Preface

Second Edition

We begin with a friendly advice to our readers. *Please*, try an online map projection forum. Do *at least* study the exercises. *Remember:* No one expects a guitarist to learn to play by going to concerts in *Central Park* or by spending hours reading transcriptions of *Jimi Hendrix* solos. Guitarists practice! Guitarists play the guitar until their fingertips are calloused. Similarly, *map projectors* solve problems. Of course, if you do not know the prerequisites, you are not be able to understand the subject.

Of course, there is the whole *world of geo-data portals* like

- *BING MAPS, Microsoft Virtual Earth*,
- *GOOGLE Earth, GOOGLE Maps, GOOGLE OCEAN* (these maps refer to *AST (Lowest Astronomic Tide*) and *IHO* (*International Hydrographic Office*) represented by the Geodesist *P. VANICEK*),
- *NASA WORLD WIRQ*,
- *YELLOW MAPS*,
- *Open Street Map.*

and many others. *But*, in order to understand the *ART of Map Projections*, read our book!

Please, read: Zero Meridian Internationally Defined for the Planet Earth

Orientation of a *World Atlas* is gained by *Longitude/Latitude maps*. The *Zero Meridian* was neatly defined: The traditional Star Observatory in *London's suburb Greenwich* was originally chosen as the *Zero Meridian.* At the time of the *Greek Geographer Ptolomäus* described about 150AC as the longitude origin the *Canary Island FERRO*, nowadays known as *El Hierro*. At *13 October 1884* a global definition of the longitude origin at the *International Meridian Conference* at *Washington, DC* was agreed upon, in addition to the numbering system of degrees/minutes/seconds. In consequence, position information was divided in Westerly and Easterly of Greenwich Observatory. In principle, modern GPS satellites for Navigation follow this definition, but with slight changes described here.

A second important result at the *World International Meridian Conference* in *Washington, DC* was a unique time measuring system: the Midday Sun over the local observatory defined the *Greenwich Mean Time* (GMT). All worldwide clocks fitted to 24 different time zones depended on the GMT. Meanwhile, over the many years passed, location and time mark lost their importance. Responsible was the progress in positioning to the millimeter accuracy and in timing to the sub-millisecond. In 1948 the Time Reference Measure GMT was changed into "*Coordinated World Time*" (UTC), *Atomic Clocks* were *measuring precisely Time.*

In addition, the *Zero Meridian* was changed, not anymore relating to the *Greenwich Meridian.* Displacements caused by continental drifts, plate tectonics and tidal effects change spatial distances. In addition, the Earth figure has changed from the plane over the sphere to *the ellipsoid of reference*, for instance the *World Ellipsoid 2000.* Following *C.F. Gauss*, the topographic Earth figure was orthogonally projected to the *World Ellipsoid of Reference.* He designed also an ellipsoid projection in strips to the plane: his *Gauss-Kruger Map/ellipsoidal Universal Transversal Mercator Projection(UTM)*, manifest of the worldwide *Ellipsoid 1984 Reference System* (*WGS84*). C.F. Gauss, in addition, defined also the proper three-dimensional Reference Coordinate System "*ellipsoidal longitude, ellipsoidal latitude, ellipsoidal height*" as the orthogonal projection to the *Reference Ellipsoid*, the worldwide basis of the GPS reference system, *a million times used easily for GPS positioning*! Nowadays, the official *Zero Meridian* passes about 100 m East of a line marked as the historical *Star Observatory in Greenwich,* relating to the *best fitted* Reference Ellipsoid, for instance the *World Geodetic Reference System 2000/World Geodetic Datum 2000.*

What topics have been added

Chapter 22 introduces optimal map projections by the variational calculus, namely generating *harmonic maps.* We solve the *LAPLACE-BELTRAMI equations* on the *International Reference Ellipsoid*, the *characteristic boundary value problem* of an arc preserving mapping. Up to now, the only solutions harmonic maps exist for the *Reference Sphere.* An important technical tool is our *distortion energy analysis* of *TISSOT type*, extended by appendices.

Alternative structure for *map projections* is the subject of *Chap. 23.* Up to now, we did analyze the mapping of sphere-to-plane, ellipsoid-to-plane as well as the double projection ellipsoid-to-sphere-to plane. *Now*, we analyze map projections of the *torus* (pneu), the *hyperboloid* (cooling tower), the *paraboloid* (parabolic mirror), the *Onion surface* (church tower) as well as the minimum distance mapping of the *clothoid* (High-Speed Railways) taking advantage of FRENEL integral. The target is always *Project Surveying*, a subject of *Engineering Geodesy.*

Finally, Chap. 24 reviews the group, $C_{10}(3)$ in a three-dimensional Euclidean space, namely the *ten parameter conformal transformation* as a datum transform. Such a differential space leaves *angles and distance ratios equivariant.*

How complex the situation with various map projection really is can be seen by the *Double Helix* illustrating *DNA*. Such a structure of *DNA* has been invented by *Francis Crick* and *James D. Watson* in 1953. It revealed how *DNA* was the substance of the genes, containing *two polynucleotide strands* wounding around each other. Both of them got the *Nobel Prize* for their research result. Read "*Genes, Girls and Gamow*" by *J.D. Watson*, A.A. Knopf Publ., New York 2002.

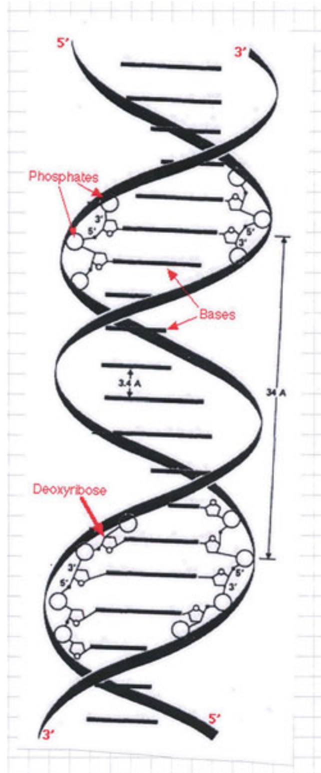

Fig. 1. Double Helix (DNA)

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First of all, we are grateful to *F. Krumm* (Stuttgart University, Germany) who produced most of the graphics. We appreciate the great help of *J. Engels* (Stuttgart, Germany) as well as *V. Schwarze* (Backnang, Germany) in designing the theoretical backgrounds.

We thank *Wolfgang Kühnel* (Stuttgart, Germany) who introduced us to the notion of a *minimal atlas* and, in particular, to the *Lusternik–Schnierelmann Category*. For example, a minimum atlas for a surface of type sphere or ellipsoid-of-revolution the *Lusternik–Schnierelmann Category* $CAT(\boldsymbol{S}^2)$or $CAT(\boldsymbol{E}^2)$ equal 2. Indeed, for a global coverage of the sphere or the ellipsoid we need at least two maps in order to avoid singularities. For instance, a Mercator as well as a transverse Mercator projection are necessary both to construct a minimal atlas for the sphere or the ellipsoid. But, for the case of the *torus* $CAT(TORUS) = 3$: *We need at least 3 maps to construct a minimum atlas* in order to avoid singularities.

An excellent source of information about cartographic coordinates and rotational elements, namely sizes and shapes of Planets, Satellites, Minor Planets and Comets, is the "*Report of the IAU/IAG Working Group on cartographic coordinates and rotational elements, 2006*," Special Report in "*Celestial Mechanics and Dynamic Astronomy 98 (2007) 155–180*" by *P. Kenneth* et al. as well as the "*Report of the IAU Working Group on Cartographic Coordinates and Rotational*

Elements, 2009," Special Report in "*Celestial Mechanics and Dynamic Astronomy 109 (2011) 101–135*" by *B.A. Archinal* et al. published by Springer Verlag. A definition of rotational elements for Planets and Satellites, the coordinate system for the Moon, the rotation elements for planets and satellites, the rotational elements of dwarf planets, minor planets, their satellites, and comets, a definition of cartographic coordinate systems for planets and satellites, cartographic coordinates for dwarf planets, minor planets, their satellites, and comets, and a list of recommendations with a full reference list of contributions are given.

We have to restrict here only on those map projections which are most commonly used, namely for the *Ellipsoid of Revolution* defining the World Geodetic Reference System. For instance, the World of Map Projections is much larger as here presented. Map projections of type *Robinson*—very popular in cartographic circles—are not treated here. Have a look into the large world of journals on Map Projections!

We appreciate the support of *Monika Sester* (Hannover, Germany) for her information about related works on the topic (a) thematic maps and map transformation by *W. Lichtner* (1979), (b) variable scale of maps by *L. Harrie, L.T. Sarjakosti* and *L. Lechto* (2002), (c) nonlinear magnification by *A. Keahey* (1997), (d) generalization and pedestrian navigation by *M. Sester* (2002), and (e) extraction and communication of way finding information by *M. Sester* and *B. Elias* (2006).

Enjoy the Wonderful World of Map Projections!

Erik W. Grafarend, Rey-Jer You, Reiner Syffus

Preface

This book is dedicated to the Memory
of US GS's J. P. Snyder (1926–1997),
genius of inventing new Map Projections.

Our review of *Map Projections* has 21 chapters and 10 appendices. Let us point out the most essential details in advance in the following passages.

Foundations.

The first four chapters are of purely introductory nature. Chapters 1 and 2 are concerned with general mappings from *Riemann manifolds to Riemann manifolds* and with general mappings from *Riemann manifolds to Euclidean manifolds* and present the important *eigenspace analysis* of types *Cauchy–Green* and *Euler–Lagrange*. Chapter 3 introduces coordinates or parameters of a Riemann manifold, *Killing vectors of symmetry*, and oblique frames of reference for the sphere and for the ellipsoid-of-revolution. A special topic is the classification of surfaces of zero *Gaussian curvature* for ruled surfaces and for developable surfaces in Chap. 4.

Next, we intend to follow the classical scheme of map projections. Consult the formal scheme above for a first impression.

The standard map projections: tangential, cylindric, conic.

Chapters 5–7 on mapping the *sphere to the tangential plane*, namely in the *polar aspect* (normal aspect)—for instance, the *Universal Polar Stereographic Projection* (UPS)—and the meta-azimuthal mapping in the *transverse* as well as the *oblique aspect*, follow. They range from equidistant mapping via conformal mapping to equal area mapping, finally to *normal perspective mappings*. Special cases are mappings of type "sphere to tangential plane" at maximal distance, at minimal distance, and at the equatorial plane (three cases). We treat the *line-of-sight*, the *line-of-contact*, and *minimal* versus *complete atlas*. The gnomonic projection, the orthographic projection, and the Lagrange projection follow. Finally, we ask the question: "what is the best projection in the class of polar and azimuthal projections of the sphere to the plane?" A special section on *pseudoazimuthal mappings*, namely the *Wiechel polar pseudoazimuthal mapping*, and

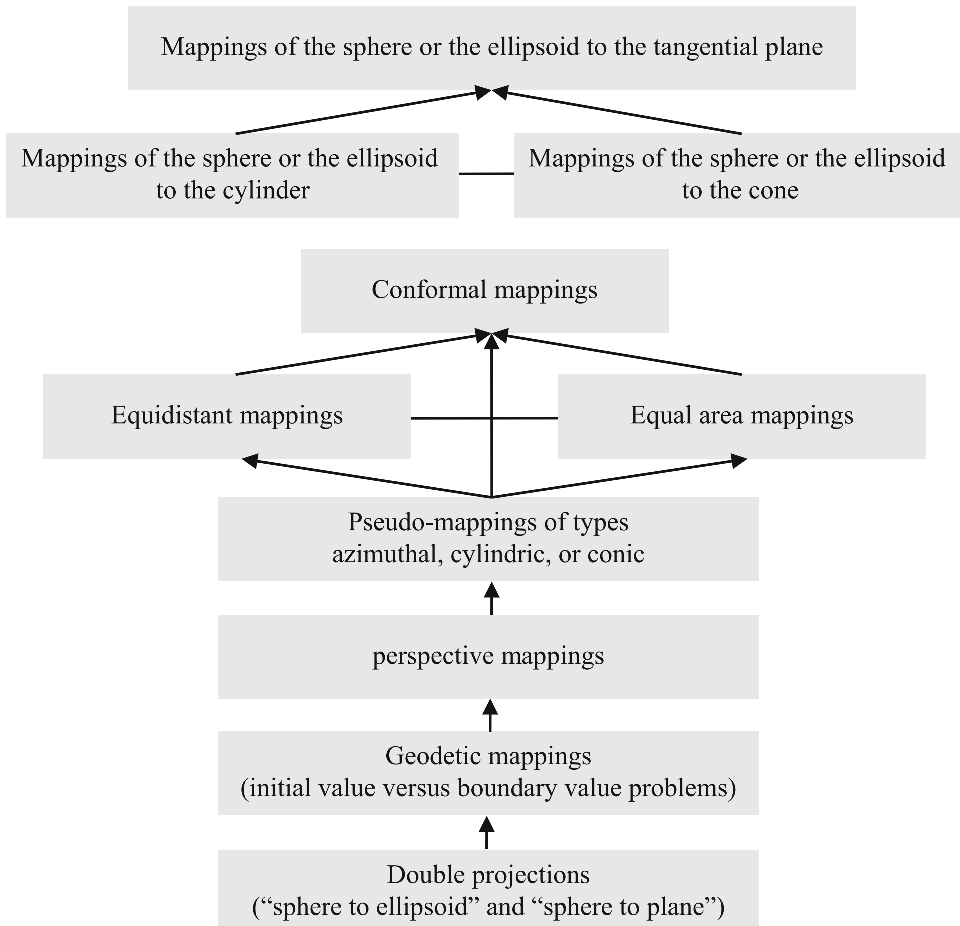

Fig. 1. The classical scheme of map projections

another special section on *meta-azimuthal projections* (stereographic, transverse Lambert, oblique UPS and oblique Lambert) concludes the important chapter on various maps “sphere to plane.”

►

Chapter 8 is the first chapter on mapping the *ellipsoid-of-revolution to the tangential plane*. We treat special mappings of type *equidistant, conformal*, and *equal area*, and of type *perspective*. Chapter 9 is the first chapter on *double projections*. First, we introduce the celebrated *Gauss double projection*. Alternatively, we introduce the *authalic equal area projection* of the ellipsoid to the sphere *and* from the sphere to the plane.

▶

Chapters 10–13 are devoted to the mapping "sphere to cylinder," namely to the polar aspect, to the meta-cylindric projections of type *transverse* and of type *oblique*, and finally to the *pseudo-cylindrical mode*. Four examples, namely from mapping the sphere to a cylinder (polar aspect, transversal aspect, oblique aspect, pseudo-cylindrical equal area projections) in Chaps. 10–13 document the power of these spherical projections. The resulting map projections are called (a) *Plate Carrée* (quadratische Plattkarte), (b) *Mercator projection* (Gerardus Mercator 1512–1594), and (c) *equal area Lambert projection*. A special feature of the Mercator projection is its property "mapping loxodromes (rhumblines, lines of constant azimuths) to a straight line crossing all meridians with a constant angle." The most popular map projection is the *Universal Transverse Mercator projection* (UTM) of the sphere to the cylinder, illustrated in Fig. 11.3. The pseudo-cylindrical equal area projections—they only exist—are widely used in the *sinusoidal* version (Cossin, Sanson–Flamsteed), in the *elliptic* version (Mollweide, very popular), in the *parabolic* version (Craster), and in the *rectilinear* version (Eckert II).

▶

In Chap. 10, a special section is devoted to the question "what is the best cylindric projection when *best* is measured by the *Airy optimal criterion* or by the *Airy–Kavrajski optimal criterion*?" We have compared three mappings: (a) conformal, (b) equal area, and (c) distance preserving in the class of "equidistance on two parallel circles." We prove that the distance preserving maps are optimal and the equal area maps are better than the conformal maps, at least until a latitude of $\Phi = 56°$, when we apply the Airy optimal criterion. Alternatively, when we measure optimality by the Airy–Kavrajski optimal criterion, we find again that the optimum is with the distance preserving maps, but conformal maps produce exactly the same equal area maps, less optimal compared to distance preserving maps.

▶

In contrast, Chaps. 14–16 are a review in mapping an *ellipsoid-of-revolution to a cylinder*. We start with the *polar aspect* of type $\{x = A\Lambda, y = f(\Phi)\}$, specialize to normal equidistant, normal conformal, and normal equiareal, in general, to a rotationally symmetric figure (for example, the torus). The *transverse aspect* is applied to the *transverse Mercator projection* and the special *Gauss–Krueger coordinates* (UTM, GK) derived from the celebrated *Korn–Lichtenstein equations* subject to an integrability condition and an optimality condition for estimating the *factor of conformality* (dilatation factor) in a given quantity range $[-l_{\mathrm{E}}, +l_{\mathrm{E}}] \times [B_{\mathrm{S}}, B_{\mathrm{N}}] = [-3.5°, +3.5°] \times [80°\mathrm{S}, 84°\mathrm{N}]$ or $[-l_{\mathrm{E}}, +l_{\mathrm{E}}] \times [B_{\mathrm{S}}, B_{\mathrm{N}}] = [-2°, +2°] \times [80°\mathrm{S}, 80°\mathrm{N}]$, namely $\rho = 0.999, 578$ or $\rho = 0.999, 864$. Due to its practical importance, we have added three examples for the transverse Mercator projection and for the Gauss–Krueger coordinate system of type {Easting, Northing}, adding the meridian zone number. Another special topic is the *strip transformation* from one meridian strip system to another one, both for Gauss–Krueger coordinates and for UTM coordinates. We conclude with two detailed examples of strip transformation (Bessel ellipsoid, World Geodetic System 84). At the end, we present to you the *oblique aspect* of type *Oblique Mercator Projection* (UOM) of the ellipsoid-of-revolution, also called *rectified skew orthomorphic* by M. Hotine. J.P. Snyder calls it "Hotine Oblique Mercator Projection (HOM)." Landsat-type data are a satellite example.

Only in the *polar aspect*, we present in Chap. 17 the maps of the *sphere to the cone*. We use Fig. 17.1 as an illustration and the setup $\{a = \Lambda \sin \Phi_0, r = f(\Phi)\}$ in terms of polar coordinates. $n := \sin \Phi_0$ range from $n = 0$ for the cylinder to $n = 1$ for the azimuthal mapping. Thus, we are left with the rule $0 < n < 1$ for conic projections. The wide variety of conic projections were already known to Ptolemy as the equidistant and conformal version on the *circle-of-contact*. If we want a *point-like image* of the North Pole, the equidistant and conformal version on the circle-of-contact is our favorite. Another equidistant and conformal version on two parallels is the *de L'Isle mapping*. Various versions of conformal mapping range from the equidistant mappings on the circle-of-contact to the equidistant mappings on two parallels (*secant cone*, J.H. Lambert). The equal area mappings range from the case of an equidistant and conformal mapping on the circle-of-contact over the case of an equidistant and conformal mapping on the circle-of-contact and a point-like image of the North Pole to the case of equidistance and conformality on two parallels (*secant cone*, H.C. Albers).

Chapter 18 is an introduction into mapping the *sphere to the cone*, namely of type *pseudo-conic*. We specialize on the *Stab–Werner projection* and on the *Bonne projection*. Both types have the shape of the heart.

▶

The polar aspect of mapping the *ellipsoid-of-revolution to the cone* is the key topic of Chap. 19. We review the line-of-contact and the principal stretches before we enter into special cases, namely of type *equidistant mappings* on the set of parallel circles of type *conformal* (variant equidistant on the circle-of-reference, variant equidistant on two parallel circles, *generalized Lambert conic projection*) and type *equal area* (variant equidistant and conformal on the reference circle, variant pointwise mapping of the central point and equidistant and conformal on the parallel circle, variant of an equidistant and conformal mapping on two parallel circles, *generalized Albers conic projection*).

▶

Geodesics and *geodetic mappings*, in particular, the *geodesic circle*, the *Darboux frame*, and the *Riemann polar and normal coordinates* are the topic of Chap. 20. We illustrate the *Lagrange* and the *Hamilton portrait* of a geodesic, introduce the *Legendre series*, the corresponding *Hamilton equations*, the notion of initial and boundary value problems, the Riemann polar and normal coordinates, *Lie series*, and specialize to the *Clairaut constant* and to the ellipsoid-of-revolution. Geodetic parallel coordinates refer to *Soldner coordinates*. Finally, we refer to *Fermi coordinates*. The deformation analysis of Riemann, Soldner, and Gauss–Krueger coordinates is presented.

Datum problems.

Datum problems, namely its analysis versus synthesis and its Cartesian approach versus curvilinear approach, are presented in Chap. 21. Examples reach from the transformation of conformal coordinates of type *Gauss–Krueger* and type *UTM* from a local datum (regional, national, European) to a global datum (WGS 84) of type *UM* (Universal Mercator).

Appendices.

Appendix A is entitled “Law and order.” It brings up *relation preserving maps*. We refer to Venn diagrams, Euler circles, power sets, Hesse diagrams, finally to *fibering*. The inversion of univariate, bivariate, in general, multivariate homogeneous polynomials is presented in Appendix B. In contrast, Appendix C reviews elliptic functions and elliptic integrals. Conformal mappings are the key subject of Appendix D. First, we treat the classical *Korn–Lichtenstein equations*. Second, we treat the celebrated *d'Alembert–Euler equations* (usually called *Cauchy–Riemann equations*) which generate conformal mapping both (a) on the basis of real algebra and (b) on the basis of complex algebra. Lemma D.1 gives three alternative formulations of the Korn–Lichtenstein equations. The fundamental solutions of the d'Alembert–Euler equations subject to the *harmonicity condition* are reviewed in Lemma D.2 in terms of a *polynomial representation* (D.15)–(D.29). An alternative solution in terms of *matrix notation* based upon the *Kronecker–Zehfuss product* is provided by (D.30) and (D.31). Lemmas D.3 and D.4 review two solutions of the d'Alembert–Euler equations subject to the *integrability conditions* of harmonicity, by separation of variables this time. Two choices of solving the basic equations of the transverse Mercator projection are presented: $x = x(q, p)$, $y = y(q, p)$. We especially estimate (a) the boundary condition for the universal transverse Mercator projection modulo an unknown dilatation factor and (b) we solve the already formulated boundary value problem with respect to the d'Alembert–Euler equations (Cauchy–Riemann equations). Finally, the unknown dilatation factor is optimally determined by optimizing the total distance distortion measure (Airy optimum) or the total areal distortion. Appendix E introduces the extrinsic terms *geodetic curvature, geodetic torsion*, and *normal curvature*, the notion of a geodesic circle, especially the *Newton form* of a geodesic in *Maupertuis gauge* on the sphere and on the ellipsoid-of-revolution. Mixed cylindrical maps of the ellipsoid-of-revolution of type *equiareal* based upon the *Lambert projection* and the sinusoidal *Sanson–Flamsteed projection*, especially as the horizontal weighted mean versus the vertical weighted mean, are the central topics of Appendix F. The *generalized Mollweide projection* and the *generalized Hammer projection* (generalized for the ellipsoid-of-revolution) are the key topics, especially of our studies in Appendices G and H. The *optimal* Mercator projection and the *optimal* polycylindric projection of type *conformal*, here developed on the ellipsoid-of-revolution, are applied to the many islands of the Indonesian Archipellagos in Appendix I. *Projection heights* in the geometry space are the topic of Appendix J. We treat the plane, the sphere, the ellipsoid-of-revolution, and the triaxial ellipsoid, and we review the solution algorithm for inverting Cartesian coordinates to projection heights. An example is the *Buchberger algorithm*. In detail, we review surface normal coordinates, for example, in the computation of the triaxial ellipsoids of type *Earth, Moon, Mars, Phobos, Amalthea, Io*, and *Mimas*.

We here would like to emphasize that our introduction into Map Projections is exclusively based upon *right-handed coordinates*. In this orientation, we particularly got support from my German colleagues *J. Engels* (Stuttgart), *V. Schwarze* (Backnang), and *R. Syffus* (Munich). We here would like to note that the software manuscript was produced by *V. Weberruß* with expertise. To all our readers, we appreciate their care for the **Wonderful World of Map Projections**. We dedicate our work to *J.P. Snyder* (1926–1997), who worked for the US Geological Survey for a lifetime. We stay on the strong shoulders of great scientists, for example, C.F. Gauss, J.L. Lagrange, B. Riemann, E. Fermi, J.H. Lambert, and J.H. Soldner. May we remember their great works.

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