developed the Special Theory of Relativity in which that equation was a logical constituent. The equation has been verified recently by comparing the mass of the stable isotope ^{28}Si with that of the unstable isotope ^{29}Si as well as of ^{32}S and ^{33}S and measuring the energy of the γ-radiation set free during the decay of the unstable atoms. Molecules of both isotopes were captured in a Penning trap and stimulated to rotation and the difference of the rotational frequencies was measured.

General Relativity is a geometric model of gravitation, a more accurate description of gravity in terms of the geometry of a curved space-time [51]. According to the General Relativity Theory, gravity is not a force acting on material particles. Instead it is identified as a curvature in space-time geometry [52]. In the absence of forces particles, including the photons of light, travel in the most straight possible way in curved space-time. This path is called 'geodesic' (Fig. 1.2a). In the absence of gravity, space-time is flat and geodesics are straight lines travelling at constant velocity. Near a star the geodesics follow the curvature of space-time and define the shortest natural paths. The presence of a large mass deforms space-time in such a way that the paths taken by particles bend towards the mass. That curved path is longer than a straight line and photons need more time to pass in dependence of the distance to the mass. This has an effect of a collecting lens (Fig. 1.2). Alternative theories of gravitation imply deviations of these geodesics.

Einstein developed the Theory of Relativity on the basis of a Gedankenexperiment: In an elevator without view to outside and without knowledge of the environment it cannot be decided whether attraction from a large mass or acceleration of the elevator is the reason of the observed gravity. The Theory of Relativity embodies the Weak (Galilean-Newtonian) Equivalence Principle:

1. Local bodies fall identically, because
2. Gravitational mass (Eq. (1.4)) is indistinguishable from inertial mass (Eq. (1.1)),
3. Regardless of composition,

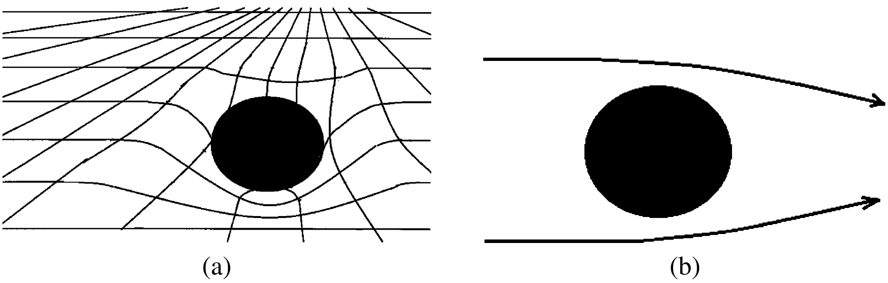

Fig. 1.2 (**a**) Schematic visualisation of geodesics in 4-dimensional space-time coordinates. In the absence of material objects space-time is flat and geodesics are straight lines. By a material accumulation geodesic will be buckled. A particle travelling along a geodesic will be deflected or even collide. (**b**) Deflection of a ray of light by a celestial body

4. Regardless of geometry (internal structure),
5. Regardless of amount of mass.

Inertial reference frames (coordinate systems) have constant relative velocity in a flat space-time manifold. Accelerating frames with consistent definitions of energy and momentum (or mass and angular momentum) require non-zero space-time curvature. Local space-time must have a unique curvature. Local test masses exhibiting non-parallel geodesic trajectories require simultaneous different values of local space-time curvature. Any paired (sets of) test masses violating the Equivalence Principle empirically falsify metric theories of gravitation at their founding postulate.

The Strong or Einstein Equivalence Principle states that all of the laws of physics (not just the laws of gravity) are the same in all small regions of space, regardless of their relative motion or acceleration.

6. Non-rotating free fall is locally indistinguishable from uniform motion absent gravitation. Linear acceleration relative to an inertial frame in Special Relativity is locally identical to being at rest in a gravitational field. A local reference frame always exists in which gravitation vanishes.
7. Local Lorentz invariance (absolute velocity does not exist) [53] and position invariance. All local free fall frames are equivalent.
8. The Strong Equivalence Principle embraces all laws of nature; all reference frames accelerated or not, in a gravitational field or not, rotating or not, anywhere at any time.

Gravity is still a central question of physics which may be expressed as "What gives an object mass (or inertia) so that it requires an effort to start it moving, and exactly the same effort to restore it to its original state?" [54]. A clarification of the existent theories could be found by search of violations of the Equivalence Principle and this requires extremely accurate measurement of the gravitational constant G in Eq. (1.4). Those which had been made are shown in Table 1.3.

Einstein predicted that ripples of low frequency in the space-time criss-cross the Universe. In order to examine the Universe by measuring intensity, properties and

Table 1.3 Test of the equivalence principle [55]

Year	Investigator	Method	Accuracy	Reference
∼500	Philoponus	Drop Tower		[56]
1585	Stevin	Drop Tower	5×10^{-2}	[57, 58]
1600	Galileo	Balls on incline, pendulum	10^{-2}	[34, 59]
1680	Newton	Pendulum	10^{-3}	[1, 2]
1832	Bessel	Pendulum	2×10^{-5}	[60]
1910	Southerns	Pendulum	5×10^{-6}	[61]
1918	Zeeman	Torsion balance	3×10^{-8}	[62, 63]
1922	Eötvös	Torsion balance	5×10^{-9}	[64]
1923	Potter	Pendulum	3×10^{-6}	[65, 66]
1935	Renner	Torsion balance	2×10^{-9}	[67]
1964	Dicke, Roll, Krotkov	Torsion balance	3×10^{-11}	[68]
1972	Braginsky, Panov	Torsion balance	10^{-12}	[69, 70]
1976	Shapiro, Counselman, King	Lunar laser ranging	10^{-12}	[71]
1981	Keiser, Faller	Fluid support	4×10^{-11}	[72]
1987	Niebauer, McHugh, Faller	Drop tower	10^{-10}	[73]
1989	Heckel, Adelberger, Stubbs, Su, Swanson, Smith	Torsion balance	10^{-11}	[74]
1990	Adelberger, Stubbs, Heckel, Su, Swanson, Smith, Gundlach	Torsion balance	10^{-12}	[75]
1999	Baeßler, Heckel, Adelberger, Gundlach, Schmidt, Swanson	Torsion balance	5×10^{-13}	[76]
2008	Schlamminger, Choi, Wagner, Gundlach, Adelberger	Rotating cryogenic torsion balance	3×10^{-14}	[77]
2010	MiniSTEP	Earth orbit	10^{-17}	[78]

direction of these gravitational waves, and likewise the correctness of the General Theory of Relativity, can be checked. The concept of falling under the influence of gravity alone follows a geodesic in space-time is at the foundation of General Relativity, our best model of gravitation. Such an experiment is planned by the LISA Pathfinder mission (Laser Interferometry Space Antenna) which was scheduled for launch at the end of 2009. The LISA Pathfinder concept is to prove geodesic motion of two test-masses in a nearly perfect gravitational free-fall through laser interferometry. As gravity is a very weak interaction an extremely low level of non-gravitational influences and parasitic accelerations is required. The test masses are cubes with 2 kg mass, 5 cm in diameter, made of a gold-platinum alloy and are surrounded with a gap of 4 mm to the electrode housing and thus can float free in the spacecraft and follow two parallel geodesics. The spacecraft is controlled by a micro-propulsion system and follows the test-masses with nanometer resolution. Violation of geodesic motion manifests itself as a relative acceleration of test-masses as measured by the laser interferometer with picometer distance resolution.

The graviton is a hypothetical elementary particle that mediates the force of gravity in the framework of quantum field theory. If it exists, the graviton must be massless because the gravitational force has unlimited range. Gravitons are postulated because of the great success of quantum field theory at modelling the behaviour of all other known forces of nature as being mediated by elementary particles. A quantum mass sometimes is postulated but never found.

Gravitation is weak but infinite ranging. Therefore aggregation of particles occurs in the huge and nearly empty space of the Universe. Then gravity + adhering forces may exceed dispersive forces and asteroids, planets and stars grow. By influence of gravity large bodies take a spherical shape, denser material concentrating in the interior.

Above a certain mass a star collapses to give a neutron star with mass of 1.35 to $2.1 \times m_{sun}$ and with diameter of 10 to 20 km [79]. In black holes the masses between 10 to $10^9 \times m_{sun}$ are concentrated. The respective diameter of such a spherical object is about 30 km for a mass of $10 \times m_{sun}$. Around a black hole there is an undetectable surface, called event horizon, which marks the point of no return. At the event horizon of a black hole, the deformation of space-time becomes so strong that all geodesic paths lead towards the black hole and not away from it. All the light is adsorbed, nothing reflected. From the interior nothing can escape, even not photons on account of its mass of motion. Karl Schwarzschild [80, 81] and independently Johannes Droste [82] gave a solution for the gravitational field of a point mass and a spherical mass. This solution has a singularity, the Schwarzschild radius, at which some of the terms in the Einstein equations become infinite.

Hess and Greiner [83, 84] made an extension of the theory of General Relativity based on pseudo-complex space-time coordinates. They constructed a pseudo-complex Schwarzschild solution, which does not suffer any more by a singularity. The solution indicates a minimal radius for a heavy mass object. As a result particles including photons may be repelled by an anti-gravitational effect when approaching closely another mass and then can leave the hole which is dark but no longer black.

1.9 Particle/Wave Duality

At the end of the 19th century the atomic theory was well established, namely, that much of nature was made of particles [85]. It was assumed that matter consists of particulate objects or atoms, and that electrons carry electric charge. Already Isaac Newton (1643–1727) argued that on account of its linear transmission light is composed of corpuscles and he could explain many optical effects by this assumption [86]. On the other hand Christiaan Huygens (1629–1695) proposed a wave theory of light [87] and later on by means of diffraction experiments the wave-like nature of light was clearly demonstrated. In the late 1800s, James Clerk Maxwell explained light as the propagation of electromagnetic waves according to the Maxwell equations [88]. By the turn of the 20th century, a duality of light was accepted.

Fig. 1.3 Light beam consisting of travelling wave packets (photons). Within the packet the oscillations have the frequency of the electromagnetic radiation. In vacuum the velocity of the photon correspond to the speed of light as a maximum. Within the packet the velocity of oscillations may be different so that the oscillations seem to move forwards or backwards, respectively. Rest mass of a photon is zero

By the photoelectric effect electrons are emitted from matter after absorption of energy from electromagnetic radiation such as X-rays or visible light. It was observed that the energy of the emitted electrons did not depend on the intensity of the incident radiation and this could not be explained by Maxwell's wave theory. In 1905 Albert Einstein described mathematically how the photoelectric effect could be described by absorption of quanta of light (now called photons) [89]. The particle-like behaviour of light was further confirmed with the discovery of the Compton scattering in 1923 [90]. The Compton Effect is the decrease in energy and, hence, increase in wavelength of an X-ray or gamma ray photon, when it interacts with matter. Because of the change in photon energy, it is an inelastic scattering process and it demonstrates that light cannot be explained purely as a wave phenomenon.

A ray of light may be regarded consisting of photons which are wave packets travelling in vacuum with speed of light $c_0 = 299{,}792.458\ \mathrm{m\,s^{-1}}$ (Fig. 1.3). The velocity of the wave within the packet may be different. The light beam can be affected by gravity. Photons, however, have no mass. Otherwise propagation with speed of light would be impossible.

In 1924, Louis-Victor Pierre Raymond de Broglie claimed that all matter, not just light, has a wave-like nature [91]. He related wavelength λ and momentum p:

$$\lambda = \frac{h}{p} \tag{1.15}$$

with the Planck constant $h = 6.626\,075\,5 \times 10^{-34}$ J s.

This is a generalisation of Einstein's equation (1.13). The wavelength of a moving body is given by:

$$\lambda = \frac{h}{mv} \tag{1.16}$$

where m = mass of particle, v = velocity of particle. When v is equal to the velocity of electromagnetic radiation c_0, and in combination with the Planck-Einstein equation,

$$E = h\nu \tag{1.17}$$

with E = Energy of the photon, h = Plank constant and ν = frequency, the Einstein equation (1.13) can be derived.

Because h represents a very small value, the wavelength (de Broglie wavelength) of such matter-wave can be observed on sufficiently small particles as photons and electrons only. Nevertheless, experiments have been conducted also with neutrons, protons and molecules. In 1999, the diffraction of C_{60} fullerenes as the biggest particle so far was reported [92]. Fullerenes are comparatively large and massive objects, having an atomic mass of about 720 u. The de Broglie wavelength is 2.5 pm, whereas the diameter of the molecule is about 1 nm, about 400 times larger.

1.10 Fundamental Interactions

The so-called Standard Model of particle physics is a theory of three of the four known fundamental interactions and the elementary particles that take part in these interactions. These particles make up all visible matter in the Universe [93]. A fundamental interaction is a mechanism by which particles interact with each other, and which cannot be explained by another more fundamental interaction. Today four fundamental interactions are known: gravitation, electromagnetism, the weak nuclear interaction, and the strong nuclear interaction. Their magnitude and behaviour vary greatly. Grand unified theories seek to unify three of these interactions as manifestations of a single, more fundamental, interaction. The concept of quantum gravity aims to unify gravitation with the other three into an interaction that is completely universal.

Gravitation is by far the weakest fundamental interaction, about 10^{37} times smaller than the electromagnetic interaction. However, because it has an infinite range and because all masses are positive, it is nevertheless very important in the Universe. Because all masses are positive, large bodies such as planets, stars and galaxies have large total masses and therefore exert large gravitational forces. In comparison, the total electric charge of these bodies is zero because half of all charges are negative. This is similar for the weak and strong interactions. Unlike the other interactions, gravity works universally on all matter and energy. There are no objects that lack a gravitational ‘charge’. Because of its long range, gravity is responsible for such large-scale phenomena such as the structure and action of galaxies. It is widely believed that in a theory of quantum gravity, gravity would be mediated by a particle which is known as the graviton. Gravitons are hypothetical particles not yet observed.

Although general relativity appears to present an accurate theory of gravity in the non-quantum mechanical limit, there are a number of alternate theories of gravity. Almost nothing is known about dark matter and dark energy and it is not clear whether it can be described completely by the present theories. Alternate theories under any serious consideration by the physics community all reduce to general relativity, and the focus of observational work is to establish limitations on what deviations from general relativity are possible.

Occasionally, physicists have postulated the existence of an additional fifth force in addition. Beyond the range of the force it is assumed to rapidly become insignificant. Many experiments have been undertaken to measure discrepancies in the measurement of the gravitational constant. This implies very sensitive measurements with pendulum [25] or weighing in the presence of varying large masses, e.g. on Earth's surface and in the shaft of a mine or at the wall of a barrage when the water level is changing [94]. Up to now there is no strong evidence for such a fifth force.

The 'standard model' describes the elementary particles and interactions between those particles, but not their mass. Peter Higgs (*1929) [95–97] and concurrently François Englert and Robert Brout [98], as well as Gerard Gouralnik, Karl Richard Hagen and Tom Kibble [99] developed field theories explaining the donation of mass to the particles. Higgs boson is a hypothetical massive scalar particle, a quantum that is postulated to be the carrier particle of the Higgs field, a theoretical field that permeates every place in the Universe at all times and endows all elementary subatomic particles with mass by interactions. Its existence is predicted by the Standard Model of particle physics and could explain how otherwise massless elementary particles still manage to construct mass in matter. In particular, it would explain the difference between the massless photon (travelling with light velocity) and other slower massive particles. On 4th July 2012 CERN informed that results of both, ATLAS and CMS experiments gave strong indications for the presence of a new particle, which could be the Higgs boson, in the mass region around 126 GeV $= 225 \times 10^{-27}$ kg. This corresponds to about 135 times the mass of a proton (hydrogen core). Such a heavy elementary particle decays very fast and it appears very seldom even under the experimental conditions which corresponded to the conditions existing after 10^{-9} s after the Big Bang.

About 99.9 % of the mass of the visible Universe is made up of protons and neutrons. Both particles are much heavier than their three quarks (about 5 %) and gluon constituents. Recently, it became possible to calculate the mass of such light hadrons using lattice quantum chromodynamics. 95 % of the mass is involved in the attractive forces between the quarks. A quantitative confirmation of this aspect of the Standard Model was represented with fully controlled uncertainties [100].

1.11 Theory of Everything

Already ancient philosophers have speculated that the apparent diversity of appearances conceals an underlying unity, and thus that the list of forces might be short, indeed might contain only a single entry [101]. Laplace formulated in 1814:

> "An intellect which at a certain moment would know all forces that set nature in motion, and all positions of all items of which nature is composed, if this intellect were also vast enough to submit these data to analysis, it would embrace in a single formula the movements of the greatest bodies of the Universe and those of the tiniest atom; for such an intellect nothing would be uncertain and the future just like the past would be present before its eyes" [102].

After Einstein's theory of gravity (general relativity) was published in 1915, the search for a unified field theory combining gravity with electromagnetism began, based on the assumption, that no other fundamental forces exist. Also Einstein in his later years was intensely occupied in finding such a unifying theory, without success.

For some decades theoretical physicists have looked for a so-called Theory of Everything. This ultimate theory should consist of a set of equations which give a connection between the currently known basic forces: gravitation, electromagnetism, the weak nuclear interaction, and the strong nuclear interaction. Unlike the point-shaped particles of quantum theory the basic particles are stretched strings, but include also more general items, called branes (derived from 'membrane') floating in space-time. Strings may be open-ended or in shape of a closed loop. It is speculated whether in a special 'supersymmetry' fermions exist as basic items of matter and bosons as particles that transmit a force to each fermion. Depending on size, shape and tension such objects form the particles known in the standard model. The average size of a string should be somewhere near the length scale of quantum gravity, called the Planck length, which is about 10^{-35} m. This means that strings are too small to be observed by today's methods and probably it will not possible at all. Although some critics concede that string theory is falsifiable in principle, they maintain that it is unfalsifiable for the foreseeable future, and so should not be called science.

Indeed such a Theory of Everything could not explain the complexity of the world and the existence of life, not even the occurrence of relatively simple inorganic structures.

1.12 International Unit of Mass

The international unit kilogram was determined as the mass of one cubic decimetre of distilled water at the temperature of its highest density at 4 °C (see Chap. 2). Kilogram weights were made of a platinum alloy and are used as prototypes (Fig. 1.4).

1.12.1 Definition of the Mass Prototype

In 1948 as a result of the General Conference on Weights and Measures it was published [103]:

> Declaration on the unit of mass and on the definition weight; conventional value of g_n (CR, 70)
> Taking into account the decision of the Comité International des Poids et Mesures of 15 October 1887, according to which the kilogram has been defined as unit of mass: Taking into account the decision contained in the sanction of the prototypes of the Metric System, unanimously accepted by the Conférence Générale des Poids et Mesures on 26 September 1889;
> Considering the necessity to an end to the ambiguity which in current practice still exists on the meaning of the word *weight*, used sometimes for *mass*, sometimes for *mechanical force*;
> The Conference declares:

Fig. 1.4 The International Prototype Kilogram (IPK) at Sèvres, France is made of an alloy of 90 % platinum and 10 % iridium (by mass) and is machined into a right-circular cylinder (height = diameter) of 39.17 mm to minimise its surface area. The IPK and its replicas are stored in air under two or more nested bell jars

1. The kilogram is the unit mass; it is equal to the mass of the international prototype of the kilogram;
2. The word 'weight' denotes a quantity of the same nature as a 'force': the weight of a body is the product of its mass and the acceleration due to gravity; in particular, the standard weight of a body is a product of its mass and the standard acceleration due to gravity;
3. The value adopted in the International Service of Weights and Measures for the standard acceleration due to gravity is 980.665 cm/s^2, a value already stated in the laws of some countries.

1.12.2 Alternative Mass Unit Definitions

Many units in the SI system are defined relative to the kilogram so its stability is important. Since the IPK and its replicas are stored in air (albeit under two or more nested bell jars), they adsorb atmospheric contamination onto their surfaces and gain mass. The relative change in mass and the instability in the IPK have prompted research into improved methods to obtain a smooth surface finish using diamond-turning on newly manufactured replicas and has intensified the search for a new definition of the kilogram. The kilogram is the last remaining base unit of the SI that is still defined by a material artefact. In 2005 the International Committee for Weights and Measures (CIPM) recommended that the kilogram be redefined in terms of fundamental constants of nature e.g. Avogadro constant.

An Avogadro constant-based approach attempts to define the kilogram as a quantity of silicon atoms. Silicon was chosen because the semiconductor industry knows processes for creating almost defect-free, ultra-pure monocrystalline silicon. To make a practical realisation of the kilogram, a one kilogram silicon globe is being produced from a rod-like, single-crystal ingot consisting almost of ^{28}Si calculated to eight decimal places. Natural silicon consists of 92 % of that isotope and some heavier atoms. By means of centrifuge that is refined to 99.99 % of ^{28}Si. The mass of the globe which is slightly more than 1 kg will be reduced by polishing down to the mass of the IPK. The surface of the ready-made globe is covered tightly with an oxide layer so that it can be handled without loss of surface molecules. The globe must be almost perfect in shape so that the volume can be determined exactly by means of an interferometer. Then the number of atoms can be calculated on the basis of the well-known value of the crystalline unit cell dimensions.

Another Avogadro-based approach, ion accumulation, would define and delineate the kilogram by creating new metal mass artefacts. It would do so by accumulating gold or bismuth ions (atoms stripped of an electron) and counting them by measuring the electrical current required to neutralise the ions. Gold and bismuth are used because they have the two greatest atomic masses of the chemical elements that have only one naturally occurring stable isotope. Because the production of enough deposits is time consuming the programme has just been cancelled.

Relativity and quantum mechanics show that even a single particle of mass m determines a Compton frequency $\omega_0 = mc^2/\hbar$, where c is the speed of light and $\hbar$ is the reduced Planck constant. A clock referenced to ω_0 would enable high-precision mass measurements and a fundamental definition of the second. By Holger Müller et al. [104] it was demonstrated that such a clock using an optical frequency comb to self-reference a Ramsey-Bordé atom interferometer and synchronise an oscillator at a subharmonic of ω_0. This directly demonstrates the connection between time and mass. It allows measurement of microscopic masses with 4×10^{-9} accuracy in the proposed revision to SI units. Together with the Avogadro project, it yields calibrated kilograms.

It is possible to compensate the force acting at a body in the gravitational field by means of the electromagnetic Lorentz force. That force acts on a point charge in an electromagnetic field. Both the electric field and magnetic field can be defined from the Lorentz force law:

$$\vec{F} = q(\vec{E} + v \times \vec{B}) \tag{1.18}$$

where F is the force [N], E is the electric field strength [V m^{-1}), B is the magnetic flux density [T = V s m^{-2}], q is the electric charge of the particle [C = A s], v is the instantaneous velocity of the particle [m s^{-1}]. Equation (1.18) is a vector equation and $\times$ means vector cross product. The electric force is straightforward, being in the direction of the electric field if the charge q is positive, and the direction of the magnetic field is perpendicular to it.

By compensation of weight forces by means of an electric current through a coil instead of a counterweight it is possible to substitute the unit mass by electrical units, supposed the geometry of the electromagnetic device is known. The Watt balance is a double-arm weighing scale with single pan for the kilogram test mass and a coil surrounding a permanent magnet at the opposite side (Fig. 1.5). The

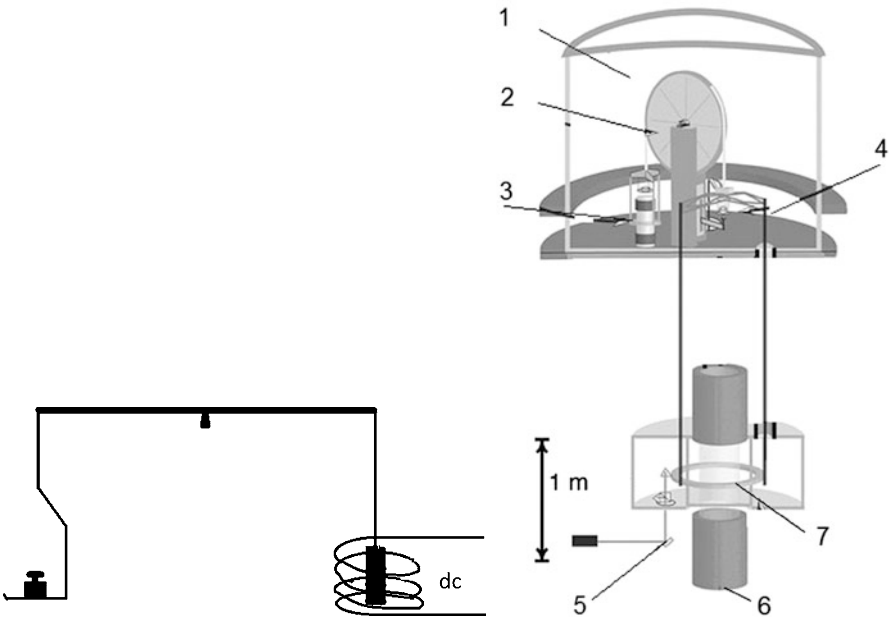

Fig. 1.5 (**a**) Diagram of an Ampère or Watt balance. (**b**) Watt balance of the UK National Physical Laboratory. *1* vacuum chamber, *2* balance wheel, *3* velocity drive coil, *4* reference mass, *5* interferometer (1 of 3), *6* superconducting solenoid, *7* induction coils. © NPL, Teddington, UK

electrical power necessary to oppose the weight of the test mass is measured as it is accelerated by gravity. However, because Eq. (1.17) contains two variables two experiments at the same balance are necessary. The Watt balance is a variation of an ampere balance in that it employs an extra calibration step that neutralises geometrical effects. Here the suspended coil is moved at defined velocity through the field of the fixed permanent magnet. In this way the electric potential in the watt balance is delineated by a Josephson voltage standard, which allows voltage to be linked to an invariant constant of nature with extremely high precision and stability. Its circuit resistance is calibrated against a quantum Hall resistance standard. The mass standard would be given by Planck's elementary quantum of action. The watt balance requires exquisitely precise measurement of gravity in the respective laboratory. Until now, the results of the various watt balances have wide scattering and the reasons of the deviations are still unclear. (See also Sect. 4.2 in Chap. 4.)

Another electrical method of defining the mass standard is the voltage balance [105–107]. Here an electrode is suspended movable at a balance and the force exerted by a fixed electrode is measured (Fig. 1.6). By application of high tension the force between the capacitor is measured using mechanical substitution balances. The balance and the capacitor are in an atmosphere of nitrogen gas.

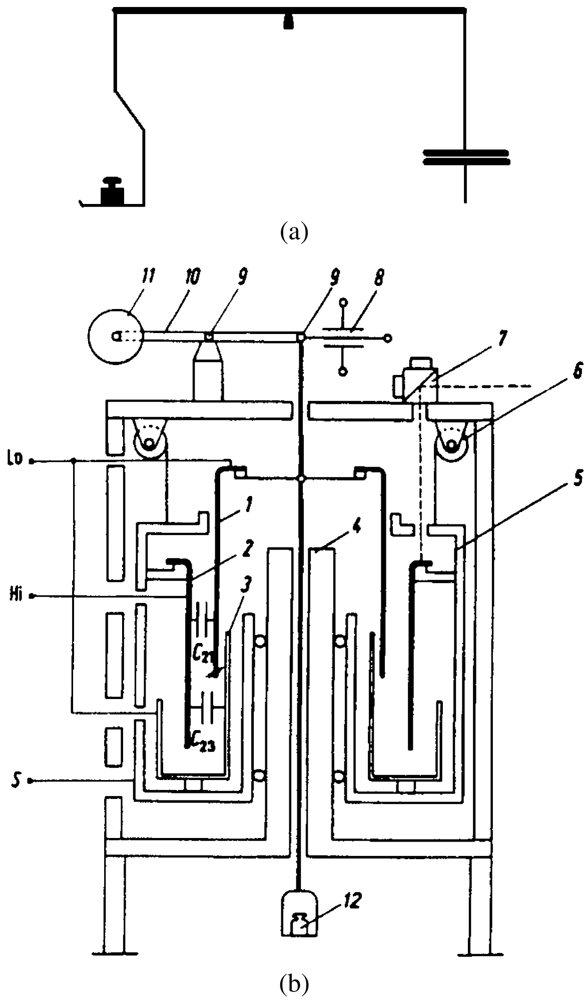

Fig. 1.6 (**a**) Diagram of a voltage balance. (**b**) Diagram of the voltage balance of the University of Zagreb, Croatia. *1* movable electrode, *2* high-voltage electrode, *3* auxiliary electrode, *4* column of the sliding unit. *5* carriage for the sliding unit, *6* drive, *7* interferometer for measuring the displacement path, *8* position sensor, *9* flexible bearings of the balance, *10* balance beam, *11* counterweight, *12* mass standard. © University of Zagreb

A liquid electrometer had been realised consisting of a horizontally arranged capacitor whereby the lower electrode is the surface of a mercury bath (Fig. 1.7) [108]. Voltage is applied between that surface and an electrode fixed at the distance d. The increase of the mercury level z is measured by means of a laser white-light interferometer in comparison to the distance of the mercury level to reference electrodes.

Proposals have been made for a new realisation of the static voltage balance for two dynamic methods: a drop method with a capacitor and an oscillation method [109].

There are also a number of proposals to replace the artificial mass standard by a natural factum. It is proposed to use the electron. However no experiment is known so far to determine the mass of an electron with the required accuracy. Furthermore it would be very difficult to extrapolate from those small mass weights for practical

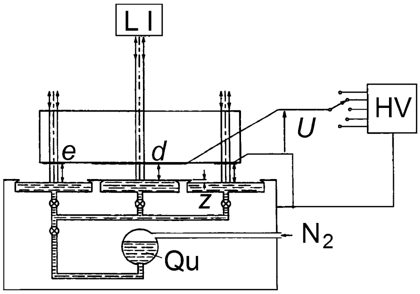

Fig. 1.7 Diagram of a liquid electrometer. *d* distance between electrodes, e_a, e_b distance between the reference electrodes, z change of the distance due to applied voltage U. The distance between electrodes is controlled by laser interferometer. *HV* high voltage source, *QU* mercury reservoir, *LI* laser with-light interferometer. © CSIRO, Australia

use. It was also suggested to use a mass derived from the Broglie wavelength [110] (see Sect. 1.10). Planck made proposals for defining mass, length and time by fundamental constants (see Sect. 1.12.2). The mass should be defined by:

$$m_{Pl} = \sqrt{\frac{\hbar c}{G}} = 2.17644(11) \times 10^{-8}\ \mathrm{kg} \approx 22\ \mu\mathrm{g} \tag{1.19}$$

where G is the gravitational constant, $\hbar$ is the Planck constant, and c is the speed of light. However, for practical weighable bodies the Planck constant cannot be realised with the sufficient accuracy.

At the next session of the general BIPM conference in 2011 it should be decided whether the kilogram will get a new definition on the basis of one of these experiments

1.13 Mass Units

To describe physically the universe and the world we need a system of four basic quantities. From these independent quantities all others can be derived. However, for convenience most systems defined some more base quantities.

1.13.1 The International System of Units

The International System of Units (Système International d'Unités SI) is the modern form of the metric system. It is the world's most widely used system of units, both in everyday commerce and in science [111–113]. The metric system was conceived by a group of scientists (among them, Lavoisier) who had been commissioned by King Louis XVI of France to create a unified and rational system of measures. After the French Revolution, the system was adopted by the new government. On August 1,

Table 1.4 SI base units

Base quantity	Quantity symbol	Dimension symbol	SI base unit	Unit symbol
Length	l	L	Meter	m
Mass	m	M	Kilogram	kg
Time	t	T	Second	s
Electric current	I or i	I	Ampere	A
Thermodynamic temperature	T	Θ	Kelvin	K
Amount of substance	n	N	Mole	mol
Luminous intensity	I_v	J	Candela	cd

Fig. 1.8 Advertisement for the metric system

1793 the National Convention adopted the new decimal ‘metre’ with a provisional length as well as the other decimal units with preliminary definitions and terms. On April 7, 1795 the terms *gramme* and *kilogramme* replaced the former terms ‘gravet’ and ‘grave’ (Loi du 18 germinal, an III). The older metric system included several groupings of units. The SI was developed in 1960 from the metre-kilogram-second (mks) system. The SI introduced several newly named units. The SI is not static; it is a living set of standards where units are created and definitions are modified with international agreement as measurement technology progresses (Fig. 1.8).

In the SI system we have two basic quantities which are used to describe matter at rest: mass and length. The corresponding SI units are kilogram or kilogramme (kg) and meter (m). Mass is a relatively new and abstract quantity which can be measured by observing effects within gravitational or acceleration fields [103]. It is an inertial property; that is, the tendency of the object to remain at constant velocity unless acted upon by an outside force. The other parameter, volume (m^3), based on length, describes the extension of a structure of matter within the three-dimensional space. It is measured usually by means of the interaction of matter with electromagnetic radiations by which the structure becomes visible.

Another useful parameter, density, is defined as the relation of mass to unit volume and the corresponding SI unit is $\mathrm{kg\,m^{-3}}$. The exact definition is relatively new, probably of Leonhard Euler [9, 114]. It should be mentioned that the notation 'specific gravity' (density related to water) is much older. Methods of determining of specific gravity are described in the books Ayin-Akbari of Muhammad Ibn-Ahmad Al-Biruni (873–1084) [115, 116] and in the Book of the Balance of Wisdom of Abd al-Rahman al-Manzur Al-Chazini 1120 [117].

A quantity whose magnitude is additive for subsystems is called 'extensive', examples are mass and volume. A quantity whose magnitude is independent of the extent of the system is called 'intensive', examples are temperature and pressure. The adjective 'specific' before the name of an extensive quantity is often used to mean 'divided by mass' [118]. When the symbol for the extensive quantity is a capital letter, the symbol used for the specific quantity is often the corresponding lower case letter.

The term 'specific' should be used only if the quantity can be (and is) divided by the total mass or partial mass of the sample in question. If it is related to another item as in the case of atomic and molecular mass the respective notation is 'relative'.

Likewise the term 'molar' before the name of an extensive quantity is used to denote 'divided by amount in moles'. Thus the heat capacity of a given quantity of a substance is the energy required to raise the temperature of that quantity of the substance by 1 °C (or 1 Kelvin), measured in joules ($\mathrm{J\,K^{-1}}$). The specific heat capacity is this quantity divided by its mass, measured in $\mathrm{J\,K^{-1}\,kg^{-1}}$. The molar heat capacity is the quantity divided by relative molar mass, measured in $\mathrm{J\,K^{-1}\,mol^{-1}}$.

1.13.2 Natural Units

The natural units or Planck units are physical units of measurement defined exclusively in terms of five universal physical constants listed in Table 1.6, in such a manner that these five physical constants take on the numerical value of 1 when expressed in terms of these natural units.

By setting to (dimensionless) 1 the five fundamental constants in Table 1.4, the base units of length, mass, time, charge, and temperature shown in Table 1.5 are also

Table 1.5 Fundamental physical constants

Constant	Symbol	Dimension	Value in SI units with uncertainties
Speed of light in vacuum	c	$\mathrm{L\,T^{-1}}$	$2.99792458108 \times 10^{8}\ \mathrm{m\,s^{-1}}$
Gravitational constant	G	$\mathrm{L^3\,M^{-1}\,T^{-2}}$	$6.67428(67) \times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}$
Reduced Planck constant	$\hbar = h/2\pi$	$\mathrm{L^2\,M\,T^{-1}}$	$1.054571628(53) \times 10^{-34}\ \mathrm{J\,s}$
Coulomb constant	$\frac{1}{4\pi\varepsilon_0}$	$\mathrm{L^3\,M\,T^{-2}\,Q^{-2}}$	$8.9875517873681764 \times 10^{9}\ \mathrm{kg\,m^3\,s^{-2}\,C^{-2}}$
Boltzmann constant	k_B	$\mathrm{L^2\,M\,T^{-2}\,\Theta^{-1}}$	$1.3806504(24) \times 10^{-23}\ \mathrm{J\,K^{-1}}$

Where h = Planck constant, ε_0 = permittivity of free space, L = length, M = mass, T = time, Q = electric charge, Θ = temperature. Speed of light and Coulomb constant are exact values by definition

Table 1.6 Natural Planck base units

Name	Dimension	Expressions	SI equivalent with uncertainties	Eq.
Planck length	L	$l_P = \sqrt{\frac{\hbar G}{c^3}}$	$1.616\,252(81) \times 10^{-35}$ m	(1.20)
Planck mass	M	$m_P = \sqrt{\frac{\hbar c}{G}}$	$2.176\,44(11) \times 10^{-8}$ kg	(1.19)
Planck time	T	$t_P = \frac{l_P}{c} = \frac{\hbar}{m_P c^2} = \sqrt{\frac{\hbar G}{c^5}}$	$5.391\,24(27) \times 10^{-44}$ s	(1.21)
Planck charge	Q	$q_P = \sqrt{4\pi\varepsilon_0 \hbar c}$	$1.875\,545\,870(47) \times 10^{-18}$ C	(1.22)
Planck temperature	Θ	$T_P = \frac{m_P c^2}{k_B} = \sqrt{\frac{\hbar c^5}{G k_B^2}}$	$1.416\,785(71) \times 10^{32}$ K	(1.23)

set to (dimensionless) 1. Particle physicists and cosmologists often use the reduced Planck mass, which is

$$\sqrt{\frac{\hbar c}{8\pi G}} \approx 4.340 \times 10^{-6}\ \mathrm{g} = 2.43 \times 10^{18}\ \mathrm{GeV}/c^2 \tag{1.24}$$

Planck units elegantly simplify particular algebraic expressions appearing in physical laws. Planck units are considered unique in that these units are not based on properties of any prototype object, or particle (that would be arbitrarily chosen) but are based only on properties of free space. However, most Planck units are many orders of magnitude too large or too small to be of any empirical and practical use, so that Planck units as a system are really only relevant to theoretical physics. That is because 1 Planck unit is often the largest or smallest value of a physical quantity that makes sense given the current state of physical theory.

1.13.3 Conventional Mass Units

By means of a balance we measure the weight but the scaling indicates mass units. The gravitational acceleration depends on the geographical location, but differences in the value of weight are almost insignificant in customary weighing. Thus in daily life the notation ‘mass’ will replace ‘weight’ only slowly or never. Furthermore ‘weight’ denotes the weight piece, which is a calibrated mass.

On the other hand, in science mass is an important physical quantity, which must be clearly distinguished from weight. Nevertheless, looking into the literature we find often the notation weight. For objects which will never be weighed by a conventional balance in the gravitational field—atoms and molecules—this notation is still familiar. Indeed, the mass of such particles is determined either indirectly or by means of mass spectrometers. These are methods in which gravity and, hence, weight is not included.

Mass is the only SI base unit with an SI prefix as part of its name and it is caused by the historical development. However, the usual prefixes in the SI system are never used additionally with kilogram (Table 1.7). With few exceptions, the system is legally being used in every country in the world, and many countries do not maintain official definitions of other units. In the United States, industrial use of SI is increasing, but popular use is still limited. In the United Kingdom, conversion to metric units is official policy but not yet complete. Those countries that still recognise non-SI units have redefined their traditional non-SI units in terms of SI units. There the avoirdupois pound is used as a unit of mass and its related unit of force is the pound-force. The European Union has a directive as a result of which non-SI markings will be banned after 31 December 2009 on any goods imported into the European Union [119]. This applies to all markings on products, enclosed directions and papers, packaging, and advertisements. Nevertheless, until now, consequences of that definition have not been completely accepted. A survey on other mass units is given in Chap. 2.

The practice of using the abbreviation ‘mcg’ rather than the SI symbol ‘µg’ was formally mandated for medical practitioners in 2004 by the Joint Commission on Accreditation of Healthcare Organisations (JCAHO) in their ‘Do Not Use’ List: Abbreviations, Acronyms, and Symbols because hand-written expressions of ‘µg’ can be confused with ‘mg’, resulting in a thousand-fold overdosing. The mandate was also adopted by the Institute for Safe Medication Practices. The metric carat or Karat is a unit of mass used for measuring gems and pearls. 1 ct = 200 mg.

Both in the British imperial system and U.S. customary units the avoirdupois pound is still used. 1 avoirdupois pound (lb) is defined as exactly 0.45359237 kg, making one kilogram approximately equal to 2.205 avoirdupois pounds.

A tonne (t) or metric ton, also referred to as a metric tonne or tonne de metrice, is a measurement of mass equal to 1000 kilograms (Table 1.8). It is not a SI unit but tolerated. Still in use are long and short tons. One long ton (2240 lb) which is 101.605 % of a tonne; one short ton (2000 lb) is 90.72 % of a tonne.

Occasionally ton describes the percentage of a metal within an alloy. The tonne of trinitrotoluene (TNT) is used as a proxy for energy of explosives (explosive force),

Table 1.7 SI multiples for gram. (Prefixes are never used additionally with kilogram!)

Value	Symbol	Name
10^{-1} g	dg	decigram
10^{-2} g	cg	centigram
10^{-3} g	mg	milligram
10^{-6} g	µg	microgram (mcg)
10^{-9} g	ng	nanogram
10^{-12} g	pg	picogram
10^{-15} g	fg	femtogram
10^{-18} g	ag	attogram
10^{-21} g	zg	zeptogram
10^{-24} g	yg	yoctogram

Value	Symbol	Name
10^{0} g	g	gram
10^{1} g	dag	decagram
10^{2} g	hg	hectogram
10^{3} g	kg	kilogram
10^{6} g	Mg	megagram = 1 tonne
10^{9} g	Gg	gigagram
10^{12} g	Tg	teragram
10^{15} g	Pg	petagram
10^{18} g	Eg	exagram
10^{21} g	Zg	zettagram
10^{24} g	Yg	yottagram

Table 1.8 Multiples for tonne

Value t	Symbol	Name
10^{0} t	t	ton, tonne
10^{1} t	dat	decaton
10^{2} t	ht	hectoton
10^{3} t	kt	kiloton
10^{6} t	Mt	megaton
10^{9} t	Gt	gigaton
10^{12} t	Tt	teraton

Value g	SI value kg	Symbol	Name
10^{6} g	10^{3}	Mg	megagram
10^{7} g	10^{4}		
10^{8} g	10^{5}		
10^{9} g	10^{6}	Gg	gigagram
10^{12} g	10^{9}	Tg	teragram
10^{15} g	10^{12}	Pg	petagram
10^{18} g	10^{15}	Eg	exagram

referred to as 4.184 GJ (gigajoules). Ton is used also used as volume unit for ships: tonnage, displacement tonnage, gross tonnage the brutto register ton BRT: 1 BRT = 100 cubic foot = 2,83164 m^3.

Obsolete is a mass unit defined in the former cgs-system: $1\ \mathrm{kp\,s^2\,m^{-1}} = 9.8$ kg.

A survey on mass units accepted by the Bureau International des Poids et Mesures (BIPM) [120] and of related units and constants is given in Table 1.9.

1.13.4 Atomic Mass

An important unit for chemists is amount of atomic or sub-atomic species expressed in moles, based on Avogadro constant. Furthermore, it is based on the number of constituent particles (protons, electrons and neutrons) of an atom of carbon-12. The mole was confirmed in 1969 as a basic SI unit.

Table 1.9 Mass and mass related units

Name	Symbol	Definition	SI unit
Number of entities (e.g. molecules, atoms, ions, formula units)	N		–
Amount of substance, amount (chemical amount)	n	$n_B = N_B/L$	mol
Avogadro constant	L, N_A	$6.022\,141\,79 \pm 0.000\,000\,30 \times 10^{23}$	mol^{-1}
Planck constant/2π	$\hbar = h/2\pi$	1.0546×10^{-34}	J s
Planck mass	m_{Pl}	$2.176\,45(16) \times 10^{-8}$	kg
Mass of atom, atomic mass	m_a, m		kg
Mass of entity (molecule, formula unit)	m, m_f		kg
Atomic mass constant	m_u	$m_u = m_a(^{12}\mathrm{C})/12 = 1$ u	kg
Dalton, unified atomic mass unit	Da, u	1 Da = 1 u = $1.66053886(28) \times 10^{-27}$	kg
Molar mass	M	$M_B = m/n_B$	$\mathrm{kg\,mol}^{-1}$
Molar mass constant	M_u	$M_u = m_u N_A$	$\mathrm{kg\,mol}^{-1}$
Relative molecular mass (relative molar mass, molecular weight)	M_r	$M_r = m_f/m_u$	–
Relative atomic mass (atomic weight)	A_r	$A_r = m_a/m_u$	–
Molar volume	V_m	$V_{m,B} = V/n_B$	$\mathrm{m}^3\,\mathrm{mol}^{-1}$
Mass fraction	w	$w_B = m_B/\sum_i m_i$	–
Electronvolt	eV	1 eV = $1.60217653(14) \times 10^{-19}$	J
Electronvolt/c^2	eV/c^2	1.783×10^{-36}	kg

In the book "The International System of Units (SI)" of the Bureau International des Poids et Mesures (BIPM) [113] we read:

> "'Atomic weights' and 'molecular weights' ... are in fact relative masses. Physicists and chemists have ever since agreed to assign the value 12, exactly, to the so-called atomic weight of the isotope carbon with mass number 12 (carbon-12, ^{12}C), correctly called the relative atomic mass A_r (^{12}C). The unified scale thus obtained gives the relative atomic and molecular masses, also known as the atomic and molecular weights, respectively."

In the IUPAC manual 'Quantities, Units and Symbols in Physical Chemistry' [118], the so-called 'Green book', it is concluded:

> "For historical reasons the terms 'molecular weight' and 'atomic weight' are still used. For molecules M_r is the 'relative molecular mass' or 'molecular weight'. For atoms M_r is the relative atomic mass or 'atomic weight', and the symbol A_r may be used. M_r may also be called the relative molar mass, $M_{r,B} = M_B/M_u$, where M_u = mass (in grams) of 1 mole of carbon-12."

The term 'atomic weight' is being phased out slowly and being replaced by 'relative atomic mass', however 'standard atomic weights' have maintained their name [121].

Relative atomic masses are listed in Table 1.7; a periodic table with standard atomic weights is depicted in Tables 1.10a and 1.10b.

The molecular mass of a substance, formerly also called molecular weight and abbreviated as MW, is the mass of one molecule of that substance, relative to the unified atomic mass unit u (equal to $1/12$ of the mass of one atom of carbon-12). This is distinct from the relative molecular mass of a molecule M_r, which is the ratio of the mass of that molecule to $1/12$ of the mass of carbon-12 and is a dimensionless number. Molar masses are almost never measured directly. They may be calculated as the sum of the standard atomic weights included in the chemical formula. The molar mass of carbon C is 12.0107 g/mol. Salts consist of ions and therefore the value is notified as formula mass instead of a molecular mass.

The unified atomic mass unit (u), or dalton (Da), is a small unit of mass used to express atomic and molecular masses. It is defined to be one-twelfth of the mass of an unbound atom of ^{12}C at rest and in its ground state. The unit is convenient because one hydrogen atom has a mass of approximately 1 u, and more generally an atom or molecule that contains N_p protons and N_n neutrons will have a mass approximately equal to $(N_p + N_n) \cdot u$. Atomic masses are often written without any unit and then the unified atomic mass unit is implied. In biochemistry, particularly in reference to proteins, the term ‘dalton’ is often used. Because proteins are large molecules, they are typically referred to in kilodaltons, or ‘kDa’ The unified atomic mass unit, or dalton, is not an SI unit of mass, although it is accepted for use with SI under either name. The symbol amu for atomic mass unit is obsolete.

The Avogadro constant $N_A \approx 6.022 \times 10^{23}\ \text{mol}^{-1}$ is the number of ‘entities’ (usually, atoms or molecules) in one mole. (The pure value 6.022×10^{23} is called Avogadro’s number.) The Avogadro constant and the mole are defined so that one mole of a substance with atomic or molecular mass 1 u have a mass of 1 g. For example, the molecular mass of a water molecule containing one ^{16}O isotope and two ^{1}H isotopes is 18.0106 u, and this means that one mole of this monoisotopic water has a mass of 18.0106 grams. Water and most molecules consist of a mixture of molecular masses due to naturally occurring isotopes. For this reason these sorts of comparisons are more meaningful and practical using molar masses which are generally expressed in g/mol, not u. The one-to-one relationship between daltons and g/mol is true but in order to be used accurately calculations must be performed with isotopically pure substances or involve much more complicated statistical averaging of multiple isotopic compositions [122].

Molar mass, symbol m_m is the mass of one mole of a substance (chemical element or chemical compound). Molar masses are almost quoted in grams per mole (g mol^{-1}). Mole is defined as the molecular mass in grams:

$$m_m = \frac{m_u}{N_A} \tag{1.25}$$

where m_u is the mass in atomic mass units and N_A is Avogadro’s number.

Table 1.10a Relative atomic masses related to $^{12}C = 12{,}0000$ [123]. The value enclosed in brackets, indicates the mass number of the most stable isotope of the element

Atomic number	Symbol	Name	Relative atomic mass
1	H	Hydrogen	1.00794(7)
2	He	Helium	4.002602(2)
3	Li	Lithium	6.941(2)
4	Be	Beryllium	9.012182(3)
5	B	Boron	10.811(7)
6	C	Carbon	12.0107(8)
7	N	Nitrogen	14.0067(2)
8	O	Oxygen	15.9994(3)
9	F	Fluorine	18.9984032(5)
10	Ne	Neon	20.1797(6)
11	Na	Sodium	22.98976928(2)
12	Mg	Magnesium	24.3050(6)
13	Al	Aluminium	26.9815386(8)
14	Si	Silicon	28.0855(3)
15	P	Phosphorus	30.973762(2)
16	S	Sulfur	32.065(5)
17	Cl	Chlorine	35.453(2)
18	Ar	Argon	39.948(1)
19	K	Potassium	39.0983(1)
20	Ca	Calcium	40.078(4)
21	Sc	Scandium	44.955912(6)
22	Ti	Titanium	47.867(1)
23	V	Vanadium	50.9415(1)
24	Cr	Chromium	51.9961(6)
25	Mn	Manganese	54.938045(5)
26	Fe	Iron	55.845(2)
27	Co	Cobalt	58.933195(5)
28	Ni	Nickel	58.6934(4)
29	Cu	Copper	63.546(3)2
30	Zn	Zinc	65.38(2)
31	Ga	Gallium	69.723(1)
32	Ge	Germanium	72.64(1)
33	As	Arsenic	74.92160(2)
34	Se	Selenium	78.96(3)
35	Br	Bromine	79.904(1)
36	Kr	Krypton	83.798(2)
37	Rb	Rubidium	85.4678(3)
38	Sr	Strontium	87.62(1)
39	Y	Yttrium	88.90585(2)

Table 1.10a (Continued)

Atomic number	Symbol	Name	Relative atomic mass
40	Zr	Zirconium	91.224(2)
41	Nb	Niobium	92.90638(2)
42	Mo	Molybdenum	95.96(2)
43	Tc	Technetium	[98]
44	Ru	Ruthenium	101.07(2)
45	Rh	Rhodium	102.90550(2)
46	Pd	Palladium	106.42(1)
47	Ag	Silver	107.8682(2)
48	Cd	Cadmium	112.411(8)
49	In	Indium	114.818(3)
50	Sn	Tin	118.710(7)
51	Sb	Antimony	121.760(1)
52	Te	Tellurium	127.60(3)
53	I	Iodine	126.90447(3)
54	Xe	Xenon	131.293(6)
55	Cs	Caesium	132.9054519(2)
56	Ba	Barium	137.327(7)
57	La	Lanthanum	138.90547(7)
58	Ce	Cerium	140.116(1)
59	Pr	Praseodymium	140.90765(2)
60	Nd	Neodymium	144.242(3)
61	Pm	Promethium	[145]
62	Sm	Samarium	150.36(2)
63	Eu	Europium	151.964(1)
64	Gd	Gadolinium	157.25(3)
65	Tb	Terbium	158.92535(2)
66	Dy	Dysprosium	162.500(1)
67	Ho	Holmium	164.93032(2)
68	Er	Erbium	167.259(3)
69	Tm	Thulium	168.93421(2)
70	Yb	Ytterbium	173.054(5)
71	Lu	Lutetium	174.9668(1)
72	Hf	Hafnium	178.49(2)
73	Ta	Tantalum	180.94788(2)
74	W	Tungsten	183.84(1)
75	Re	Rhenium	186.207(1)
76	Os	Osmium	190.23(3)1
77	Ir	Iridium	192.217(3)
78	Pt	Platinum	195.084(9)

Table 1.10a (Continued)

Atomic number	Symbol	Name	Relative atomic mass
79	Au	Gold	196.966569(4)
80	Hg	Mercury	200.59(2)
81	Tl	Thallium	204.3833(2)
82	Pb	Lead	207.2(1)
83	Bi	Bismuth	208.98040(1)
84	Po	Polonium	[209]
85	At	Astatine	[210]
86	Rn	Radon	[222]
87	Fr	Francium	[223]
88	Ra	Radium	[226]
89	Ac	Actinium	[227]
90	Th	Thorium	232.03806(2)
91	Pa	Protactinium	231.03588(2)
92	U	Uranium	238.02891(3)
93	Np	Neptunium	[237]
94	Pu	Plutonium	[244]
95	Am	Americium	[243]
96	Cm	Curium	[247]
97	Bk	Berkelium	[247]
98	Cf	Californium	[251]
99	Es	Einsteinium	[252]
100	Fm	Fermium	[257]
101	Md	Mendelevium	[258]
102	No	Nobelium	[259]
103	Lr	Lawrencium	[262]
104	Rf	Rutherfordium	[267]
105	Db	Dubnium	[268]
106	Sg	Seaborgium	[271]
107	Bh	Bohrium	[272]
108	Hs	Hassium	[270]
109	Mt	Meitnerium	[276]
110	Ds	Darmstadtium	[281]
111	Rg	Roentgenium	[280]
112	Cp	Copernicum	[285]
113	Uut	Ununtrium	[284]
114	Uuq	Ununquadium	[289]
115	Uup	Ununpentium	[288]
116	Uuh	Ununhexium	[293]
118	Uuo	Ununoctium	[294]

Table 1.10b Periodic table with standard atomic weights

G → P ↓	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	H 1.008																	He 4.003
2	Li 6.941	Be 9.012											B 10.81	C 12.01	N 14.01	O 16.00	Fe 19.00	Ne 20.18
3	Na 22.99	Mg 24.31											Al 26.98	Si 28.09	P 30.97	S 32.07	Cl 35.45	Ar 39.95
4	K 39.10	Ca 40.08	Sc 44.96	Ti 47.87	V 50.94	Cr 52.00	Mn 54.94	Fe 55.84	Co 58.93	Ni 58.69	Cu 63.55	Zn 65.39	Ga 69.72	Ge 72.61	As 74.92	Se 78.96	Br 79.90	Kr 83.80
5	Rb 85.74	Sr 87.62	Y 88.91	Zr 91.22	Nb 82.91	Mo 95.94	Tc [99]	Ru 101.1	Rh 102.9	Pd 106.4	Ag 107.9	Cd 112.4	In 114.8	Sn 118.7	Sb 121.8	Te 127.6	I 126.9	Xe 131.3
6	Cs 132.9	Ba 137,3	*	Hf 178.5	Ta 180.9	W 183.8	Re 186.2	Os 190.2	Ir 192.2	Pt 195.1	Au 197.0	Hg 200.6	Tl 204.4	Pb 207.2	Bi 209.0	Po [209]	At [210]	Rn [222]
7	Fr [223]	Ra [224]	**	Rf [263]	Db [262]	Sg [266]	Bh [264]	Hs [269]	Mt [268]	Ds [272]	Rg [272]	Uub [277]	Uut [284]	Uuq [289]	Uup [288]	Uuh [292]	Uus [291]	Uuo [293]

*Lanthanides	La 138.9	Ce 140.1	Pr 140.9	Nd 144.2	Pm [145]	Sm 150.4	Eu 152.0	Gd 157.3	Tb 158.9	Dy 162.5	Ho 164.9	Er 167.3	Tm 168.93	Yb 173.0	Lu 175.0
**Actinides	Ac [227]	Th 232.0	Pa 231.0	U 238.0	Np [237]	Pu [244]	Am [243]	Cm [247]	Bk [247]	Cf [251]	Es [252]	Fm [257]	Md [258]	No [259]	Lr [262]

P = period, G = group

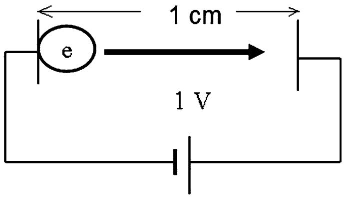

Fig. 1.9 Definition electronvolt

Table 1.11 Multiples for electronvolt$/c^2$

Symbol	Value
1 EeV/c^2	1.783×10^{-18} kg
1 PeV/c^2	1.783×10^{-21} kg
1 TeV/c^2	1.783×10^{-24} kg
1 GeV/c^2	1.783×10^{-27} kg
1 MeV/c^2	1.783×10^{-30} kg
1 keV/c^2	1.783×10^{-33} kg
1 eV/c^2	1.783×10^{-36} kg

Table 1.12 Mass of some objects in rest

Object	Mass	Mass kg
Photon	0	0
Electron	510 998.9 eV c^{-2} = 5.485 799 110(12) $\times 10^{-4}$ u	91.09 381 88(72) $\times 10^{-30}$
Quark Up	1.5–4.0 MeV c^{-2}	2.7–7 $\times 10^{-30}$
Quark Down	4–8 MeV c^{-2}	7–14 $\times 10^{-30}$
Quark Strange	80–130 MeV c^{-2}	0.14–0.23 $\times 10^{-27}$
Quark Charm	1150–1350 MeV c^{-2}	2–2.41 $\times 10^{-27}$
Quark Bottom	4100–4400 MeV c^{-2}	7.3–7.8 $\times 10^{-27}$
Quark Top	170900 $\pm$ 1800 MeV c^{-2}	134 $\times 10^{-27}$
Higgs particle	126 GeV c^{-2}	225 $\times 10^{-27}$
Proton	1.007 276 466 88(13) u	1.672 621 58(13) $\times 10^{-27}$
Mist particle	$\sim$0.01–0.3 g	$\sim$0.01–0.3 $\times 10^{-3}$
Rain droplet	$\sim$0.05 g	$\sim$0.05 $\times 10^{-3}$
1 l Water at 4 °C		1
Water on Earth		1.384 $\times 10^{21}$ kg

Object	Mass kg
Sun	1.989×10^{30}
Moon	7.349×10^{22}
Mercury	3.302×10^{23}
Venus	4.869×10^{24}
Earth	5.9736×10^{24}
Mars	6.419×10^{23}
Jupiter	1.899×10^{27}
Saturn	5.685×10^{26}
Uranus	8.683×10^{25}
Neptun	1.0243×10^{26}
Neutron star	2.7–4.2 $\times 10^{30}$
Black holes	$\sim 5 \times 10^{30}$–10^{40}
Galaxy Milky Way	3.6×10^{41}
Universe (visible)	10^{52}–10^{54}

Nearly 99.9 % of the mass of the solar system is concentrated in the Sun. The solar mass $M_{Sun} = 1.989 \times 10^{30}$ kg is used as an astronomical unit. Occasionally also the Earth mass is used as a reference, $M_{Sun} = 332\,946\ M_{Earth}$. Whereas the mass of the Sun decreases almost imperceptibly due to radiation, the mass of the Earth increases slightly because of the cosmic debris (dust, meteorites) 10^6–10^8 kg/day

1.13.5 Electronvolt

One eV is a very small amount of energy widely used in solid state, nuclear and particle physics. For large energies, the million electronvolt (MeV) is used. One electronvolt is based on a notional experiment as the amount of energy that could be gained by a single unbound electron when it would be accelerated through an electrostatic potential difference of one volt, in vacuo (Fig. 1.9, Table 1.11) [124]. It is equal to one volt times the (unsigned) charge of a single electron $C_{\text{el.}}$:

$$1\ \text{eV} = 1\ \text{J}\,\text{C}^{-1} \times C_{\text{el.}} = 1.602\,176\,53(14) \times 10^{-19}\ \text{J} \approx 0.160\ \text{aJ} \tag{1.26}$$

The unit electronvolt is accepted (but not encouraged) for use with SI (Table 1.4). A single atom is such a small entity that to talk about its energy in joules would be inconvenient. But instead of taking the appropriate SI unit attojoule physicists (and chemists) have unfortunately chosen, arbitrarily, a non-conformist unit called an electronvolt (eV). In mitigation, one can acknowledge the convenience of this unit when it comes to experimental measurement of ionisation energies of elements, an important parameter for chemists. The experimentally measurable parameter is the potential difference (measured in volts) required to dislodge the electron from the atom, leading to the erroneous description of the energy as the 'ionisation potential'. Whereas ionisation energies in eV are convenient numerical quantities, the energies for dislodging electrons from single atoms are very small. This can be overcome if ionisation energies are expressed in kJ mole^{-1}, the recommended unit.

According to Einstein's relation [13] mass can converted into energy (Eq. (1.13)) and

$$E/m = c^2 = (299\,792\,458\ \text{m/s})^2 = 89.875 \times 10^{15}\ \text{J/kg} = 89.875\ \text{PJ/kg} \tag{1.27}$$

So one gram of mass is equivalent to the energy of 89.9 terajoules (TJ).

It is common in particle physics, where mass and energy are often interchanged, to use eV/c^2 as a unit of mass.

$$1\ \text{eV}/c^2 = 1.783 \times 10^{-36}\ \text{kg} \tag{1.28}$$

Obsolete: Bevatron (BeV) was used for 'billion-electron-volt; occasionally; it is equivalent to the GeV.

1.13.6 Summary

The mass as an inertial quantity of a body should be clearly distinguished from its weight, which is the product of mass and the acceleration due to gravity. Therefore, use of the terms wt%, atomic and molecular weight should be avoided and instead mass %, relative atomic mass, relative molecular mass or relative molar mass and formula mass should be used [125]. If electronvolt is used it would be appropriate to mention in addition the respective value in SI units because that would facilitate further assessments and calculations.

Table 1.13 Symbols used

Symbol	Explanation	SI unit
a	Acceleration	$\mathrm{m\,s^{-2}}$
d	Distance	m
E	Energy	$\mathrm{J = kg\,m^2\,s^{-2}}$
W	Weight = force due to the gravitational field.	N
F_I	Applied force	$\mathrm{N = kg\,m\,s^{-2}}$
g	Acceleration due to gravity	$\mathrm{m\,s^{-2}}$
g_n	Standard acceleration due to gravity = 9.80665	$\mathrm{m\,s^{-2}}$
G	Gravitational constant = $6.672\,59(85) \times 10^{-11}$	$\mathrm{kg^{-1}\,m^3\,s^{-2}}$
h	Planck constant = $6.626\,075\,5 \times 10^{-34}$	J s
m	Mass	kg
m_p	Mass of a planet	kg
m_S	Mass of the Sun or a star	kg
m_0	Mass of a body at rest	kg
N_A	Avogadro constant = $6.022\,141\,79 \times 10^{23}$	$\mathrm{mol^{-1}}$
p	Momentum	$\mathrm{kg\,m\,s^{-1}}$
t	Time	s
t_u	Time of a sidereal period	s
c_0	Speed of light in vacuum = 2.99792458×10^8	$\mathrm{m\,s^{-1}}$
v	Velocity	$\mathrm{m\,s^{-1}}$
λ	Wavelength	m
ν	Frequency	$\mathrm{s^{-1}}$

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