

Chapter 1

Quantum Ring: A Unique Playground for the Quantum-Mechanical Paradigm

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“The Ring is... not just a story... it’s a cosmos.”
(R. Lepage about the tetralogy *The Ring of the Nibelung*
of R. Wagner)
<http://wagnersdream.metoperafamily.org/robert-interview.html>

Abstract The physics of quantum rings is reviewed from basic concepts rooted in the quantum-mechanical paradigm—via unprecedented challenges brilliantly overcome by both theory and experiment—to promising application perspectives.

1.1 Prologue

Doubly-connected (ring-like) structures at the scale of nanometers (nanoscale) are generally termed *Quantum Rings* (QRs). They exhibit a unique density of states for charge carriers and quantum fields and hence a vast variety of physical properties, which are cardinally different from those of singly-connected structures (like quantum dots).

Circular electric currents prophetically introduced by Ampère [1, 2] to explain the origin of magnetism: “... un aimant doit être considéré comme un assemblage de courants électriques qui ont lieu dans des plans perpendiculaires à son axe...”¹ were an essential precursor of persistent currents in the modern physics of QRs. A magnetic field was related to the currents circulating along concentric paths: “... à chacun des pôles d’un aimant, les courants électriques dont il se compose sont dirigés suivant des courbes fermées concentriques...”² Quantum mechanics predicts that small enough ring-like structures threaded by a magnetic flux, in the

¹“... a magnet should be considered as an assembly of electric currents that occur in planes perpendicular to its axis...” (Translation by V. M. F.)

²“... at each of the poles of a magnet, the electrical currents, of which it consists, are directed along concentric closed curves...” (Translation by V. M. F.)

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equilibrium state, carry *persistent* (dissipationless) circulating electron currents that do not require an external power source. A prerequisite is that the electron state keeps quantum coherence over the whole doubly-connected system.

There have been a number of reviews representing various aspects of physics of QRs, for example, effects of a finite width of the QRs [3], mesoscopic phenomena in QRs with strongly coupled polarons [4], possible types of III–V semiconductor QRs [5], equilibrium properties of mesoscopic metal rings [6], ring-like nanostructures as a leitmotif in plasmonics and nanophotonics [7], theoretical modeling of the self-organized QRs on the basis of the modern characterization of those nanostructures [8], theoretical analysis and experimental observations of persistent currents by virtue of the magnetic flux quantization phenomenon [9], and advancements in experimental and theoretical physics of QRs [10]. In the present Chapter, we discuss a number of contributions to the physics of QRs, essential for the topics of the present book—(i) fundamentals of physics of QRs and (ii) semiconductor QRs—without any claim for an exhaustive presentation of the extensive literature in this vigorously developing field.

1.2 At Dawn

The following studies, commenced already at the very early stage of the quantum physics, unraveled the key properties of persistent currents in ring-like quantum structures.

For calculating the magnetically induced current densities of aromatic hydrocarbon ring molecules, Pauling [11] advanced a hypothesis that the external electrons in the benzene molecule can *circulate freely* and provide a very large contribution to the diamagnetic susceptibility with the magnetic field normal to the plane of the carbon hexagon: “We may well expect that in these regions the potential function representing the interaction of an electron with the nuclei and other electrons in the molecule would be approximately cylindrically symmetrical with respect to the hexagonal axis of the molecule, the electron, some distance above or below the plane of the nuclei, passing almost imperceptibly from the field of one carbon atom to that of the next.”

Within the framework of a quantum-mechanical derivation, London [12] demonstrated that the diamagnetic susceptibility of aromatic ring molecules was related to a *current circulating around the opening* induced by the magnetic field: “La susceptibilité... correspond à des courants induits qui circulent d’un atome à l’autre autour de la chaîne cyclique.”³ This current belonged to the *ground state*, in analogue with superconducting currents: “Nous pouvons... disant que les combinaisons aromatiques se comportent comme des supraconducteurs.”⁴

³“The susceptibility... corresponds to the induced currents that flow from one atom to another around the cyclic chain.” (Translation by V.M.F.)

⁴“We can... say that the aromatic combinations behave as superconductors.” (Translation by V.M.F.)

Calculating the magnetic response of ultrasmall magnetic ring-shaped particles on the basis of the Schrödinger equation, Hund [13] showed that both at zero temperature and in thermodynamic equilibrium at temperature $T > 0$, there existed a *total current circulating around the annulus*, which was *dissipationless*: “...ein wesentlicher Teil des der diamagnetischen Magnetisierung entsprechenden Stromes um das Loch herumfließt; dieser Strom hat keine Joulesche Wärme, da die Besetzung der Zustände dem Temperaturgleichgewicht entspricht.”⁵ Further, it was demonstrated that a set of eigenstates found for an electron in a ring in a magnetic field B led to jumps in the magnetization from negative (diamagnetic) to positive (paramagnetic) at certain values of the applied magnetic field: “...es tritt zu der negativen (diamagnetischen) Magnetisierung plötzlich eine konstante positive Magnetisierung hinzu, und dies wiederholt sich nach einem gewissen Zuwachs von B .”⁶ As a result, the current circulating around the opening of the ring acquires a *zigzag form* as a function of the applied magnetic field (shown in Fig. 4 of [13]).

Systematically developing the earlier ideas, Dingle found [14] that the equilibrium properties calculated for small *free-electron systems* in a perfect ring and in a perfect infinite cylinder were sensitive to the *magnetic flux* Φ threading the system, the magnetic permittivity consisting of a steady part and periodic in the magnetic flux terms. The fundamental dimensionless quantity, which determined the *periodic* dependence, was (in the modern notation) the ratio Φ/Φ_0 , where the magnetic flux quantum $\Phi_0 = h/e$ was determined by universal constants: the Planck constant h and the elementary charge e . Dingle already noticed the challenges in observing those periodic terms: “...a single cylinder would possess only a very small magnetic moment, whilst it would be difficult to ensure a uniform radius for a bundle of cylinders”—a conclusion that has remained very urgent for experimentalists ever since then. The key challenges in detecting persistent currents experimentally are twofold: they produce exceptionally small signals and they are very sensitive to the environment [15].

1.3 Fundamentals of Topological Effects

A fundamental role of the ring-topology for the quantum-mechanical paradigm was unraveled within the theory of a geometric phase [16–19]. Berry [19] provided a simple but intuitively appealing derivation of the geometric (Berry) phase, which will be recalled below. A system is considered whose Hamiltonian H depends on a set of varying parameters $\mathbf{R} \equiv \mathbf{R}(t)$ forming a closed path C between the instant $t = 0$ and the instant $t = T$ such that $\mathbf{R}(0) = \mathbf{R}(T)$.

⁵“...an essential part of the current corresponding to the diamagnetic magnetization flows around the annulus; this current produces no Joule heat, as the population of states corresponds to the thermal equilibrium.” (Translation by V.M.F.)

⁶“...a constant positive magnetization occurs to be suddenly added to the negative (diamagnetic) magnetization, and this is repeated after a certain increase in B .” (Translation by V.M.F.)

The evolution of the state of the system is governed by the Schrödinger equation:

$$i\hbar|\dot{\psi}(t)\rangle = H(\mathbf{R}(t))|\psi(t)\rangle. \quad (1.1)$$

At any instant t , the eigenstates satisfy the stationary Schrödinger equation

$$H(\mathbf{R})|n(\mathbf{R})\rangle = E_n(\mathbf{R})|n(\mathbf{R})\rangle, \quad (1.2)$$

where $|n(\mathbf{R})\rangle$ is single-valued in the region that includes C . Within the *adiabatic* approximation [20], the system prepared in one of these states $|n(\mathbf{R}(0))\rangle$ will evolve with the Hamiltonian $H(\mathbf{R}(t))$ and be in the state $|n(\mathbf{R}(t))\rangle$ at the instant t . A gauge-invariant generalization to the phase-coherence phenomena in *nonadiabatically* evolving quantum systems was proposed by Aharonov and Anandan [21]. The solution to the Schrödinger equation (1.1) is sought in the form

$$|\psi(t)\rangle = \exp\left[-\frac{i}{\hbar} \int_0^t d\tau E_n(\mathbf{R}(\tau))\right] \exp[i\gamma_n(t)] |n(\mathbf{R}(t))\rangle. \quad (1.3)$$

Substituting (1.3) into the Schrödinger equation (1.1) and taking into account (1.2), we find the equation for the *geometric phase* $\gamma_n(t)$:

$$\dot{\gamma}_n(t) = i\langle n(\mathbf{R}(t)) | \nabla_{\mathbf{R}} n(\mathbf{R}(t)) \rangle \cdot \dot{\mathbf{R}}(t).$$

(A gauge can be chosen so that the Aharonov-Bohm phase is included in the dynamical phase instead of the geometric phase, see, e.g., [22, 23].) The total phase change of the state of (1.3) on the path C

$$|\psi(T)\rangle = \exp\left[-\frac{i}{\hbar} \int_0^T d\tau E_n(\mathbf{R}(\tau))\right] \exp[i\gamma_n(C)] |\psi(0)\rangle \quad (1.4)$$

is then determined by the *geometric phase change*

$$\gamma_n(C) = i \oint_C \langle n(\mathbf{R}) | \nabla_{\mathbf{R}} n(\mathbf{R}) \rangle \cdot d\mathbf{R}. \quad (1.5)$$

A generalization of the phase factor $i\gamma_n(C)$ in (1.4) (which was initially derived for a non-degenerate Hamiltonian) to the Hamiltonian with degenerate energy levels, was provided in terms of the path-ordered integrals involving non-Abelian gauge fields [24].

Magnetic field is an important tool revealing physical effects due to the doubly-connected topology. Consider a magnetic flux line [25] (or tube) carrying a flux Φ . For positions \mathbf{R} outside of the flux line (tube), the magnetic field is zero, but there exists a set of gauge-equivalent vector potentials $\mathbf{A}(\mathbf{R})$ such that for any closed path C threaded by the magnetic flux line (tube)

$$\oint_C \mathbf{A}(\mathbf{R}) \cdot d\mathbf{R} = \Phi.$$

Further, let a particle carrying a charge q be confined to a box at \mathbf{R} , which is not penetrated by the flux line (tube). Without a flux, the Hamiltonian of the particle $H(\mathbf{p}, \mathbf{r} - \mathbf{R})$ depends on the momentum \mathbf{p} and the relative position $\mathbf{r} - \mathbf{R}$ and possesses the eigenfunctions $\psi_n(\mathbf{r} - \mathbf{R})$ that satisfy (1.2) with eigenenergies independent of \mathbf{R} . With non-zero flux, the states $|n(\mathbf{R})\rangle$ satisfy

$$H(\mathbf{p} - q\mathbf{A}(\mathbf{r}), \mathbf{r} - \mathbf{R})|n(\mathbf{R})\rangle = E_n|n(\mathbf{R})\rangle \quad (1.6)$$

with the eigenenergies unaffected by the vector potential. Solutions of (1.6) are obtained in terms of the Dirac phase factor [26]

$$\langle \mathbf{r} | n(\mathbf{R}) \rangle = \exp \left[i \frac{q}{\hbar} \int_{\mathbf{R}}^{\mathbf{r}} d\boldsymbol{\rho} \cdot \mathbf{A}(\boldsymbol{\rho}) \right] \psi_n(\mathbf{r} - \mathbf{R}). \quad (1.7)$$

Within a thought experiment, the box is transported round a closed doubly-connected path C threaded by the flux line (tube). Any such path is topologically equivalent to a *ring*. The integrand in the geometric phase change of (1.5) is then

$$\begin{aligned} \langle n(\mathbf{R}) | \nabla_{\mathbf{R}} n(\mathbf{R}) \rangle &= \int d^3r \psi_n^*(\mathbf{r} - \mathbf{R}) \left[-i \frac{q}{\hbar} \mathbf{A}(\mathbf{R}) \psi(\mathbf{r} - \mathbf{R}) + \nabla_{\mathbf{R}} \psi_n(\mathbf{r} - \mathbf{R}) \right] \\ &= -i \frac{q}{\hbar} \mathbf{A}(\mathbf{R}). \end{aligned}$$

The integral of the second term in the integrand vanishes because of the wave function normalization. Consequently, the geometric phase change

$$\gamma_n(C) = \frac{q}{\hbar} \oint_C \mathbf{A}(\mathbf{R}) \cdot d\mathbf{R} = \frac{q\Phi}{\hbar} \quad (1.8)$$

is independent of n . Thus, a charged particle *gains a phase* as it moves over a closed path about the flux line (tube):

$$|\psi(\Phi)\rangle = \exp \left[i \frac{q\Phi}{\hbar} \right] |\psi(\Phi = 0)\rangle. \quad (1.9)$$

The geometric phase occurring in (1.9) leads to a *quantum interference* between the states of the particles in the transported box and those in a box that was not moved about the flux line (tube). There are numerous manifestations of this quantum interference, which is known as the Aharonov-Bohm effect [25, 27]. They are revealed in the electronic spectra, magnetization, optical and transport properties of QRs and, in particular, represented in the present book. Observation of the Aharonov-Bohm effect was significantly facilitated by nano-scale fabrication and low-temperature detection techniques, which minimize dephasing, as demonstrated in the beautiful experiment on dephasing in electron interference by a ‘which-path’ detector [28]. The phase acquired by a particle with nonzero spin can also follow from spin-orbit-coupling instead of a magnetic field (Aharonov-Casher effect) [29].

Oscillating persistent currents were extensively investigated in superconductor QRs, which are beyond the scope of the present book; see [30–32] for references.

1.4 Renaissance

In their works dealing with the flux quantization in *superconducting* rings, Byers and Yang [22] and Bloch [23] showed that “the magnetic flux through any surface whose boundary loop lies entirely in superconductors is quantized in units” $\Phi_0^{\text{sup}} = h/(2e)$, where $2e$ is the charge of a Cooper pair [22]. As a result, a general theorem follows: all physical properties of a doubly-connected system are periodic in the magnetic flux through the opening Φ with the period Φ_0^{sup} . The experimental detection of the periodicity of the magnetization as a function of magnetic flux (“magnetic flux quantization”) in superconducting rings [33] and cylinders [34] was used to demonstrate that charge in superconductors was carried in units of $2e$ [34].

Gunther and Imry [35] analyzed persistent currents in a hollow, cylindrically shaped superconductor taking into account that the magnetic flux consists of two parts: that due to the external magnetic field and that due to the current. The flux quantization was shown to be exhibited when the cylinder was thick enough as compared to the penetration depth of the superconductor and thin enough as compared to the temperature-dependent coherence length as to exclude the off-diagonal long-range order.

Kulik [36, 37] discussed the persistent currents and the flux quantization in a hollow thin-walled normal metallic cylinder and ring threaded by a tube of magnetic flux-lines that were confined within an inner cylinder (a magnetic coil) with a radius smaller than the radius of the outer cylinder, in which no electric or magnetic field was present.

In cylindrical bismuth single-crystal whiskers 200–800 nm thick, oscillations in the longitudinal magnetoresistance with the period $\Phi_0/\cos\theta$ (θ was the angle of the tilt of the magnetic field with respect to the cylinder axis) observed by Brandt et al. [38, 39] were interpreted as a possible manifestation of the Aharonov-Bohm effect.

Büttiker, Imry and Landauer [40] were the first to consider persistent currents in a strictly one-dimensional *normal-metal* ring with disorder. It was concluded that “Small and strictly one-dimensional rings of normal metal, driven by an external magnetic flux, act like superconducting rings with a Josephson junction, except that $2e$ is replaced by e .” These authors noticed a fundamental analogy between the energy spectrum of an electron traversing the ring and that of an electron in a periodic potential: it consisted of bands of width V with band gaps Δ . Such band states carried persistent currents. The heuristic value of the possibility to conduct a persistent-current calculation using the widely developed solid-state band-structure theory could be hardly overestimated. It was pointed out, that the band energy in a ring oscillated periodically as a function of the enclosed flux: $E_n(\Phi) = E_n(\Phi + \Phi_0)$ and carried the (*single-band*) *persistent current* $I_n = -dE_n(\Phi)/d\Phi$. For a geometrically perfect ring, the persistent currents carried by consecutive bands had opposite signs.

The key criteria for a possible observation of the persistent current are represented in Chap. 4 of [41]. Firstly, the electron level width (determined as \hbar/τ_ϕ through the inelastic scattering time τ_ϕ) must be much smaller than the typical values of the band gap Δ and the bandwidth V . The latter condition is equivalent

to the requirement that *phase coherence be maintained along the whole ring*, i.e., the phase-coherence length l is larger than the mean circumference L of the ring (ballistic regime). With increasing disorder, when the electron free path is smaller than the ring circumference (diffusive regime) the period of the Aharonov-Bohm effect becomes $\Phi_0/2$ [42]. Secondly, the *temperature must be low* enough: $k_B T \leq \Delta$. Otherwise, the sum of the persistent currents (with alternating signs) carried by the occupied levels would lead to a strong reduction of the overall persistent current.

The seminal work by Büttiker et al. [40] initiated a tremendous interest in the persistent current problem, starting with the papers on the resistance of small one-dimensional rings of normal metal driven by an external time-dependent magnetic flux [43] and the persistent currents and the absorption of power in the ring that was coupled via a single current lead to a dissipative electron reservoir [44].

The first evidence for persistent currents in mesoscopic rings was provided in the following three pioneering experiments. The persistent current in the *diffusive regime*, where the *elastic* (non-dephasing) *mean free path* l was much smaller than the mean circumference L of the ring, was measured in an ensemble of 10^7 copper rings [45] with a SQUID magnetometer and for a single (isolated) gold ring [46] using a highly sensitive thin-film miniature dc-SQUID magnetometer. In semiconductors, the persistent current was first detected for a lithographically prepared single GaAs ring in the *ballistic regime*, i.e., for $L < l$, [47] using a special technique, where the sample and the SQUID were made on the same chip. Further measurements of persistent currents were made on arrays of gold QRs [48, 49] and an ensemble of 10^5 disconnected silver rings [50].

The problem of matching theoretical predictions with the emerging experimental evidence stimulated the further intensive research aimed at a development of more realistic models of QRs, taking into account effects due to the finite size, disorder of different nature, and the electron-electron interaction.

For the metallic QRs (in the diffusive regime) the magnitudes of the persistent currents occurred much larger (by two orders of magnitude) than those predicted using the model of non-interacting electrons [51, 52], while for the semiconductor QRs in the ballistic regime this simple theory seemed to agree with experiment. This stimulated investigations (see [3, 41] for details) of the following issues: (i) the role of the choice of the statistical ensemble (canonical versus grand canonical) to calculate average values of persistent currents [53–55], (ii) the role of spin in producing the fractional Aharonov-Bohm effect [56–58], (iii) the role of the electron-electron interaction [59–61], (iv) the role of correlations due to the electron-electron interaction beyond the first-order perturbation approach [62–65].

Important conceptual ingredients to resolve the discrepancy between the measured and observed values of the magnitude of persistent currents in metallic QRs were (i) the argument of local charge neutrality in volume elements larger than the screening length [60, 66] and (ii) the fact that the effect of disorder may be strongly reduced by the electron-electron interaction [67–69]. Another interesting way to get agreement was based on the diamagnetic sign of the persistent currents observed in metal QRs, e.g., by [45], which suggested that the materials were weak superconductors [62] with a very low critical temperature [65]. Attractive electron-electron

interaction may enhance the magnetic response of a QR due to the contribution of high energy levels [62]. Resolving the contradiction between experiment and theory in what concerns the magnitude of persistent currents in metallic QRs has been recognized as a major open challenge in mesoscopic physics [49, 63, 64, 70].

A rigorous quantum-mechanical theory of persistent currents developed for QRs in the ballistic regime revealed that the coupling between the different channels of the electron motion caused the occurrence of higher harmonics of Φ_0 in the persistent current. In particular, the halving of the fundamental period of the persistent current may occur in a single finite-width [71] or finite-height [72] QR due to the coupling of the azimuthal and, correspondingly, radial or paraxial electron motions by virtue of the impurity scattering.

If the magnetic field penetrated the conducting region of the finite-width QR, the Aharonov-Bohm-type oscillations due to the magnetic flux threading the opening coexisted with the diamagnetic shift of energy levels due to the magnetic field in the QR and were aperiodic [73, 74].

The role of the electron-electron interaction in a finite-width QR for a sufficiently low density, at which the correlation energy is much larger than the Fermi energy, consisted in formation of an N -electron Wigner molecule with relative angular motions of the electrons in the form of harmonic oscillations and radial motions depending on the shape of the confining potential [75–77]. The results for highly correlated electrons were, generally speaking, distinct from those for free electrons, except for the case of low temperatures, when a high-symmetry equilibrium configuration of electrons occurred by virtue of the strong repulsion between them. A Wigner molecule determined the ring-specific rich spectra of absorption, photoluminescence (PL), and Raman scattering [78, 79], which were significantly distinct from those of free electrons in a QR. Study of electronic transitions in QRs caused by a high-frequency inhomogeneous piezoelectric field accompanying a surface acoustic wave unveiled another possibility to distinguish the Wigner-molecule-regime from that of the free electrons by virtue of a different mechanism of the electronic absorption: for free electrons the dipole matrix element was other than zero, while in the Wigner molecule the absorption occurred due to quadruple and higher multipolar transitions [80].

Effect of the spin-orbit interaction was shown to dramatically change persistent currents in QRs as a function of the magnetic flux as compared to the case without the spin-orbit coupling; in particular, it may suppress the first Fourier harmonic in the persistent current and thus simulate the $\Phi_0/2$ -periodicity [81].

The presence of magnetic impurities in a QR may induce bistability of the persistent current of two interacting electrons and a hysteresis in its dependence on the magnetic flux [82].

Interesting effects were unveiled in systems, where quantum rings were coupled to quantum dots. For a mesoscopic ring with a quantum dot inserted in one of its arms, it was shown that the phase of Aharonov-Bohm oscillations was not related to the dot charge alone but instead to the total charge of the system [83]. In the presence of the Aharonov-Bohm flux, a charge response of a mesoscopic ring coupled to a side-branch quantum dot revealed a sequence of plateaus of diamagnetic and

paramagnetic states, while a mesoscopic ring containing an embedded quantum dot with leads exhibited a number of sharp peaks in the persistent current depending on the parity of the total number of electrons in the system [84].

Emergence of novel materials, e.g., carbon nanotubes, provided a new playground for observation and investigation of the Aharonov-Bohm effect [85].

1.5 Florescence

As the cornerstone of high-tech industry of the twenty-first century, nanostructures [86] are known as the cradle of new fabrication technologies [87], new characterization instruments [88, 89], and new theoretical insights [41].

A remarkable breakthrough in the physics of QRs was related to the discovery of the self-organized formation of QRs of a few tens of nm in diameter in 1997 by García, Medeiros-Ribeiro, Schmidt, Ngo, Feng, Lorke, Kotthaus and Petroff for the InAs/GaAs system [90]. It opened unprecedented perspectives to fabricate, characterize and investigate large arrays of semiconductor QRs as well as to control their size and shape.

1.5.1 Self-assembly Through Partial Overgrowth

It was demonstrated that by using a partial capping process the shape and size of InAs self-assembled quantum dots grown by Molecular Beam Epitaxy (MBE) may be modified in a way that led to the fabrication of self-assembled QRs [90]. The fabrication process was monitored using the *in situ* Reflection High-Energy Electron Diffraction (RHEED) technique [91], cross-section Transmission Electron Microscopy (TEM) [90, 92] and Atomic Force Microscopy (AFM) [90, 93, 94] measurements.

Two mechanisms were revealed, which mainly contributed to the self-assembled formation of QRs via partial overgrowth technique. One of them was *kinetic diffusion*: the In atoms, due to their higher diffusion mobility at the interface as compared to the diffusion mobility of the Ga atoms, could diffuse out of the partially capped quantum dots outwards onto the surface of the surrounding GaAs forming a ring-shaped InGaAs island [95]. Another mechanism was based on the thermodynamically driven *dewetting*: the imbalance of surface and interface forces acting upon the partially capped islands InAs/GaP [93]. Formation of liquid In droplets on the top of the InAs quantum dots under partial capping due to the *stress-induced melting* effect was established experimentally [91] and theoretically [96].

Unlike mesoscopic QRs defined lithographically, the self-assembled QRs, embedded in a GaAs matrix, could function in the quantum limit, *free of decoherence*, owing to scattering processes [97, 98]. The energy spectra of self-assembled QRs were thoroughly studied through their peculiar optical properties using PL [94, 99],

Time-Resolved PL and PL Excitation [100], as well as Photoemission Microscopy [101] in both single QRs and QR-arrays.

Being embedded in a heterostructure, the QRs can be electrically tuned by an electric field, and carriers can be injected with single-electron/single-hole precision. The detailed energy structure of electrons (holes) in QRs was obtained using the following three spectroscopic techniques. The PL (optical emission) of a single QR changed as electrons were added one-by-one. The emission energy changed abruptly whenever an electron was added, the sizes of the jumps revealing a shell structure [98]. Capacitance-voltage measurements allowed for probing the single-particle and many-particle ground states as a function of the applied electric field. Far-infrared absorption spectra demonstrated the effect of flux quantization on the intraband transitions.

1.5.2 Characterization

Cross-Sectional Scanning Tunneling Microscopy (X-STM) and Scanning-Gate Microscopy (SGM) belong to the advanced characterization methods that can access the intimate behavior of buried electronic systems and have been successfully exploited to get insight into the geometric structure of QRs.

X-STM of self-assembled InGaAs/GaAs QRs revealed the remaining quantum dot material whereas the Atomic Force Microscopy (AFM) represented the erupted QD material [102]. Based on this structural information from the X-STM measurements, a model of a self-assembled QR as a *singly-connected* “quantum volcano” (with a strong dip rather than opening in the center) was substantiated [102–104]. The electron magnetization was calculated as a function of the applied magnetic field for single-electron [74, 103] and two-electron [105] QRs. Quite surprisingly, even though those nanostructures were *singly-connected* and *anisotropic*, they exhibited the Aharonov-Bohm behavior, which was generally considered to be restricted to *doubly-connected* topologies. This was due to the fact that the electron wave functions in a “quantum volcano” were decaying towards the center so rapidly (exponentially) that they were *topologically identical* to the electron wave functions in doubly-connected QRs. The theory allowed for a quantitative explanation of the Aharonov-Bohm oscillations in the magnetization observed using the torsion magnetometry on those ring-like structures [106]. For measurements of the persistent currents in metal QRs, a micromechanical detector based on cantilever torsion magnetometry was proved to provide orders of magnitude greater sensitivity than SQUID-based detectors [15].

The Aharonov-Bohm oscillations of conductance in a mesoscopic ring defined by dry etching in a two-dimensional electron gas below the surface of an $\text{Al}_x\text{Ga}_{1-x}\text{As}/\text{GaAs}$ heterostructure and interrupted by two tunnel barriers were modified by a perpendicular magnetic field and a bias voltage [107]. As a result, the nonequilibrium electron dephasing time was found to be significantly shortened at high voltages and magnetic fields.

Studies of the lithographically patterned InGaAs-based QRs by means of SGM provided unique imaging of Aharonov-Bohm interferences in real space and the electronic local density-of-states at low magnetic fields [108] and Coulomb islands in the quantum Hall regime at high magnetic fields and very low temperatures [109]. This allowed for unveiling the spatial structure of transport inside a quantum Hall interferometer and, subsequently, for deciphering the high-magnetic field magnetoresistance oscillations. Scanning-probe technique unraveled, also, a counter-intuitive behavior of a two-path network patterned from a GaInAs heterojunction in the form of a rectangular QR-structure connected to a source and a drain via two openings [110]. The antidot in the initial rectangular QR-structure could then be bypassed by a third path for the electrons. Partially blocking the electron transport through this additional branch by using SGM resulted in an *increased* current through the whole device. This counter-intuitive effect was interpreted as a mesoscopic analog of the Braess paradox known for classical networks.

1.5.3 Various Materials Systems

The self-assembly was proved to be an efficient method of QR formation also in diverse materials systems, for instance, InAs/InP [111], Ge/Si [112] and GaSb/GaAs [113]. Capping the InAs or InGaAs quantum dots by a GaAs/AlAs layer before annealing allowed for impeding the inward diffusion of the Ga and Al atoms and resulted in nicely shaped self-assembled QR-structures [114].

In contrast to the InGaAs/GaAs materials system, where capping of quantum dots followed by a growth interruption was necessary to initiate the quantum-dot to QR transformation, GaSb QRs occurred just after the deposition of GaSb on GaAs(001). Ring-shaped GaSb/GaAs quantum dots, grown by MBE, were characterized using X-STM [115]. These QRs, as distinct from the self-assembled InGaAs/GaAs QRs, possessed a *clear central opening* extending over about 40 % of the outer base length and were therefore truly doubly-connected objects.

A distinct series of quantized modes in the vortex state observed in the spin excitations of ferromagnetic rings at the micrometer scale, fabricated using electron beam lithography, was attributed to spin waves that circulate around the ring and interfere constructively [116]. This is a representative example of the vigorously developing spin-wave physics in devices with topologically nontrivial magnetization profile.

1.5.4 Droplet Epitaxy and Lithography

Besides the above-described partial overgrowth technique within the Stranski-Krastanov growth mode, another technique of the QRs fabrication was developed, which allowed for preparation of strain free GaAs/AlGaAs QRs and QR-complexes—droplet epitaxy [117–119]. This method started with formation of

group-III liquid-metal droplets within a Volmer-Weber growth mode on the substrate surface by supplying their pure molecular beam. The nanostructures were subsequently formed by being exposed to a group-V element (As, Sb, P). The temporal evolution of these nanometer-scale objects was tracked in situ during the growth process using the RHEED technique [120].

Droplet epitaxy has demonstrated a unique ability to assemble QR nanostructures of complex morphologies ranging from single QRs [121], concentric double QRs [118, 122] and double-QR complexes [123] to concentric higher-order multiple QRs [124] and coupled QR/disks [125]. In GaAs double QRs formed by the droplet epitaxy, the size and height of the QRs were shown to depend on the supplied As flux. At a low As flux, larger and flatter rings were obtained. The formation of outer and inner rings was attributed to crystallization of out-diffused Ga and nanodrilling of Ga on the GaAs surface, correspondingly [118].

There has been a continuing insightful analysis of lithographically determined QRs. Magnetotransport experiments in the Coulomb blockade regime [126] and magnetoresistance measurements [127] on closed rings, fabricated with AFM oxidation lithography, confirmed that a microscopic understanding of energy levels of band charge carriers in QRs with the spin-orbit interaction could be extended to a many-electron system.

1.5.5 Novel Manifestations of the Aharonov-Bohm Effect

New conditions for manifestation of the Aharonov-Bohm effect through *neutral composite entities* consisting of charged particles in QRs were actively sought for starting with the seminal paper by Chaplik [81]. For an exciton in a one-dimensional QR placed in a perpendicular magnetic field, he found a Φ_0 -periodic dependence of the exciton binding energy on the magnetic flux. In [128] the same result for the exciton ground-state energy was obtained within another analytical approach.

Extending the Berry's analysis of the phase evolution for a charge carrier, given in (1.9), consider the case when a particle (exciton) composed of an electron ($q = -e$) and a hole ($q = e$) is confined to a box that is transported around an opening of a doubly-connected system threaded by a magnetic flux line (tube). The wave function of the exciton

$$|\Psi(\Phi_h, \Phi_e)\rangle = \exp\left[i\frac{e(\Phi_h - \Phi_e)}{\hbar}\right] |\Psi(\Phi_h = 0, \Phi_e = 0)\rangle \quad (1.10)$$

gains a phase, which is determined by a difference between the magnetic fluxes through the paths C_h and C_e encircled by the hole and the electron, respectively:

$$\Phi_h = \frac{e}{\hbar} \oint_{C_h} \mathbf{A}(\mathbf{R}) \cdot d\mathbf{R}, \quad \Phi_e = \frac{e}{\hbar} \oint_{C_e} \mathbf{A}(\mathbf{R}) \cdot d\mathbf{R}. \quad (1.11)$$

If the exciton is *polarized*, the paths C_h and C_e are different from each other (cp. [129]), the quantum interference, according to (1.10), is caused by the magnetic

flux ($\Phi_h - \Phi_e$) through the area between the two paths. Being manifested mainly through optical response of QRs, it is called *excitonic (or optical) Aharonov-Bohm effect* [129–132].

An example of the occurrence of the excitonic Aharonov-Bohm effect in transport phenomena was provided by the following feature of the vertical transport through a QR, which was immersed in a dielectric matrix: the tunnel current, as a function of magnetic flux for a given voltage across the structure, had the form of modulated oscillations with a characteristic period Φ_0 [133]. The optical Aharonov-Bohm effect became more prominent if the dc electric field was applied in the plane containing a QR [134] or in the vertical direction [135], because of the enhanced polarization of the exciton. Control over the Aharonov-Bohm oscillations in the energy spectrum of a QR could be realized also using low-frequency electromagnetic radiation [136].

The first experimental verification of the excitonic Aharonov-Bohm effect in self-assembled QRs was obtained by tracing patterns of the PL intensity under increasing magnetic field at different temperatures [137]. The role of the built-in piezoelectric fields in strained QR-systems consisted in changing the sequence of maxima and minima of the Aharonov-Bohm oscillations. For those observations, a correlation between the electron and hole due to the Coulomb interaction was shown to be a necessary condition.

The existence of the optical Aharonov-Bohm effect was first demonstrated through PL for type-II InP/GaAs quantum dots [138] and Zn(SeTe) quantum dots in ZnTe/ZnSe superlattice [139, 140]. Large and persistent oscillations in both the energy and the intensity of the PL unveiled the presence of coherently rotating exciton states. These remarkably robust Aharonov-Bohm oscillations were shown to persist until 180 K. The magnitude of the observed effects was attributed to the geometry of the columnar type-II structures investigated, which created a ring-like topology of the electron state. The advantage of this geometry to favor the optical Aharonov-Bohm effect was demonstrated in magnetic (ZnMn)Te quantum dot structures, where the strength of the Aharonov-Bohm interference effect could be controlled by the spin disorder in the system [140, 141].

The first MPL study of single neutral excitons was performed in single self-assembled InGaAs/GaAs QRs, which were fabricated by MBE combined with AsBr₃ *in situ* etching [135]. Oscillations in the neutral exciton radiative recombination energy and in the emission intensity were detected as a function of the applied magnetic field. Effective control over the period of the oscillations was achieved through a gate potential that modified the exciton confinement [135, 142]. Strain was shown to play a crucial role to govern the localization of electrons and holes in type-I semiconductor QRs, eventually leading to spatially separated charge carriers [143].

Singly charged excitons (trions) and multiply charged excitons in QRs were extensively studied, both theoretically and experimentally. The period of oscillations of the binding energy of charged complexes in magnetic flux was shown to differ from Φ_0 , being determined by the number of electrons and the ratio of effective masses of the electron and the hole [144]. The diamagnetic shift of the exciton

PL line was found to be positive for a neutral exciton and negative for a trion and other negatively charged complexes [145]. Circularly polarized magnetophotoluminescence (MPL) spectra of a single QR, fabricated with the modulated-barrier approach, were dominated by two features: a high-energy line due to neutral exciton recombination and a low-energy line owing to emission from charged excitons [146]. Measured photon energy from charged exciton recombination as a function of the magnetic field clearly revealed the oscillatory behavior, which was in antiphase with the calculated electron's energy [147].

The excitonic Aharonov-Bohm effect, originally considered for a one-dimensional model, was shown to remain essentially unchanged in QRs of finite width [148]. Though the Aharonov-Bohm oscillations of the oscillator strength as a function of the magnetic flux for the ground state of the exciton decreased with increasing the QR width, their amplitude remained finite down to radius-to-width ratios less than unity due to the non-simply-connectedness of the confinement potential. That implied that the key condition needed for the observation of the excitonic Aharonov-Bohm effect was the avoidance of the QR center.

The exciton energy spectra and optical transitions spectra calculated for excitons with the realistic confinement potential of self-assembled QRs [102] taking into account the strain revealed a very high sensitivity to the size, anisotropic shape, and composition of a QR [149]. Photoluminescence spectroscopy of a large ensemble of InAs/GaAs QRs in magnetic fields up to 30 T for different excitation densities unveiled that the confinement of an electron and a hole along with the Coulomb interaction suppressed the excitonic Aharonov-Bohm effect in these QRs [99, 150]. This suppression was confirmed also by MPL studies of type-II self-assembled GaSb/GaAs QRs [151].

Another manifestation of the Aharonov-Bohm effect in neutral formations was related to magnetoplasmon oscillations in QRs [152]. The plasmon frequency in a finite-width QR was constituted of a monotonous part superposed with Aharonov-Bohm oscillations. Their period and amplitude were found to vary with the magnetic field.

Polaron shift in QRs revealed the non-monochromaticity of the Aharonov-Bohm oscillations, which was attributed to the difference in the magnetic fluxes that are encircled by different electron trajectories [153]. When an exciton was generated, the contributions of the electron and the hole to the polarization of the medium had opposite signs, and it was therefore important to take the finiteness of the ring into account when calculating the net effect determined by the wave functions of the particles.

1.5.6 Advancements of Theory

Embedding QRs in various multilayer structures is an important tool to control their physical properties. The non-trivial role of strain in QR multilayer systems was theoretically revealed in Ref. [154]. In GaAs-capped InAs/In_{0.53}Ga_{0.47}As QRs, there

occurred an anomalous strain relaxation: GaAs embedded in the $\text{In}_{0.53}\text{Ga}_{0.47}\text{As}$ matrix considerably weakened each strain component and biaxial strain by providing enough room for the atomic relaxation of InAs. GaAs embedded in $\text{In}_{0.53}\text{Ga}_{0.47}\text{As}$ acted as a potential barrier for both electrons and heavy holes and as a potential well for light holes. The weak positive biaxial strain of InAs along with the strong negative biaxial strain of GaAs in a QR led to an enhancement of the light-hole character of the states in the valence band of a QR as compared to those in a quantum dot.

Calculating the strain profile as well as the charge carriers energy and other properties (shell filling, spin polarization, exciton fine structure, magnetization...) in QRs requires the extensive use of a great variety of the advanced tools of the modern theoretical physics, of which we name below only a few.

Exact diagonalization method revealed the fractional Aharonov-Bohm effect of a few-electron system in a one-dimensional QR taking into account spin, disorder and the Coulomb interaction [155]. A great challenge for the theory—to find analytical solutions for quantum states in QRs—was addressed for two electrons on a one-dimensional QR for particular values of the radius [156]. Many-electron QRs were studied using a number of versions of the Density Functional Theory [157, 158].

After calculating the strain with the atomistic Valence Force Field method, the electronic properties were derived in the framework of the Empirical Pseudopotential method [159] or the Empirical Tight-Binding method [81, 160–162]. A continuum description of the QR system was assumed within the single-band Effective-Mass approximation [163, 164] and its diversified generalizations onto multiband $\mathbf{k} \cdot \mathbf{p}$ approaches [165]. Of importance for modeling the self-assembled QRs was the finding [166], that the 14-band $\mathbf{k} \cdot \mathbf{p}$ model can accommodate for the correct symmetry of the underlying GaAs zincblende lattice, which was not reflected in the standard 8-band model. The ground-state energy of the few-particle systems in QRs was calculated within the Configuration Interaction method [135, 159].

Transfer-Matrix method was employed to account for the mutual influence of the radial and azimuthal motions in the presence of impurities in the finite-width QR [71]. The Landauer-Büttiker formalism was used to analyze transport properties of QRs [167, 168]. Using the Keldysh Green's function formalism enabled unveiling two contributions, thermodynamic and kinetic, to the disorder-averaged magnetization of QRs [169].

Path-Integral Quantum-Monte-Carlo method was applied for investigation of the energy spectra of few-electron systems in QRs as a function of their geometry [58]. The interplay between the confinement geometry and the Coulomb interaction was pronouncedly manifested through the electronic properties of a QR. The ground state of a perfect QR containing a small number of interacting electrons was analyzed as a function of its geometric parameters: ring radius, radial confinement, and eccentricity. A Path-Integral Quantum-Monte-Carlo calculation demonstrated a strong dependence of the total spin of the ground state on the structure geometry. For instance, for a three-electron QR, changing the radius produced a spin polarization of the ground state, while an elliptical deformation resulted in a spin-depolarized ground state [170].

A finite mixing of the heavy-hole subband with the light-hole subband in self-assembled InAs/GaAs QRs was shown to be larger than in quantum dots and critical

in determining the hole spin properties. The large light-hole component in QRs underpinned their perspectives for applications requiring enhanced tunneling rates [171] and spin-orbit mediated control [172].

1.6 Multi-Faceted Horizons

In view of the emerging high-tech realizations, finding and exploiting novel phenomena in QR-structures will be the key issues for the future development in the theoretical and experimental physics of QRs. At the time of writing this chapter, the perspective research directions in the field range from non-trivial topologies, new materials, alignment and assembly of QRs arrays—through engineering QR-based metamaterials—to device design, manufacturing, and application.

1.6.1 Novel Topological Structures

Nanostructure fabrication techniques have allowed for generating topologically non-trivial manifolds at the micro- and nanoscale with manmade space metrics, which determine the energy spectrum and other physical properties of electrons confined in such objects. For instance, when spooling a single crystalline NbSe₃ ribbon on a selenium droplet, surface tension produces a twist in the ribbon, leading to the formation of a one-sided Möbius ring [173]. Analytical and computational differential geometry methods have been developed to examine particle quantum eigenstates and eigenenergies in curved and strained nanostructures [174, 175]. Significant changes in eigenstate symmetry and eigenenergy are revealed due to the interplay between curvature and strain effects for bending radii of a few nanometers. Curvature effects become negligible at bending radii above ~ 50 nm.

Symbiosis of the geometric potential and an inhomogeneous twist renders an observation of the topology effect on the electron ground-state energy in microscale Möbius rings into the realm of experimental verification. A “*delocalization-to-localization*” transition for the electron ground state is unveiled in inhomogeneous Möbius rings. This transition can be quantified through the Aharonov-Bohm effect on the ground-state persistent current as a function of the magnetic flux threading the Möbius ring [176]. The theoretical analysis of such topologically nontrivial manifolds at the nanoscale will have practical relevance, as any pertinent fabrication techniques are likely to generate geometric and structural inhomogeneities.

1.6.2 Graphene QRs

Electronic quantum interference in QR-structures based on graphene has been investigated with a focus on the interplay between the Aharonov-Bohm effect and the

peculiar electronic and transport properties of this material [168]. The first experimental realization of a graphene ring structure has been provided by [177]. In this work, the authors investigate the Aharonov-Bohm oscillations in diffusive single-layer graphene as a function of the magnetic field, which is applied perpendicularly to the graphene plane in a two-terminal setup. They find clear magnetoconductance oscillations with the expected period of Φ_0 on the top of a low-frequency background signal due to universal conductance fluctuations. A significant increase in the oscillation amplitude at strong magnetic fields close to the onset of the quantum Hall regime is strong enough to make the second harmonic (oscillations of period $\Phi_0/2$) visible in the frequency spectrum. Such a behavior, observed in smaller rings using a two-terminal as well as four-terminal geometry is attributed to scattering on magnetic impurities [178].

Additional tunability is introduced into the graphene ring device by applying a side gate potential to one of the ring arms. Investigation of the influence of such side gates on a four-terminal geometry in the diffusive regime reveals phase shifts of the Aharonov-Bohm oscillations as a function of the gate voltage as well as phase jumps of π at zero magnetic field—direct consequences of the electrostatic Aharonov-Bohm effect as well as the generalized Onsager relations [179]. The electrostatic Aharonov-Bohm effect appears to be more feasible in graphene QRs than in metal QRs due to the low screening of this material [180, 181]. Voltage-driven charge-carrier states ranging from metallic to semiconductor ones are theoretically revealed for QRs determined by a set of concentric circular gates over a graphene sheet placed on a substrate [182].

1.6.3 Ordering of QRs. Metamaterials

Great efforts have been devoted to achieve vertical and lateral alignment of QRs. Stacking of three InGaAs/GaAs QRs is demonstrated to provide a broad-area laser [183]. In QR complexes and stacks, novel correlations occur, which allow for control over their electronic and magnetic properties.

One-dimensional ordered QR-chains have been fabricated on a quantum-dot superlattice template by MBE. The quantum-dot superlattice template is prepared by stacking multiple quantum-dot layers. The lateral ordering is introduced by engineering the strain field of a multi-layer InGaAs quantum-dot superlattice. QR chains are then formed by partially capping InAs quantum dots with a thin layer of GaAs which introduces a morphological change from quantum dots to QRs [184]. It is shown that two-dimensional periodically aligned QR-arrays can be fabricated on GaAs high-index [(311)B and (511)B] surfaces [185].

An alternative approach to self-assembly of aligned QRs is to create an artificially ordered template by pre-patterning. Nanosphere lithography is used to create ordered GeSi quantum dots, and ordered GeSi QRs are subsequently formed by capping the quantum dots with a thin Si capping layer [186]. When the Si capping layer is more than 3 nm thick, most quantum dots are converted into QRs. Additional

fabrication techniques, such as Ar^+ sputter redeposition using porous alumina templates [187] and laser-interference lithography in conjunction with electrochemical deposition [188], are promising, low-cost, and scalable tools for producing ordered QR-arrays.

QRs are a very promising building block for metamaterials. High-density assemblies of QRs may contain clusters of close or even partially overlapping QRs [114]. A moderate coupling between adjacent QRs in a cluster is shown to significantly influence the energy spectrum: while its lowest part may preserve the single-QR behavior, the high-energy part is strongly modified [189]. Metamaterials consisting of split nanosized gold QRs are found to possess unusual electromagnetic response properties like a negative index of refraction for wavelengths in the micrometer region [190, 191], where the resonance wavelength scales linearly with the size of the circuit. A possible control over the electromagnetic response in QR composite metamaterials made from metals and semiconductors is an attractive goal for further investigations.

1.6.4 Photonic Sources and Detectors

Using QRs as photon sources and detectors is based on their unique optical properties associated with the excitonic Aharonov-Bohm effect [192]. Using QRs, a technique is theoretically devised to completely freeze and release individual photons at will by tuning magnetic and electric fields that enable QRs *to trap and store light* [192]. Application of these QRs as light capacitors or buffers is expected in the fields of photonic computing and communications technologies [192, 193]. The shallow bound-state energy levels of the InGaAs QRs are shown to be feasible for detecting photons in the terahertz regime [194].

Single InAs/GaAs QRs embedded in a photonic crystal lattice are demonstrated to allow for single-photon emission and photon antibunching between the exciton and biexciton emissions [195]. This extends the realm of the QRs investigations towards quantum electrodynamics. The antibunching of photons observed in a double QR [196] is a clear signature of a single photon emitter.

1.6.5 Spintronics. Magnetic Memory

An effective confinement-governed wave function engineering is explored theoretically in systems with QRs: two-dimensional complex nanostructures in the form of double concentric QRs and dot-QR nanostructures that consist of a QR with a quantum dot inside. The higher spin stability in a QR than in a quantum dot makes QRs attractive for the realization of spin qubits, because the relaxation and decoherence processes take place in the time scale that is sufficiently long for spin manipulations and readout [197]. The dot-QR nanostructure allows, by changing the potential barrier separating the dot from the QR and the potential well offset between the dot and

the QR, for a significant alteration of coherent, optical and transport properties of the structure. In particular, the spin relaxation time of dot-QR nanostructures, used as spin qubits or spin memory devices, can be modified by orders of magnitude [198]. A crossover from the ballistic to the resonant tunneling transport, unveiled for an ideal one-dimensional QR with spin-orbit interaction, underpins the suggestion to use QRs for fabricating one-qubit spintronic quantum gate and thus, for quantum information processing [167]. An Aharonov-Bohm interferometer consisting of a QR with two quantum dots embedded in its arms reveals sensitive spin-polarized electron transmission that might be useful for spintronics applications [199].

QR arrays offer superior prospects in high density magnetic memory applications as magnetic random access memory, recording medium, and other spintronic devices [200].

In conclusion, a great variety of semiconductor QR-systems, in particular, single and multiple QRs, ordered arrays of QRs, complexes of QRs in combination with other nanostructures, Möbius QRs, have been fabricated with advanced high-tech methods, characterized with cutting-edge technologies, and analyzed with innovative theoretical tools. Their unique *doubly-connected topology* and the *ring-like density of states* for charge carriers, spins, plasmon and photon fields provide a veritable cornucopia of fascinating properties and possibilities to boost development of the strategic domains of technology: quantum computing based on photon and spin manipulations, photonic emitters and detectors, magnetic memory, and engineering of novel metamaterials.

Acknowledgements I wish to express my greatest gratitude to Axel Lorke, Ian R. Sellers, Paul M. Koenraad, and Alexander O. Govorov, who have read the manuscript of the present chapter, for their critical comments and valuable suggestions.

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<http://www.springer.com/978-3-642-39196-5>

Physics of Quantum Rings

Fomin, V.M. (Ed.)

2014, XXIV, 487 p., Hardcover

ISBN: 978-3-642-39196-5