Chapter 2
Fuzzy Sets and Possibility Theory

2.1 Introduction

In many real-world applications, information is often imprecise and uncertain. Many sources can contribute to the imprecision and uncertainty of data or information. We face, for example, increasingly large volumes of data generalized by non-traditional means (e.g., sensors and RFID), which is almost always imprecise and uncertain. It has been pointed out that in the future, we need to learn how to manage data that is imprecise or uncertain, and that contains an explicit representation of the uncertainty (Dalvi and Suciu 2007).

Over the years, imprecise and uncertain information has been widely investigated in lots of application domains, including database, information system, and so on (Klir and Yuan 1995; Ma and Yan 2008). Also, this is a well-known problem especially for semantics-based applications of the Semantic Web, such as knowledge management, e-commerce, and web portals (Calegari and Ciucci 2006). As the examples mentioned by Stoilos et al. (2010), a task like a "holiday organization" could involve a request like: "Find me a *good* hotel in a place that is *relatively hot* and with *many* attractions", or a "doctor appointment" could look like: "Make me an appointment with a doctor *close to* my home not *too early* and of *good* references". Moreover, several intermediate processes, like information extraction or retrieval, matching user preferences with data and more, might involve imperfect information due to their automatic nature (Stoilos et al. 2010). Last but not least, several modern applications that have adopted Semantic Web technologies in order to enhance their performance and "connect" with the Semantic Web require the management of such knowledge (Stoilos et al. 2010). For example there are schemes for using Semantic Web technologies in multimedia applications for multimedia analysis, in Semantic Portals, ontology alignment and semantic interoperability, Semantic Web Services matching, word-computing based systems and many more, all of which require the management of some form of fuzzy information. For example, in image analysis one has to map low-level numerical values that are extracted by analysis algorithms for the color, shape and texture of a region into more high-level symbolic features like concepts.

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For example, values of the RGB color model would need to be mapped into concepts like *Blue*, *Green*, etc. or values of special shape and texture (signal) transforms need to be mapped to concepts like *Rectangular_Shaped*, *Coarse_Textured*, *Smooth_Textured* and more, all of which are obviously fuzzy concepts and need proper handling (Stoilos et al. 2010).

The conceptual formalism supported by typical Semantic Web techniques (such as ontologies, Description Logics, and rules mentioned in Chap. 1) may not be sufficient to represent such information and knowledge as mentioned above. In particular, the problem to deal with fuzzy information has been addressed in several decades ago by Zadeh (1965), who gave bird in the meanwhile to the so-called fuzzy set and fuzzy logic theory and a huge number of real life applications are based on it. Fuzzy set theory (Zadeh 1965), which is interchangeably referred as fuzzy logic, is a generalization of the set theory and provides a means for the representation of imprecision and vagueness. The *fuzzy set theory* has been identified as a successful technique for modeling the fuzzy information in many application areas such as text mining, multimedia information system, database (Klir and Yuan 1995; Bosc et al. 2005; Ma and Yan 2008). Also, in the context of the Semantic Web as well as applications using ontologies, Description Logics, and rules, the fuzzy set theory has been considered as the main theory basis for extensions to them to handle fuzzy information (Sanchez 2006; Ma et al. 2012). In this chapter, we briefly introduce some notions about imperfect information and fuzzy sets and possibility theory, which are of interest or relevance to the discussions of successive chapters.

2.2 Imperfect Information

In order to satisfy the need for modeling fuzzy information, the following several kinds of imperfect information have been extensively introduced into real-world applications (Bosc et al. 2005; Klir and Yuan 1995; Ma and Yan 2008; Smets 1996):

- Inconsistency is a kind of semantic conflict, meaning the same aspect of the real world is irreconcilably represented more than once in a database or in several different databases. For example, the *age* of *George* is stored as 34 and 37 simultaneously.
- Imprecision is relevant to the content of an attribute value and means that a choice must be made from a given range (interval or set) of values without knowing which one to choose. For example, the *age* of *Michael* is a set {18, 19, 20, 21}.
- Vagueness is like imprecision but which is generally represented with linguistic terms. For example, the *age* of *John* is a linguistic value "*old*".
- Uncertainty is related to the degree of truth of its attribute value, meaning that we can apportion some, but not all, of our belief to a given value or group of values. For example, the possibility that the *age* of *Chris* is 35 right now should

be 0.98. The random uncertainty, described using probability theory, is not considered here.

- Ambiguity means that some elements of the model lack complete semantics, leading to several possible interpretations.

Generally, several different kinds of imperfection can co-exist with respect to the same piece of information. For example, the *age* of *John* is a set {18, 19, 20} and their possibilities are 0.70, 0.95, and 0.98, respectively. Imprecision, vagueness, and uncertainty are three types of imperfect information (Ma and Yan 2008; Smets 1996). Imprecision are essential properties of the information itself, whereas uncertainty is a property of the relation between the information and our knowledge about the world.

Moreover, imprecise values generally denote a set of values in the form of [ai1, ai2,…, aim] or [ai1, ai2] for the discrete and continuous universe of discourse, respectively, meaning that exactly one of the values is the true value for the single-valued attribute, or at least one of the values is the true value for the multivalued attribute. So, imprecise information here has two interpretations: disjunctive information and conjunctive information.

One kind of imprecise information that has been studied extensively is the well-known null values (Codd 1986, 1987; Motor 1990; Parsons 1996; Zaniolo 1984), which were originally called incomplete information. The possible interpretations of null values include: (a) "*existing but unknown*", (b) "*nonexisting*" or "*inapplicable*", and (c) "*no information*". A null value on a multivalued object, however, means an "open null value" (Gottlob and Zicari 1988), i.e., the value may not exist, has exactly one unknown value, or has several unknown values. Null values with the semantics of "existent but unknown" can be considered as the special type of partial values that the true value can be any one value in the corresponding domain, i.e., an applicable null value corresponds to the whole domain.

The notion of a partial value is illustrated as follows (Grant 1979). A partial value on a universe of discourse *U* corresponds to a finite set of possible values in which exactly one of the values in the set is the true value, denoted by $\{a_1, a_2, \ldots, a_m\}$ for discrete *U* or $[a_1, a_n]$ for continua *U*, in which $\{a_1, a_2, \ldots, a_m\} \subseteq U$ or $[a_1, a_n] \subseteq U$. Let η be a partial value, then $sub(\eta)$ and $sup(\eta)$ are used to represent the minimum and maximum in the set.

Note that crisp data can also be viewed as special cases of partial values. A crisp data on discrete universe of discourse can be represented in the form of {p}, and a crisp data on continua universe of discourse can be represented in the form of [p, p]. Moreover, a partial value without containing any element is called an *empty partial value*, denoted by $\perp$. In fact, the symbol $\perp$ means an inapplicable missing data (Codd 1986, 1987). Null values, partial values, and crisp values are thus represented with a uniform format.

2.3 Representation of Fuzzy Sets and Possibility Distributions

In 1965, L. A. Zadeh published his innovating paper "Fuzzy Set" in the journal of *Information and Control* (Zadeh 1965). Since then fuzzy set has been infiltrating into almost all branches of pure and applied mathematics that are set-theory-based. This has resulted in a vast number of real applications crossing over a broad realm of domains and disciplines. Over the years, many of the existing approaches dealing with imprecision and uncertainty are based on the theory of fuzzy sets (Zadeh 1965) and possibility theory (Zadeh 1978).

Fuzzy data is originally described as fuzzy set (Zadeh 1965). A fuzzy set, say {0.6/18, 0.7/19, 0.8/20, 0.9/21} for the age of *Michael*, is more informative because it contains information imprecision (the age may be 18, 19, 20, or 21 and we do not know which one is true) and uncertainty (the degrees of truth of all possible age values are respectively 0.6, 0.7, 0.8, and 0.9) simultaneously.

Let U be a universe of discourse. A fuzzy value on U is characterized by a fuzzy set F in U. A membership function

$$\mu_F : U \rightarrow [0, 1]$$

is defined for the fuzzy set F, where $\mu_F(u)$, for each $u \in U$, denotes the degree of membership of u in the fuzzy set F. For example, $\mu_F(u) = 0.75$ means that u is "likely" to be an element of F by a degree of 0.75. For ease of representation, a fuzzy set F over universe U is organized into a set of ordered pairs:

$$F = \{\mu_F(u_1)/u_1, \mu_F(u_2)/u_2, \ldots, \mu_F(u_\mathrm{n})/u_\mathrm{n}\}$$

When the membership function $\mu_F(u)$ above is explained to be a measure of the possibility that a variable X has the value u in this approach, where X takes values in U, a fuzzy value is described by a possibility distribution π_X (Zadeh 1978).

$$\pi_X = \{\pi_X(u_1)/u_1, \pi_X(u_2)/u_2, \ldots, \pi_X(u_n)/u_n\}$$

Here, $\pi_X(u_i)$, $u_i \in U$, denotes the possibility that u_i is true.

Moreover, the extension principle introduced by Zadeh (1975) has been regarded as one of the most basic ideas of fuzzy set theory. By providing a general method, the extension principle has been extensively employed to extend non-fuzzy mathematical concepts. The idea is to induce a fuzzy set from a number of given fuzzy sets through a mapping.

Zadeh's extension principle can also be referred to maximum-minimum principle sometimes. Let X_1, X_2, …, X_n and Y be ordinary sets, f be a mapping from $X_1 \times X_2 \times \ldots \times X_n$ to Y such that $y = f(x_1, x_2, \ldots, x_n)$, P (X_i) and P (Y) be the power sets of X_i and Y ($0 \le i \le n$), respectively. Here, P $(X_i) = \{C | C \subseteq X_i\}$ and P (Y) $= \{D | D \subseteq Y\}$. Then f induces a mapping from P $(X_1) \times$ P $(X_2) \times \ldots \times$ P (X_n) to P (Y) with

$$f(\mathrm{C_1}, \mathrm{C_2}, \ldots, \mathrm{C_n}) = \{f(\mathrm{X_1}, \mathrm{X_2}, \ldots, \mathrm{X_n}) | \mathrm{X_i} \in \mathrm{C}_i, 0 \le i \le \mathrm{n}\}$$

Here $\mathrm{C_i} \subseteq \mathrm{X_i}$ and $0 \le i \le \mathrm{n}$. Now, let F $(\mathrm{X_i})$ be the class of all fuzzy sets on $\mathrm{X_i}$, i.e., F $(\mathrm{X_i}) = \{\}$, $0 \le i \le$ n and F (Y) be the class of all fuzzy sets on Y, i.e., F (Y) $= \{\}$, then f induces a mapping from F $(\mathrm{X_1}) \times$ F $(\mathrm{X_2}) \times \ldots \times$ F $(\mathrm{X_n})$ to F (Y) such that for all $\mathrm{A_i} \in$ F $(\mathrm{X_i})$, f $(\mathrm{A_1}, \mathrm{A_2}, \ldots, \mathrm{A_n})$ is a fuzzy set on Y with

$$f(\mathrm{A_1}, \mathrm{A_2}, \ldots, \mathrm{A_n})(y) = \begin{cases} \sup\limits_{f(x1,x2,\ldots,xn)=y} & (\min(\mu_{\mathrm{A1}}(x1), \mu_{\mathrm{A2}}(x2), \ldots, \mu_{\mathrm{An}}(xn)), f^{-1}(y)) \neq \Phi \\ xi \in Xi\,(i = 1, 2, \ldots, n) & \\ 0, f^{-1}(y) = \Phi & \end{cases}$$

2.4 Operations on Fuzzy Sets

In order to manipulate fuzzy sets (as well as possibility distributions), several operations, including *set operations*, *arithmetic operations*, *relational operations*, and *logical operations*, should be defined. The usual *set operations* (such as union, intersection and complementation) have been extended to deal with fuzzy sets (Zadeh 1965). Let A and B be fuzzy sets on the same universe of discourse U with the membership functions μ_A and μ_B, respectively. Then we have

Union. The union of fuzzy sets A and B, denoted A U B, is a fuzzy set on U with the membership function $\mu_{A \cup B}$: $U \to [0, 1]$, where

$$\forall u \in U, \mu_{A \cup \bar{B}}(u) = \max(\mu_A(u), \mu_B(u)).$$

Intersection. The intersection of fuzzy sets A and B, denoted $A \cap B$, is a fuzzy set on U with the membership function $\mu_{A \cap B}$: $U \to [0, 1]$, where

$$\forall u \in U, \mu_{A \cap B}(u) = \min(\mu_A(u), \mu_B(u)).$$

Complementation. The complementation of fuzzy set $\bar{A}$, denoted by $\bar{A}$, is a fuzzy set on U with the membership function $\mu_{\bar{A}}$: $U \to [0, 1]$, where

$$\forall u \in U, \mu_A(u) = 1 - \mu_A(u).$$

Based on these definitions, the *difference* of the fuzzy sets B and A can be defined as:

$$B - A = B \cap \overline{\mathrm{A}}.$$

Also, most of the properties that hold for classical set operations, such as DeMorgan's Laws, have been shown to hold for fuzzy sets. The only law of ordinary set theory that is no longer true is the law of the excluded middle, i.e.,

$$A \cap \overline{\mathrm{A}} \neq \emptyset \text{ and } A \cup \overline{\mathrm{A}} \neq U.$$

Let A, B, and C be fuzzy sets in a universe of discourse U, then the operations on fuzzy sets satisfy the following conditions:

- Commutativity laws: $\mathrm{A} \cup \mathrm{B} = \mathrm{B} \cup \mathrm{A}, \mathrm{A} \cap \mathrm{B} = \mathrm{B} \cap \mathrm{A}$
- Associativity laws: $(\mathrm{A} \cup \mathrm{B}) \cup \mathrm{C} = \mathrm{A} \cup (\mathrm{B} \cup \mathrm{C}), (\mathrm{A} \cap \mathrm{B}) \cap \mathrm{C} = \mathrm{A} \cap (\mathrm{B} \cap \mathrm{C})$
- Distribution laws: $(\mathrm{A} \cup \mathrm{B}) \cap \mathrm{C} = (\mathrm{A} \cap \mathrm{C}) \cup (\mathrm{B} \cap \mathrm{C}), (\mathrm{A} \cap \mathrm{B}) \cup \mathrm{C} = (\mathrm{A} \cup \mathrm{C}) \cap (\mathrm{B} \cup \mathrm{C})$
- Absorption laws: $(\mathrm{A} \cup \mathrm{B})\ \mathrm{A} = \mathrm{A}, (\mathrm{A} \cap \mathrm{B}) \cup \mathrm{A} = \mathrm{A}$
- Idempotency laws: $\mathrm{A} \cup \mathrm{A} = \mathrm{A}, \mathrm{A} \cap \mathrm{A} = \mathrm{A}$
- De Morgan laws: $\overline{\mathrm{A} \cup \mathrm{B}} = \overline{\mathrm{A}} \cap \overline{\mathrm{B}}$, $\overline{\mathrm{A} \cap \mathrm{B}} = \overline{\mathrm{A}} \cup \overline{\mathrm{B}}$

Given two fuzzy sets A and B in U, B is a fuzzy subset of A, denoted by $B \subseteq A$, if

$$\mu_B(u) \leq \mu_A(u)$$

for all $u \in U$.

Two fuzzy sets A and B are said to be equal if $A \subseteq B$ and $B \subseteq A$.

In order to define Cartesian product of fuzzy sets, let $U = U_1 \times U_2 \times \ldots \times U_n$ be the Cartesian product of n universes and $A_1, A_2, \ldots, A_n$ be fuzzy sets in $U_1, U_2, \ldots, U_n$, respectively. The Cartesian product $A_1 \times A_2 \times \ldots \times A_n$ is defined to be a fuzzy subset of $U_1 \times U_2 \times \ldots \times U_n$, where

$$\mu_{A1 \times \ldots \times An}(u_1 \ldots u_n) = \min(\mu_{A1}(u_1), \ldots, \mu_{An}(u_n))$$

and $u_i \in U_i, i = 1, \ldots, n$.

Moreover, Let U be a universe of discourse and F a fuzzy set in U with the membership function μ_F: $U \rightarrow [0, 1]$. We have then the following notions related to fuzzy sets.

Support. The set of the elements that have non-zero degrees of membership in F is called the support of F, denoted by

$$supp(F) = \{u | u \in U \text{ and } \mu_F(u) > 0\}$$

Kernel. The set of the elements that completely belong to F is called the kernel of F, denoted by

$$ker(F) = \{u | u \in U \text{ and } \mu_F(u) = 1\}.$$

α-Cut. The set of the elements which degrees of membership in F are greater than (greater than or equal to) α, where $0 \leq \alpha < 1$ ($0 < \alpha \leq 1$), is called the strong (weak) α-cut of F, respectively denoted by

$$F_{\alpha+} = \{u|\, u \in U \text{ and } \mu_F(u) > \alpha\}$$

and

$$F_{\alpha} = \{u|\, u \in U \text{ and } \mu_F(u) \geq \alpha\}.$$

The relationships among the support, kernel, and α-cut of a fuzzy set can be illustrated in Fig. 2.1.

Consider the example of a preliminary product design. The value of the performance parameter *capacity* is "about 2.5e + 03", which is represented by the following fuzzy set

$$F = \{1.0/2.5\text{e}+03,\ 0.96/5.0\text{e}+03,\ 0.88/7.5\text{e}+03,\ 0.75/1.0\text{e} \\ +04,\ 0.57/1.25\text{e}+04,\ 0.32/1.5\text{e}+04,\ 0.08/1.75\text{e}+04\}.$$

Then, we have

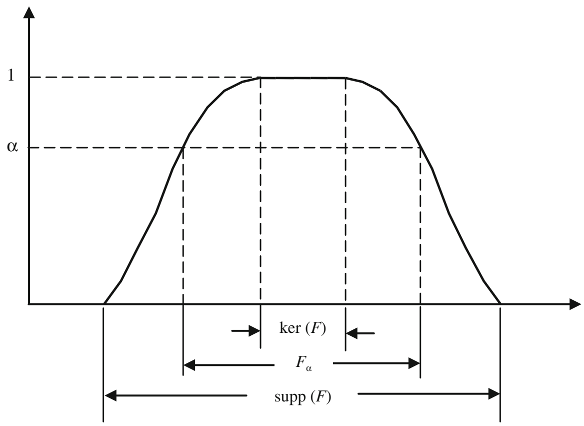
$$\begin{aligned} \text{supp}(F) &= \{2.5\text{e}+03, 5.0\text{e}+03, 7.5\text{e}+03, 1.0\text{e}+04, 1.25\text{e}+04, 1.5\text{e} \\ &\quad +04, 1.75\text{e}+04\}, \\ \ker(F) &= \{2.5\text{e}+03\}, \\ F_{0.88+} &= \{2.5\text{e}+03, 5.0\text{e}+03\}, \text{ and } F_{0.88} = \{2.5\text{e}+03,\ 5.0\text{e}+03,\ 7.5\text{e}+03\}. \end{aligned}$$

Moreover, utilizing Zadeh's extension principle, which can also be referred to maximum-minimum principle sometimes, *arithmetic operations* can be defined. Let A and B be fuzzy sets on the same universe of discourse U with the membership functions μ_A and μ_B, respectively, and "θ" be an infix operator. $A\ \theta\ B$ is a fuzzy set on U with the membership function $\mu_{A\ \theta\ B}$: $U \to [0, 1]$, where

$$\forall z \in U, \mu_{A\theta B}(z) = \max_{z=x\theta y}(\min(\mu_A(x), \mu_B(y))).$$

In addition, *fuzzy relational operations* are various kinds of comparison operations on fuzzy sets, namely, *equal* (=), *not equal* ($\neq$), *greater than* (>), *greater*

Fig. 2.1 Support, kernel, and α-cut of fuzzy sets



than or equal ($\geq$), *less than* ($<$), and *less than or equal* ($\leq$). The definitions of these fuzzy relational operations are essentially related to the closeness measures between fuzzy sets and the given thresholds.

Let A and B be fuzzy sets on the same universe of discourse U with the membership functions μ_A and μ_B, respectively, and β be a given threshold value. Then we have

(a) $A \approx_\beta B$ if $SE\,(A, B) \geq \beta$,
(b) $A \not\approx_\beta B$ if $SE\,(A, B) < \beta$,
(c) $A \succ_\beta B$ if $SE\,(A, B) < \beta$ and max (supp (A)) $>$ max (supp (B)),
(d) $A \prec_\beta B$ if $SE\,(A, B) < \beta$ and max (supp (A)) $<$ max (supp (B)),
(e) $A \succeq_\beta B$ if $A \succ_\beta B$ or $A \approx_\beta B$, and
(f) $A \preceq_\beta B$ if $A \prec_\beta B$ or $A \approx_\beta B$.

Now let us have a close look at the fuzzy sets being operators. Three kinds of fuzzy sets can be identified: simple (atomic) fuzzy set, modified (composite) fuzzy set, and compound fuzzy set.

Simple (atomic) fuzzy set. A simple fuzzy set such as *young* or *tall* is defined by a fuzzy number with membership function.

Modified (composite) fuzzy set. A modified fuzzy set such as *very young* or *more or less tall* is described by a fuzzy number with membership function. Note that its membership function is not defined but computed through the membership function of the corresponding simple fuzzy set. In order to compute the membership function of modified fuzzy set, some semantic rules should be used. Let F is a simple fuzzy set represented by a fuzzy number in the universe of discourse U and its membership function is μ_F: $U \to [0,1]$, then we have the following rules.

Concentration rule: $\mu_{\text{very } F}\,(\text{u}) = (\mu_F\,(\text{u}))^2$
More generally, $\mu_{\text{very very } \ldots \text{ very } F}\,(\text{u}) = (\mu_F\,(\text{u}))^{2 \times \text{(times of very)}}$
Dilation rule: $\mu_{\text{more or less } F}\,(\text{u}) = (\mu_F\,(\text{u}))^{1/2}$.

Compound fuzzy set. A compound fuzzy set such as *young* $\cap$ *very tall* is represented by simple fuzzy sets or modified fuzzy sets connected by union ($\cup$), intersection ($\cap$) or complementation connectors.

Generally speaking, the results of fuzzy relational operations are fuzzy Boolean. They can be combined with logical operators such as *not* ($\neg$), *and* ($\wedge$), and *or* ($\vee$) to form complicated logical expression. Such expression can be used to represent logical conditions for information retrieval and so on. In the above definitions of the fuzzy relational operations, classical two-valued logic (2VL), namely, true (T) and false (F), is used because of the use of threshold values.

In the relational operations of fuzzy sets, there may be some fuzzy relations such as (*not*) *close to*/*around*, (*not*) *at lease*, and (*not*) *at most* etc. with crisp values in addition to the traditional operators such as $>$, $<$, $=$, $\neq$, $\geq$, and $\leq$. Now consider fuzzy relations as operators and crisp values as operands. For $A\ \tilde{\theta} Y$, where A is an attribute, $\tilde{\theta}$ is a fuzzy relation, and Y is a crisp value, $\tilde{\theta} Y$ is a fuzzy number.

First, let's focus on fuzzy relation "*close to* (*around*)". According to (Chen and Jong 1997), the membership function of the fuzzy number "*close to Y* (*around Y*)" on the universe of discourse can be defined by

$$\mu_{close\ to\ Y}(\mathrm{u}) = \frac{1}{1 + \left(\frac{\mathrm{u}-\mathrm{Y}}{\beta}\right)^2}$$

The membership function of the fuzzy number "*close to Y*" is shown in Fig. 2.2.

It should be noted that the fuzzy number above is a simple fuzzy term. Based on it, we have the following modified fuzzy terms: "*very close to Y*", "*very very … very close to Y*", and "*more or less close to Y*", which membership functions can be defined as

$$\mu_{\text{very close to Y}}(\mathrm{u}) = (\mu_{\text{close to Y}}(\mathrm{u}))^2,$$

$$\mu_{\text{very very}\ldots\text{very close to Y}}(\mathrm{u}) = (\mu_{\text{close to Y}}(\mathrm{u}))^{2\times(\text{times of very})}, \text{ and}$$

$$\mu_{\text{more or less close to Y}}(\mathrm{u}) = (\mu_{\text{close to Y}}(\mathrm{u}))^{1/2}.$$

Based on fuzzy number "*close to Y*", a compound fuzzy term "*not close to Y*" can be defined. Its membership function is as follows.

$$\mu_{\text{not close to Y}}(\mathrm{u}) = (1 - \mu_{\text{close to Y}}(\mathrm{u}))$$

Second, the membership function of the fuzzy number "*at least Y*" on the universe of discourse (Bosc and Pivert 1995) can be defined by

$$\mu_{at\ least\ Y}(\mathrm{u}) = \begin{cases} 0, u \leq \omega \\ \dfrac{u - \omega}{Y - \omega}, \omega < u < Y \\ 1, u \geq Y \end{cases}.$$

The membership function of the fuzzy number "*at least Y*" is shown in Fig. 2.3.

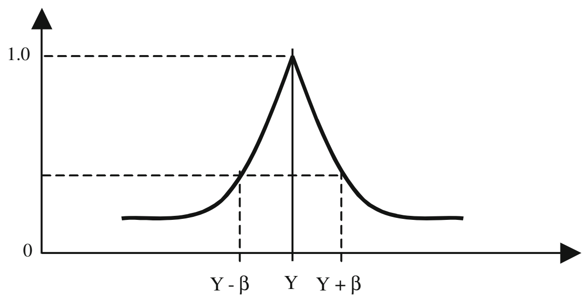


Fig. 2.2 Membership function of the fuzzy number "*close to Y*"

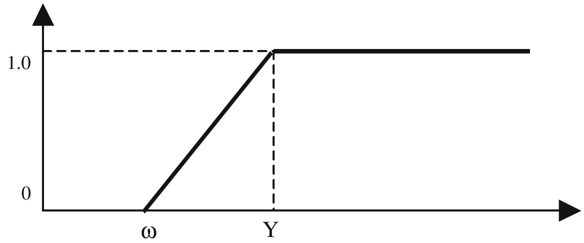


Fig. 2.3 Membership function of the fuzzy number "*at least Y*"

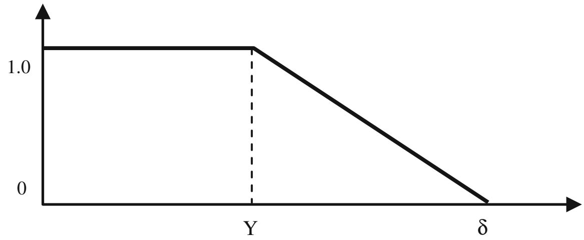


Fig. 2.4 Membership function of the fuzzy number "*at most Y*"

Based on fuzzy number "*at least Y*", a compound fuzzy term "*not at least Y*" can be defined. Its membership function is as follows.

$$\mu_{not\ at\ least\ Y}(\mathrm{u}) = (1 - \mu_{\mathrm{at\ least\ Y}}(\mathrm{u}))$$

Finally let's focus on fuzzy relation "*at most*". The membership function of the fuzzy number "*at most Y*" on the universe of discourse can be defined by

$$\mu_{at\ most\ Y}(\mathrm{u}) = \begin{cases} 1, u \leq Y \\ \dfrac{\delta - u}{\delta - Y}, Y < u < \delta \\ 0, u \geq \delta \end{cases}.$$

The membership function of the fuzzy number "*at most Y*" is shown in Fig. 2.4.

Based on fuzzy number "*at most Y*", a compound fuzzy term "*not at most Y*" can be defined. Its membership function is as follows.

$$\mu_{\mathrm{not\ at\ most\ Y}}(\mathrm{u}) = (1 - \mu_{\mathrm{atmostY}}(\mathrm{u}))$$

The fuzzy relations "close to", "not close to", "at least", "at most", "not at least", and "not at most" can be viewed as "fuzzy equal to", "fuzzy not equal to", "fuzzy greater than and equal to", "fuzzy less than and equal to", "fuzzy greater than", and "fuzzy less than", respectively. Using these fuzzy relations and crisp values, the fuzzy query condition with fuzzy operators, which has the form $\mathrm{A}\tilde{\theta}\mathrm{Y}$, is formed.

Fuzzy logical operations dependent on the representation of fuzzy Boolean values as well as fuzzy logic. Three logical operations *fuzzy not* ($\tilde{\neg}$), *fuzzy and* ($\tilde{\wedge}$), and *fuzzy or* ($\tilde{\vee}$), which operands are fuzzy Boolean value(s) represented by fuzzy sets, are defined in the following.

Fuzzy and. The result of *fuzzy and* is a fuzzy Boolean value. *Fuzzy and* can be defined with "intersection" kinds of operations such as "min" operation. Let A: μfi$_A$ (u) and B: μ_B (u) be two fuzzy Boolean values represented by fuzzy sets on the same universe of discourse U. Then

$$A\tilde{\wedge}B : \min(\mu_A(u), \mu_B(u)), u \in U.$$

Fuzzy or. The result of *fuzzy or* is a fuzzy Boolean value. *Fuzzy or* can be defined with "union" kinds of operations such as "max" operation. Let A: μ_A (u) and B: μ_B (u) be two fuzzy Boolean values represented by fuzzy sets on the same universe of discourse U. One has

$$A\tilde{\vee}B : \max(\mu_A(u), \mu_B(u)), u \in U.$$

Fuzzy not. The result of *fuzzy not* is a fuzzy Boolean value. *Fuzzy not* can be defined with "complementation" kinds of operations such as "subtraction" operation. Let A: μ_A (u) be a fuzzy Boolean values represented by fuzzy sets on the universe of discourse U. One has

$$\tilde{\neg}A : (1 - \mu_A(u)), u \in U.$$

A fuzzy set A w.r.t U is called *convex*, iff for all $u_1, u_2 \in U$, $\mu_A(\lambda u_1 + (1-\lambda)u_2) \geq min(\mu_A(u_1), \mu_A(u_2))$, where $\lambda \in [0, 1]$. A fuzzy set A w.r.t U is called *normal*, if $\exists u \in U$, s.t. $\mu_A(u) = 1$. A *fuzzy number* is a convex and normal fuzzy set. The set of elements whose membership degrees in A are greater than (greater than or equal to) α, where $0 \leq \alpha < 1\,(0 < \alpha \leq 1)$, is called the *strong (weak) α-cut* of A, denoted by $A_{\alpha+} = \{u \in U | \mu_A(u) > \alpha\}$ and $A_\alpha = \{u \in U | \mu_A(u) \geq \alpha\}$. The α-cut of a fuzzy number corresponds to an interval of U. Let A and B be two fuzzy numbers of U, and $A_\alpha = [x_1, y_1]$ and $B_\alpha = [x_2, y_2]$ the α-cuts of A and B, respectively. Then we have $(A \cup B)_\alpha = A_\alpha \underline{\cup} B_\alpha$, $(A \cap B)_\alpha = A_\alpha \underline{\cap} B_\alpha$, where $\underline{\cup}$ and $\underline{\cap}$ denote the union and the intersection operators between two intervals, respectively. They are defined as follows,

$$A_\alpha \underline{\cup} B_\alpha = \begin{cases} [x_1, y_1]\underline{\cup}[x_2, y_2], & \mathit{if}\ A_\alpha \cap B_\alpha = \emptyset \\ [\min(x_1, x_2), \max(y_1, y_2)], & \mathit{if}\ A_\alpha \cap B_\alpha \neq \emptyset \end{cases}$$

$$A_\alpha \underline{\cap} B_\alpha = \begin{cases} \emptyset, & \mathit{if}\ A_\alpha \cap B_\alpha = \emptyset \\ [\max(x_1, x_2), \min(y_1, y_2)], & \mathit{if}\ A_\alpha \cap B_\alpha \neq \emptyset \end{cases}$$

2.5 Summary

In real-world applications, information is often imprecise or uncertain. Many sources can contribute to the imprecision and uncertainty of data or information. It is particular true in the knowledge representation and reasoning in the Semantic Web as well as applications using Semantic Web techniques such as ontologies, Description Logics, and rules. In particular, many of the existing approaches dealing with imprecision and uncertainty are based on the theory of fuzzy sets and possibility theory. Zadeh's fuzzy set theory has been identified as a successful technique for modelling the fuzzy information in many application areas. For example, as we have known, fuzzy set theory has been extensively applied to extend various database models and resulted in numerous contributions as will be introduced in the next Chap. 3.

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