

# One-Shot Decision Theory: A Fundamental Alternative for Decision Under Uncertainty

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**Abstract** The attempts of this paper are as follows: clarifying the fundamental differences between the one-shot decision theory which was initially proposed in the paper [16] and other decision theories under uncertainty to highlight that the one-shot decision theory is a scenario-based decision theory instead of a lottery-based one; pointing out the instinct problems in other decision theories to show that the one-shot decision theory is necessary to solve one-shot decision problems; manifesting the relation between the one-shot decision theory and the probabilistic decision methods. As regret is a common psychological experience in one-shot decision making, we propose the one-shot decision methods with regret in this paper.

**Keywords** Decision making · One-shot decision · Regret · Regret focus points · Scenario-based decision theory · Human-centric decision-making · Behavioral operations research

## 1 Introduction

In many decision problems encountered in practice, a decision maker has one and only chance to make a decision under uncertainty. Such decision problems are called one-shot decision problems. Let us begin with several real examples to show the features of one-shot decision problems. An article in NIKKAN SPORTS (10-28-2005) stated that Hanshin Electric Railway Co., Ltd., which owns Hansin Tiger baseball team, lost nearly 500 thousand dollars because Hansin Tiger was beaten by Chiba Lotte Marines in Japanese National Baseball Championship in 2005. The huge loss resulted from the production cost of commemorative goods. The Hanshin

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Electric Railway Co., Ltd. had one and only one chance to make a decision whether to prepare the commemorative goods and decide how many goods to be produced before the final result of the game was known. Another example is the Great Sichuan Earthquake that occurred at 14:28:01 CST on May 12, 2008. Official figures stated that 69,197 people were confirmed dead. Amongst many serious problems caused by the earthquake, Tangjiashan Lake particularly drew the attention of the world because it was seriously threatening the lives of 1300,000 people, Lanchengyu Oil Pipeline and one of the arterial railways in China, Chengbao Railway. To prevent damage to the dam, the water in the lake needed to be drained away as soon as possible by building a sluice channel. There were only two alternatives for building a sluice channel, using explosives or digging by excavators. It was a one-shot decision to decide which method should be utilized in the face of the uncertainties from rain, aftershock, dam stability, land slip and time.

Quoting from King ([23], p. 102) “There is a strong basis for the belief that the decision to outsource-particularly offshore-is a “one-time and-never-return” decision because the loss of capability by the client in activities that are outsourced is well known and the cost of re-creating those capabilities may be prohibitive.” Clemen and Kwit ([7], p. 74) stated that “Because of the one-time nature of typical decision-analysis projects, organizations often have difficulty identifying and documenting their value. Based on Eastman Kodak Company’s records for 1990 to 1999, we estimated that decision analysis contributed around a billion dollars to the organization over this time.” Fine [11] emphasized that technological innovation and competitive intensity have been acting as two major drivers to speed up the rates of evolution i.e. “industry clock speeds”, with regard to the product, the process, and the organization of each industry. Accelerated industry clock speed makes one-shot decision problem highly relevant. Lastly, the growing dominance of service industries makes one-shot decision problems especially applicable.

It can be seen that one-shot decision is a kind of irreversible action for problems with partially known information. Such decision problems are commonly encountered in business, social systems and economics.

Guo [16] proposed the one-shot decision theory (OSDT) for solving one-shot decision problems. In OSDT, we argue that a person makes a one-shot decision based on some particular scenario which is regarded as the most appropriate one for him/her while considering the satisfaction level incurred by this scenario and its possibility degree. The one-shot decision process involves two steps. The first step is to identify which state of nature should be taken into account for each alternative. The identified state of nature is called focus point. The second step is to evaluate the alternatives based on the outcomes brought by the focus points to obtain the optimal alternative. As an application, a duopoly market of a new product with a short life cycle is analyzed where three kinds of firms, i.e. normal, active and passive firms are considered. Possibilistic Cournot equilibriums are obtained for different kinds of pairs of firms in a duopoly market. The results of analysis are quite in agreement with the situations encountered in the real business world [14]. Private real estate investment is a typical one-shot decision problem for personal investors due to the huge investment expense and the fear of substantial loss.

In Guo [15], private real estate investment problem is analyzed using one-shot decision framework. The analysis demonstrates the relation between the amount of uncertainty and the investment scale for different types of personal investors. The proposed model provides insights into personal real estate investment decisions and important policy implications in regulating urban land development.

In this research, we attempt to clarify the fundamental differences between OSDT and other decision theories under uncertainty, types of instinct problems in other decision theories that make OSDT necessary to solve one-shot decision problems, and the kind of relation that OSDT holds with the probabilistic decision methods. Realizing that regret is a common psychological experience in one-shot decision making, we propose the one-shot decision methods with regret in this paper.

The remainder of the paper is organized as follows. In Sect. 2, we address the fundamental differences between OSDT and other decision theories under uncertainty and why OSDT is necessary to solve certain types of problems. In Sect. 3, the one-shot decision methods with regret are proposed. In Sect. 4, a numerical example of a newsvendor problem is addressed. Finally, the relationship between OSDT and other decision theories under uncertainty is clarified and the future research directions are provided in Sect. 5.

## 2 The Need for the One-Shot Decision Theory

### *2.1 The Same Framework of Weighting Average for the Existing Decision Theories Under Uncertainty*

In general, before taking an action a decision maker cannot know which outcome will occur. Such unknown situations can be divided into three categories: risk, uncertainty and ignorance. According to Knight [24], risk involves situations where the probabilities of all possible outcomes can be exactly calculated whereas uncertainty is related to the status when exact probabilities cannot be obtained due to inadequate information. Ignorance occurs when no information is available to distinguish which outcome is more likely to occur.

Different unknown situations require different decision theories. Decision rules for situations involving ignorance include maximin, maximax, minmax regret and Hurwicz criterion. The expected utility (EU) theory of Von Neumann and Morgenstern is appropriate for decision making under risk and the subjective expected utility (SEU) theory of Savage is appropriate for decision making under uncertainty where subjective probabilities are used to reflect an individual's belief. There is evidence that people systematically violate EU theory while making decisions [21, 25]. Most criticism of the Von Neumann-Morgenstern's and Savage's axioms mainly focus on independence axiom or sure thing principle [1, 10], transitivity axiom [26] and completeness axiom [5, 30].

Let us discuss the completeness axiom. Quoting Von Neumann and Morgenstern ([33], p. 17) “Let us for the moment accept the picture of an individual whose system of preferences is all-embracing and complete, i.e. who, for any two objects or rather for any two imagined events, possesses a clear intuition of preference. More precisely we expect him, for any two alternative events which are put before him as possibilities, to be able to tell which of the two he prefers.” In fact, in the real world the decision maker does not have the capability to distinguish which alternative is better so that he/she asks a decision analyst to help solving the problem. Nevertheless, with the assumption that the completeness axiom holds for the decision maker the decision analyst builds decision models based on (subjective) expected utility theory. Obviously, it is logically inconsistent. It is natural to raise the questions: who is the protagonist? Is it the decision maker or the decision analyst?

Many theories have been proposed to react to such empirical evidence that human behavior often contradicts expected utility theory. One such theory, i.e. prospect theory developed by Kahneman and Tversky [20] is a non-additive probability model. In prospect theory, value is assigned to gains and losses based on a reference point rather than to the final asset as in EU and SEU. Also, probabilities are replaced by decision weights which do not satisfy the probability additivity. The value function is defined on deviations from a reference point. Value functions are normally concave for gains (implying risk aversion), and convex for losses (implying risk seeking). Regret theory [26] uses modified utility of choosing one alternative instead of another which consists of a choiceless utility and a regret-rejoice function.

Other models such as, second-order probabilities models [19, 29] and non-additive probability models [13, 28] have also been proposed in this empirical challenge. It should be noted that these decision theories follow the same framework of weighting average of all outcomes no matter how they revise their models. In the context of fuzzy decision making, Yager [35] proposed the optimistic utility and Whalen ([34]) gave the pessimistic utility. These two utilities were axiomatized in the style of Savage by Dubois et al. [9]. Giang and Shenoy [12] generalized them by introducing an order on a class of canonical lotteries. In fact, the optimistic utility is a sort of a weighted average where multiplication and addition is replaced by T-norm, min and Co-norm, max, respectively. The pessimistic utility is a counterpart of the optimistic utility in the sense of possibility and necessity measures. Brandstatter et al. [6] proposed the priority heuristic where the lotteries are chosen by lexicographic rules for the four reasons, i.e. minimum gain, maximum gain and their respective probabilities. Katsikopoulos and Gigerenzer [22] showed that the priority heuristic can predict human decision-making better than the most popular modifications of utility theory, such as cumulative prospect theory, and is, in this sense, close to human psychology.

It can be concluded that decision theories under uncertainty are theories of choice under uncertainty where the objects of choice are lotteries. In light of the features of the one-shot decision problem, this raises two problems: Is the probability distribution suitable for characterizing the uncertainty? Is the expected utility a reasonable index for evaluating the performance of a one-shot decision? The answers are given in the following two subsections.

## 2.2 *Is the Probability Distribution Suitable for Characterizing the Uncertainty in the One-Shot Decision Problem?*

In general, the one-shot decision problem involves the situation that has seldom or never happened so far so that the decision maker can not obtain the objective probability distribution. Subjective probability enters as a means of describing the belief about how likely a particular event is to occur. Mainly, there are two kinds of approaches for obtaining subjective probabilities, i.e. the lottery method and the exchangeability method. The lottery method determines the subjective probability of an event in terms of simple betting odds [27]. The exchangeability method consists in the subsequent splitting of the state space into equally likely events via binary choices between binary prospects. Baillon [2] argued that subjective probabilities elicited by the exchangeability method might violate the additivity. The lottery method also was examined by the following experiment.

### **Subjects:**

Fifty subjects participated in the experiment conducted on May 25, 2011. All the participants were undergraduate students who took the course of Decision Sciences, at Faculty of Business Administration, Yokohama National University. The experiment started at the beginning of the lesson. None of them were aware of the true goal of the experiment. The experiment was conducted before teaching them what the subjective probability and the additivity of probability measure are.

### **Procedure:**

The following questions are independently asked:

Q1: If it rains next Wednesday, you will get 10,000Yen. However, if it does not rain, you will get nothing. How much would you be willing to pay for this proposition?

Q2: If it does not rain next Wednesday, you will get 10,000Yen. However, if it rains, you will get nothing. How much would you be willing to pay for this proposition?

### **Results:**

The prices for Q1 and Q2 are denoted as  $X_1$  and  $X_2$ , respectively. The mean of  $X_1 + X_2$  is 4773.98 and the standard deviation of  $X_1 + X_2$  is 2858.73. If the additivity property holds, then the mean of  $X_1 + X_2$  should be 10,000. We set up the null hypothesis: the mean of  $X_1 + X_2$  is 10,000. The value of the test statistic is calculated as  $-12.93$  so that this null hypothesis is rejected at the 0.01 level of significance by two-tail test. It means that the additivity can not always be guaranteed while using the lottery method.

Possibility is an alternative for characterizing the uncertain situation. It can be explained from three semantic aspects, i.e. ease of achievement, plausibility referring to the propensity of events to occur (which relates to the concept “potential surprise”) and logical consistency of available information. Possibility distribution is a function whose value shows the degree to which an element is to occur, as defined as follows.

**Definition 1** Given a function  $\pi : S \rightarrow [0, 1]$  if  $\max_{x \in S} \pi(x) = 1$ , then  $\pi(x)$  is called a possibility distribution where  $S$  is the sample space.  $\pi(x)$  is the possibility degree of  $x$ .

$\pi(x) = 1$  means that it is normal that  $x$  occurs and  $\pi(x) = 0$  means that it is abnormal that  $x$  occurs. The smaller the possibility degree of  $x$ , the more surprising the occurrence of  $x$ . Obtaining the possibility distribution always poses a fundamental problem for decision with possibilistic information. Guo and Tanaka [17] proposed the method for identifying the possibility distribution of the stock returns with the idea of similarity. Guo et al. [18] obtained the possibility distribution of the demand for a new product with the idea of potential surprise. Guo [16] presented a general method for identifying the possibility distribution by voting described as follows:

Suppose  $S = \{x_1, x_2, \dots, x_n\}$ . We ask multiple experts to select the most possible events from  $S$ . In other words, if an expert selects the event  $x_i$ , the expert will not be surprised by its occurrence. The number of experts who select  $x_i$  is denoted as  $k_i$ . Setting  $K = \max_{i=1, \dots, n} k_i$ , the possibility degree of  $x_i$  is obtained as  $k_i / K$  in the sense that each expert has equal reliability for judging which event will occur.

It is a valid question to ask which is better, probability or possibility. To answer this question, let us take a look at the following example.

**Example 1** [15] Who is guilty?

A car has been destroyed by somebody in a parking lot. After careful investigation, it is sure that one and only one of three suspects  $A$ ,  $B$  and  $C$  must be guilty of the crime. However, who is guilty of the crime is still unknown. Suppose, based on the currently obtained evidence subjective probabilities are used to characterize the belief about who is guilty amongst the three suspects and given as e.g.  $P(A) = 0.4$ ,  $P(B) = 0.4$  and  $P(C) = 0.2$ . Considering the relation  $P(A) = 1 - P(\bar{A})$  where  $\bar{A}$  is the complement of  $A$ , it can be concluded that none of these three suspects is guilty in the context of probability ( $P(A) < P(\bar{A})$ ,  $P(B) < P(\bar{B})$ ,  $P(C) < P(\bar{C})$ ). This conclusion is in conflict with the antecedent one, i.e. one and only one of three suspects  $A$ ,  $B$  and  $C$  must be guilty. This conflict originates from the existence of incomplete information. In this example, the possibility distributions showing the degrees to which a person is guilty might be given as e.g.  $\pi(A) = 1$ ,  $\pi(B) = 1$  and  $\pi(C) = 0.7$ .  $\pi(A) = \pi(B) = 1$  means that based on the obtained evidence,  $A$  or  $B$  is most possible to be guilty. The relation  $\pi(A) \neq 1 - \pi(\bar{A})$  implies that the possibility degree of  $A$  being guilty does not provide any information on  $A$  not being guilty.

It follows from this example that the possibility distribution is a less restricted framework than single probability measures and hence can be used for encoding ill-known subjective probability information. The answer to the question which is better, probability or possibility is that the possibility distribution might be effective for representing the rough knowledge or judgment of human being when the information is not rich enough.

### ***2.3 Is the Expected Value a Reasonable Index for Evaluating the Performance of a One-Shot Decision?***

To answer this question, let us consider the following example.

**Example 2** Is Mr. Smith is taller than Mr. Tanaka?

Let us consider two populations:

Population A: The heights of male undergraduate students in Yokohama National University (YNU)

Population B: The heights of male undergraduate students in University of Alberta (UA)

For instance, we take 100 samples from the populations A and B, respectively. The sample mean of A, say 175cm is less than the sample mean of B, say 180cm. You randomly select one male undergraduate student from UA, say Mr. Smith and select one from YNU, say Mr. Tanaka. Can you say Mr. Smith is taller than Mr. Tanaka? The answer will be “no” because the statistical property by itself does not imply anything about what might happen in just one sample. Next, let us take into account two other populations as follows:

Population I: The outcomes generated by an alternative C

Population II: The outcomes generated by an alternative D

Suppose that the mean of the population I is larger than the one of II. Randomly select one outcome from I, that is,  $x$ , and one outcome from II, that is,  $y$ . Can you say  $x$  is larger than  $y$ ? Can you say C is better than D? Both of answers will be “no”. From the above examples, it is easy to understand that for the one-shot decision problem the expected value might not be a suitable index for evaluating the performance of an alternative.

In conclusion, a new decision theory is needed to solve one-shot decision problems featured by partially known information and the occurrence of only one outcome. Guo [16] initially proposed the one-shot decision theory (OSDT) which is scenarios-based instead of lotteries-based as in other decision theories under uncertainty. In OSDT, we argue that a person makes a one-shot decision based on some particular scenario which is regarded as the most appropriate one for him/her while considering the satisfaction level incurred by this scenario and its possibility degree. Because regret is a common emotion in one-shot decision problems, we propose one-shot decision methods with regret in the following section.

## **3 One-Shot Decision Methods with Regret**

Some people find decision making under uncertainty difficult because they fear making the “wrong decision”, wrong in the sense that the outcome of their chosen alternative proves to be worse than could have been achieved with another alternative ([3], p. 1156). This kind of situation can be described by the word “regret” which is “the painful sensation of recognizing that ‘what is’ compares unfavorably with

‘what might have been’” ([32], p. 77). Shimanoff pointed out that regret was the most frequently named negative emotion in a study of verbal expressions of emotions in everyday conversation [31]. Decision with regret has been researched by Savage [27], Loomes and Sugden [26], Bell [3], Sugden [32] and so on. In one-shot decision problems, the decision maker has one and only opportunity to make a decision so that there is no chance to correct his/her decisions once the decision has been made. Hence, regret emotion is an especially important factor that affects the decision maker’s behavior.

### 3.1 Regret Function

Denote the set of an alternative  $a$  as  $A$  and the set of a state of nature  $x$  as  $S$ . The degree to which a state of nature is to occur in the future is characterized by a possibility distribution  $\pi(x)$  defined by the definition 1. The consequence resulting from the combination of an alternative  $a$  and a state of nature  $x$  is refereed to as a payoff, denoted as  $v(x, a)$ . Suppose that after a decision maker chooses an alternative  $a$ , a state of nature  $x$  appears. The decision maker might regret his/her choice. The regret value is  $p(x, a) = \max_{b \in A} v(x, b) - v(x, a)$ . Then the regret quantile denoted as  $w(x, a)$ , is calculated as follows:

$$w(x, a) = p(x, a) / \max_{d \in A} p(x, d). \quad (1)$$

The regret level of a decision maker for a regret quantile can be expressed by a regret function, as defined below.

**Definition 2** Denote the set of a regret quantile  $w(x, a)$  as  $W$ . The following function

$$r : W \rightarrow [0, 1] \quad (2)$$

with

$$r(w_1) > r(w_2) \quad \text{for } w_1 > w_2, \quad (3)$$

is called a regret function. Because the regret quantile is the function of  $x$  and  $a$ , we can rewrite the regret function as  $r(w(x, a))$ . For the sake of simplification, we write  $r(w(x, a))$  as  $r(x, a)$  in this paper. Regret function is a nonlinear transformation of the regret quantile and represents the relative position of the regret.

The information for one-shot decision with regret can be summarized as a quadruple  $(A, S, \pi, r)$ . One-shot decision is to choose one alternative based on  $(A, S, \pi, r)$  when only one decision chance is given.

It is well recognized that when you ask some person why he/she makes such a one-shot decision with little information, he/she always tells you just one scenario which is crucial to him/her and is the basis for achieving some conclusion. For



instance, empirical evidence suggests that insurance buyers focus on the potential large loss even at the low probabilities; lottery ticket buyers focus on the big gains even at small probabilities [8]. Interestingly, Bertrand and Schoar [4] found out that financial decision depended not just on the nature of the firm and its economic environment, but also the personalities of the firm's top management. For instance, while older CEOs tended to be more conservative and pushed their firms towards lower debt, CEOs with MBA degrees tended to be more aggressive.

For the one-shot decision methods with regret, we think that a person make a one-shot decision based on some particular scenario while considering the possibility degree and the regret level. Selecting the scenario depends on the personalities of the decision maker for example one person may be active whereas another may be passive. The one-shot decision making procedure consists of the following three steps. In Step 1, a decision maker identifies some state of nature (particular scenario), called regret focus point for each alternative according to his/her own characteristic. In Step 2, the validity of the regret focus points is checked. In Step 3, the decision maker evaluates the alternatives based on the regret level brought by regret focus point to obtain the best alternative. These three steps are addressed in detail in the following subsections.

### ***3.2 Identifying Regret Focus Points***

Since one and only one state of nature will come up for a one-shot decision problem, a decision maker needs to decide which state of nature ought to be considered for making a one-shot decision. Each state of nature is equipped with a pair of possibility and regret so that how to determine the states of nature depends on his/her attitudes about possibility and regret. The selected state of nature is call regret focus point. Twelve types of regret focus points are provided to help a decision maker in finding out his/her own appropriate one. The characteristics of these focus points are depicted below (shown in Tables 1, 2, 3). Type I and II regret focus points are the states of nature that have the highest and the lowest regret levels, respectively, amongst the ones that have high possibility degrees. Type III and IV regret focus points are the states of nature that have the highest and the lowest regret levels, respectively, amongst the ones that have low possibility degrees. Type V and VI regret focus points are the states of nature that have the highest and the lowest possibility degrees, respectively, amongst the ones that have high regret levels. Type VII and VIII regret focus points are the states of nature that have the highest and lowest possibility degrees, respectively, amongst the ones that have low regret levels. Type IX regret focus point is the state of nature with the higher possibility degree and the higher regret level. Type X regret focus point is the state of nature that has the lower possibility degree and the lower regret level. Type XI regret focus point is the state of nature with the higher possibility degree but the lower regret level. Type XII regret focus point is the state of nature that has the lower possibility degree but the higher regret level.

**Table 1** The characteristics of regret focus points (types I–IV)

|                    | High possibility           | Low possibility             |
|--------------------|----------------------------|-----------------------------|
| The highest regret | Type I regret focus point  | Type III regret focus point |
| The lowest regret  | Type II regret focus point | Type IV regret focus point  |

**Table 2** The characteristics of regret focus points (types V–VIII)

|                         | High regret                | Low regret                   |
|-------------------------|----------------------------|------------------------------|
| The highest possibility | Type V regret focus point  | Type VII regret focus point  |
| The lowest possibility  | Type VI regret focus point | Type VIII regret focus point |

**Table 3** The characteristics of regret focus points (types IX–XII)

|                    | Higher regret               | Lower regret               |
|--------------------|-----------------------------|----------------------------|
| Higher possibility | Type IX regret focus point  | Type XI regret focus point |
| Lower possibility  | Type XII regret focus point | Type X regret focus point  |

In the following we will provide mathematical formulas to find out the above mentioned twelve types of regret focus points. For establishing the focus points, we use the operators

$$\min[b_1, b_2, \dots, b_n] = [\bigwedge_{i=1, \dots, n} b_i, \bigwedge_{i=1, \dots, n} b_i, \dots, \bigwedge_{i=1, \dots, n} b_i], \quad (4)$$

and

$$\max[b_1, b_2, \dots, b_n] = [\bigvee_{i=1, \dots, n} b_i, \bigvee_{i=1, \dots, n} b_i, \dots, \bigvee_{i=1, \dots, n} b_i]. \quad (5)$$

$\min[b_1, b_2, \dots, b_n]$  and  $\max[b_1, b_2, \dots, b_n]$  are lower and upper bounds of  $[b_1, b_2, \dots, b_n]$ , respectively. For example,  $\min[0.3, 0.8] = [0.3, 0.3]$  and  $\max[0.3, 0.8] = [0.8, 0.8]$ . Twelve kinds of regret focus points are as follows:

Type I:  $x_\alpha^{1*}(a) = \arg \max_{x \in X^{\geq \alpha}} r(x, a)$  where  $X^{\geq \alpha} = \{x | \pi(x) \geq \alpha\}$ .

The given parameter  $\alpha$  is a level used to distinguish whether the possibility degree is evaluated as ‘high’ by a decision maker. If  $\alpha = 1$  then only the normal case ( $\pi(x) = 1$ ) is considered. The states of nature belonging to  $X^{\geq \alpha} = \{x | \pi(x) \geq \alpha\}$  are regarded as having the equivalent possibility to occur.  $x_\alpha^{1*}(a)$  is a state of nature with high occurrence possibility. Once it occurs, the decision maker will most regret his/her choice of the alternative  $a$ .  $x_\alpha^{1*}(a)$  is Type I regret focus point.

Type II:  $x_\alpha^{2*}(a) = \arg \min_{x \in X^{\geq \alpha}} r(x, a)$  where  $X^{\geq \alpha} = \{x | \pi(x) \geq \alpha\}$ .

$x_\alpha^{2*}(a)$  is a state of nature with high occurrence possibility. Its occurrence will lead to the lowest regret level of the decision maker for choosing the alternative  $a$ .  $x_\alpha^{2*}(a)$  is Type II regret focus point.

Type III:  $x_\alpha^{3*}(a) = \arg \max_{x \in X^{\leq \alpha}} r(x, a)$  where  $X^{\leq \alpha} = \{x | \pi(x) \leq \alpha\}$ .

The occurrence of  $x_\alpha^{3*}(a)$  will make the decision maker most regret his/her choice of the alternative  $a$ . However, the possibility of its occurrence is low.  $x_\alpha^{3*}(a)$  is Type III regret focus point.

Type IV:  $x_\alpha^{4*}(a) = \arg \min_{x \in X^{\leq \alpha}} r(x, a)$  where  $X^{\leq \alpha} = \{x | \pi(x) \leq \alpha\}$ .

The occurrence of  $x_\alpha^{4*}(a)$  will make the decision maker have the lowest regret level for choosing the alternative  $a$ . However, the possibility of its occurrence is low.  $x_\alpha^{4*}(a)$  is Type IV regret focus point.

Type V:  $x_\beta^{5*}(a) = \arg \max_{x \in X^{\geq \beta}(a)} \pi(x)$  where  $X^{\geq \beta}(a) = \{x | r(x, a) \geq \beta\}$ .

The given parameter  $\beta$  is the level to distinguish whether the regret level is evaluated as ‘high’ by a decision maker. The states of nature belonging to  $X^{\geq \beta}(a) = \{x | r(x, a) \geq \beta\}$  are regarded as having the same regret level generated by the alternative  $a$ .  $x_\beta^{5*}(a)$  is an undesirable (the regret level is high) state of nature that has the highest possibility to occur.  $x_\beta^{5*}(a)$  is Type V regret focus point.

Type VI:  $x_\beta^{6*}(a) = \arg \min_{x \in X^{\geq \beta}(a)} \pi(x)$  where  $X^{\geq \beta}(a) = \{x | r(x, a) \geq \beta\}$ ,

which called Type VI regret focus point, is an undesirable state of nature that has the smallest possibility to occur.

Type VII:  $x_\beta^{7*}(a) = \arg \max_{x \in X^{\leq \beta}(a)} \pi(x)$  where  $X^{\leq \beta}(a) = \{x | r(x, a) \leq \beta\}$ ,

which called Type VII regret focus point, is a desirable (the regret level is low) state of nature that has the highest possibility to occur.

Type VIII:  $x_\beta^{8*}(a) = \arg \min_{x \in X^{\leq \beta}(a)} \pi(x)$  where  $X^{\leq \beta}(a) = \{x | r(x, a) \leq \beta\}$ ,

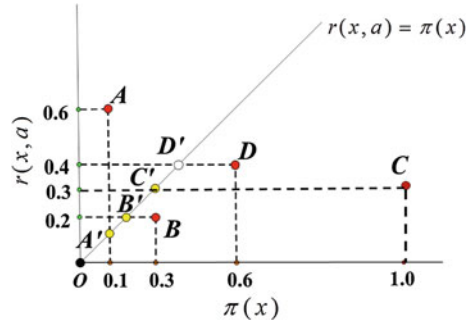
which called Type VIII regret focus point, is a desirable state of nature that has the smallest possibility to occur.

Type IX:

$$x^{9*}(a) = \arg \max_{x \in S} \min[\pi(x), r(x, a)]. \quad (6)$$

It follows from (6) that  $x = x^{9*}(a)$  maximizes  $g(x, a) = \min[\pi(x), r(x, a)]$ . In consideration of (4), we know that  $\min[\pi(x), r(x, a)]$  represents the lower bound of the vector  $[\pi(x), r(x, a)]$ . Increasing  $\min[\pi(x), r(x, a)]$  ( $\max_{x \in S} \min[\pi(x), r(x, a)]$ ) will increase the possibility degree and the regret level simultaneously. Therefore,  $\arg \max_{x \in S} \min[\pi(x), r(x, a)]$  is for seeking a state of nature that has the higher possibility degree and brings the higher regret level due to the choice of the alternative

**Fig. 1** The explanation of the formula (6)



$a$ .  $x^{9*}(a)$  is Type IX regret focus point. For easily understanding (6), let us have a look at Fig. 1. There are four states of nature  $x_1, x_2, x_3$  and  $x_4$  whose  $[\pi(x), r(x, a)]$  are respectively  $[0.1, 0.6]$ ,  $[0.3, 0.2]$ ,  $[1.0, 0.3]$  and  $[0.6, 0.4]$  represented by  $A, B, C$  and  $D$ .  $\min[\pi(x), r(x, a)]$  transfers  $A, B, C$  and  $D$  into  $A', B', C'$  and  $D'$ , which are  $[0.1, 0.1]$ ,  $[0.2, 0.2]$ ,  $[0.3, 0.3]$  and  $[0.4, 0.4]$ , respectively.  $\max_{x \in S} \min[\pi(x), r(x, a)]$ , that is,  $\max([0.1, 0.1], [0.2, 0.2], [0.3, 0.3], [0.4, 0.4]) = [0.4, 0.4]$  corresponds to  $D'$ .  $\arg \max_{x \in S} \min[\pi(x), r(x, a)]$  chooses  $x_4$ . It follows from Fig. 1 that  $x_4$  is a state of nature with a higher possibility degree and a higher regret level.

Type X:

$$x^{10*}(a) = \arg \min_{x \in S} \max[\pi(x), r(x, a)]. \quad (7)$$

(7) shows that  $x = x^{10*}(a)$  minimizes  $h(x, a) = \max[\pi(x), r(x, a)]$ . In consideration of (5), we know that  $\max[\pi(x), r(x, a)]$  represents the upper bound of the vector  $[\pi(x), r(x, a)]$ . Decreasing  $\max[\pi(x), r(x, a)]$  ( $\min \max_{x \in S} [\pi(x), r(x, a)]$ ) will decrease the possibility degree and the regret level simultaneously. Therefore,  $\arg \min_{x \in S} \max[\pi(x), r(x, a)]$  is for seeking a state of nature that has the lower possibility degree and generates the lower regret level due to the choice of the alternative  $a$ .  $x^{10*}(a)$  is Type X regret focus point.

Type XI:

$$x^{11*}(a) = \arg \min_{x \in S} \max[1 - \pi(x), r(x, a)]. \quad (8)$$

Likewise, we understand that  $x^{11*}(a)$  is the state of nature that has the higher possibility degree and causes the lower regret level when choosing the alternative  $a$ .  $x^{11*}(a)$  is Type XI regret focus point.

Type XII:

$$x^{12*}(a) = \arg \min_{x \in S} \max[\pi(x), 1 - r(x, a)]. \quad (9)$$

Following (9), we know that  $x^{12*}(a)$  is the state of nature that has the lower possibility degree and incurs the higher regret level when choosing the alternative  $a$ .  $x^{12*}(a)$  is Type XII regret focus point.

For one alternative, more than one state of nature might exist as one type of regret focus point. We denote the sets of twelve types of regret focus points of the alternative  $a$  as  $X_\alpha^1(a)$ ,  $X_\alpha^2(a)$ ,  $X_\alpha^3(a)$ ,  $X_\alpha^4(a)$ ,  $X_\beta^5(a)$ ,  $X_\beta^6(a)$ ,  $X_\beta^7(a)$ ,  $X_\beta^8(a)$ ,  $X^9(a)$ ,  $X^{10}(a)$ ,  $X^{11}(a)$ , and  $X^{12}(a)$ , respectively. It should be noted that  $X_\alpha^3(a)$  and  $X_\alpha^4(a)$  are empty sets when  $X^{\leq\alpha} = \emptyset$ ;  $X_\beta^5(a)$  and  $X_\beta^6(a)$  are empty sets when  $X^{\geq\beta}(a) = \emptyset$ ;  $X_\beta^7(a)$  and  $X_\beta^8(a)$  are empty sets when  $X^{\leq\beta}(a) = \emptyset$ . The relationships between different focus points are shown in the following theorem.

**Theorem 1**

$$(I) \quad X_\alpha^1(a) \cup X_\beta^5(a) \subseteq X^9(a), \quad (10)$$

where

$$\alpha = \beta = \max_{x \in S} \min(\pi(x), r(x, a)). \quad (11)$$

$$(II) \quad X_\alpha^4(a) \cup X_\beta^8(a) \subseteq X^{10}(a), \quad (12)$$

where

$$\alpha = \beta = \min_{x \in S} \max(\pi(x), r(x, a)). \quad (13)$$

$$(III) \quad X_\alpha^2(a) \cup X_\beta^7(a) \subseteq X^{11}(a), \quad (14)$$

where

$$\alpha = 1 - \beta = \max_{x \in S} \min(\pi(x), 1 - r(x, a)). \quad (15)$$

$$(IV) \quad X_\alpha^3(a) \cup X_\beta^6(a) \subseteq X^{12}(a), \quad (16)$$

where

$$1 - \alpha = \beta = \max_{x \in S} \min(1 - \pi(x), r(x, a)). \quad (17)$$

**Proof** The proof is similar to the proof of Theorem 1 in the paper [16].

Theorem 1 shows the relationships between the different types of regret focus points. The inclusion relations (10), (12), (14) and (16) hold by choosing the suitable values of parameters  $\alpha$  and  $\beta$  shown in (11), (13), (15) and (17). Expressed in detail, the set of regret focus points with the higher regret and the higher possibility ( $X^9(a)$ ) includes the set of regret focus points with the highest regret and the high possibility ( $X_\alpha^1(a)$ ) and the set of regret focus points with the highest possibility and the high regret ( $X_\beta^5(a)$ ). The set of regret focus points with the lower regret and the lower possibility ( $X^{10}(a)$ ) includes the set of regret focus points with the lowest regret and the low possibility ( $X_\alpha^4(a)$ ) and the set of regret focus points with the lowest

possibility and the low regret ( $X_\beta^8(a)$ ). The set of focus points with the lower regret and the higher possibility ( $X^{11}(a)$ ) includes the set of regret focus points with the lowest regret and the high possibility ( $X_\alpha^2(a)$ ) and the set of regret focus points with the highest possibility and the low regret ( $X_\beta^7(a)$ ). The set of regret focus points with the higher regret and the lower possibility ( $X^{12}(a)$ ) include the set of regret focus points with the highest regret and the low possibility ( $X_\alpha^3(a)$ ) and the set of regret focus points with the lowest possibility and the high regret ( $X_\beta^6(a)$ ).

**Comments:** It raises one question how a decision maker would choose among the twelve focus points. The answer is choosing which type focus point completely depends on which kind of the combination of possibility and regret, for example, the higher possibility and the higher regret, is most worth taking into account for his/her making a one-shot decision. It should be decided by the decision maker himself/herself instead of a decision analyst. Sometimes, a decision maker may consider several types or all types of focus points to make a final decision.

### 3.3 Checking the Validity of Regret Focus Points (Type IX, X, XI and XII)

In Step 1, twelve types of regret focus points are identified. These regret focus points will be used for determining the optimal alternative. Before that, the validity of Type IX, X, XI and XII regret focus points needs to be checked.

**Definition 3** Given the thresholds of the possibility degree  $\alpha$  and the regret level  $\beta$ , we say that  $x^{9*}(a)$ ,  $x^{10*}(a)$ ,  $x^{11*}(a)$  and  $x^{12*}(a)$  are acceptable for  $\alpha$  and  $\beta$  if  $x^{9*}(a) \in X^{\geq\alpha} \cap X^{\geq\beta}(a)$ ,  $x^{10*}(a) \in X^{\leq\alpha} \cap X^{\leq\beta}(a)$ ,  $x^{11*}(a) \in X^{\geq\alpha} \cap X^{\leq\beta}(a)$  and  $x^{12*}(a) \in X^{\leq\alpha} \cap X^{\geq\beta}(a)$  hold, respectively.

We denote the sets of Type IX, X, XI and XII acceptable regret focus points for  $\alpha$  and  $\beta$  as  $X_{\alpha,\beta}^9(a)$ ,  $X_{\alpha,\beta}^{10}(a)$ ,  $X_{\alpha,\beta}^{11}(a)$  and  $X_{\alpha,\beta}^{12}(a)$ , respectively. For easily understanding the definitions 3, let us consider the following example.

**Example 3** The sets of alternatives and states of nature are  $A = \{a_1, a_2\}$  and  $S = \{x_1, x_2\}$ , respectively. For illustrative purposes, let us assume that the estimated possibility degrees of states of nature and the regret levels for two alternatives on each state of nature are shown in Table 4. We set  $\alpha$  and  $\beta$ , e.g. as 0.5 and 0.5, respectively.  $x^{9*}(a_2)$ ,  $x^{10*}(a_2)$ ,  $x^{11*}(a_1)$ , and  $x^{12*}(a_1)$  are not acceptable because  $x^{9*}(a_2) = x_1 \notin X^{\geq\alpha} \cap X^{\geq\beta}(a_2) = \emptyset$ ,  $x^{10*}(a_2) = x_1, x_2 \notin X^{\leq\alpha} \cap X^{\leq\beta}(a_2) = \emptyset$ ,  $x^{11*}(a_1) = x_1 \notin X^{\geq\alpha} \cap X^{\leq\beta}(a_1) = \emptyset$  and  $x^{12*}(a_1) = x_1 \notin X^{\leq\alpha} \cap X^{\geq\beta}(a_1) = \emptyset$  hold. We can always obtain Type IX, X, XI and XII regret focus points by (6), (7), (8) and (9). However, in some cases, they are not intuitively accepted as the states of nature with the higher possibility and the higher regret, the lower possibility and the lower regret, the higher possibility and the lower regret, the lower possibility and the higher regret as shown in this example.

**Table 4** Data of the example 3

|               | $x_1$ | $x_2$ |
|---------------|-------|-------|
| $\pi(x_i)$    | 0.2   | 1     |
| $r(x_i, a_1)$ | 0.3   | 0.85  |
| $r(x_i, a_2)$ | 1     | 0.1   |

### 3.4 Obtaining Optimal Alternatives

A decision maker identifies the valid regret focus points of each alternative according to his/her own attitude about possibility and regret as shown in Sects. 3.2 and 3.3. He/she contemplates that the regret focus points are the most appropriate states of nature (scenarios) for him/her and then chooses the alternative which can bring about the best consequence (the lowest regret level) once the regret focus point (scenario) comes true. The procedure for choosing the optimal alternative with regret focus points are given below. Since there are twelve types of regret focus points, there are twelve types of optimal alternatives.

Type I optimal alternative  $a^{1*}(\alpha)$ :  $a^{1*}(\alpha) = \arg \min_{a \in A} r(x_\alpha^{1*}(a), a)$ .

Type II optimal alternative  $a^{2*}(\alpha)$ :  $a^{2*}(\alpha) = \arg \min_{a \in A} r(x_\alpha^{2*}(a), a)$ .

Type III optimal alternative  $a^{3*}(\alpha)$ : If  $X^{\leq \alpha} = \emptyset$ , then  $a^{3*}(\alpha) \in \emptyset$ ; else  $a^{3*}(\alpha) = \arg \min_{a \in A} r(x_\alpha^{3*}(a), a)$ .

Type IV optimal alternative  $a^{4*}(\alpha)$ : If  $X^{\leq \alpha} = \emptyset$ , then  $a^{4*}(\alpha) \in \emptyset$ ; else  $a^{4*}(\alpha) = \arg \min_{a \in A} r(x_\alpha^{4*}(a), a)$ .

Type V optimal alternative  $a^{5*}(\beta)$ : If  $\forall a X_\beta^5(a) \neq \emptyset$ , then  $a^{5*}(\beta) = \arg \min_{a \in A} \max_{x_\beta^{5*}(a) \in X_\beta^5(a)} r(x_\beta^{5*}(a), a)$ ; if  $\forall a X_\beta^5(a) = \emptyset$ , then  $a^{5*}(\beta) \in \emptyset$ ; else  $a^{5*}(\beta) \in \{a | X_\beta^5(a) = \emptyset\}$ .

The minmax operator is needed for the cases where multiple focus points of an alternative  $a$  exist. It reflects the conservative attitude of a decision maker.

Type VI optimal alternative  $a^{6*}(\beta)$ : If  $\forall a X_\beta^6(a) \neq \emptyset$ , then  $a^{6*}(\beta) = \arg \min_{a \in A} \max_{x_\beta^{6*}(a) \in X_\beta^6(a)} r(x_\beta^{6*}(a), a)$ ; if  $\forall a X_\beta^6(a) = \emptyset$ , then  $a^{6*}(\beta) \in \emptyset$ ; else  $a^{6*}(\beta) \in \{a | X_\beta^6(a) = \emptyset\}$ .

Type VII optimal alternative  $a^{7*}(\beta)$ : If  $\forall a X_\beta^7(a) = \emptyset$ , then  $a^{7*}(\beta) \in \emptyset$ ; else  $a^{7*}(\beta) = \arg \min_{a \in A^-} \max_{x_\beta^{7*}(a) \in X_\beta^7(a)} r(x_\beta^{7*}(a), a)$  where  $A^- = \{a | X_\beta^7(a) \neq \emptyset\}$ .

Type VIII optimal alternative  $a^{8*}(\beta)$ : If  $\forall a X_\beta^8(a) = \emptyset$ , then  $a^{8*}(\beta) \in \emptyset$ ; else  $a^{8*}(\beta) = \arg \min_{a \in A^-} \max_{x_\beta^{8*}(a) \in X_\beta^8(a)} r(x_\beta^{8*}(a), a)$  where  $A^- = \{a | X_\beta^8(a) \neq \emptyset\}$ .

Type IX optimal alternative  $a^{9*}(\alpha, \beta)$ : If  $\forall a X_{\alpha, \beta}^9(a) \neq \emptyset$ , then  $a^{9*}(\alpha, \beta) = \arg \min_{a \in A} \max_{x^{9*}(a) \in X_{\alpha, \beta}^9(a)} r(x^{9*}(a), a)$ ; if  $\forall a X_{\alpha, \beta}^9(a) = \emptyset$ , then  $a^{9*}(\alpha, \beta) \in \emptyset$ ; else  $a^{9*}(\alpha, \beta) \in \{a | X_{\alpha, \beta}^9(a) = \emptyset\}$ .

Type X optimal alternative  $a^{10*}(\alpha, \beta)$ : If  $\forall a X_{\alpha, \beta}^{10}(a) = \emptyset$ , then  $a^{10*}(\alpha, \beta) \in \emptyset$ ; else  $a^{10*}(\alpha, \beta) = \arg \min_{a \in A^-} \min_{x^{10*}(a) \in X_{\alpha, \beta}^{10}(a)} r(x^{10*}(a), a)$  where  $A^- = \{a | X_{\alpha, \beta}^{10}(a) \neq \emptyset\}$ . The minmin operator is used for the cases where multiple focus points of an alternative  $a$  exist. It reflects the aggressive attitude of a decision maker.

Type XI optimal alternative  $a^{11*}(\alpha, \beta)$ : If  $\forall a X_{\alpha, \beta}^{11}(a) = \emptyset$ , then  $a^{11*}(\alpha, \beta) \in \emptyset$ ; else  $a^{11*}(\alpha, \beta) = \arg \min_{a \in A^-} \min_{x^{11*}(a) \in X_{\alpha, \beta}^{11}(a)} r(x^{11*}(a), a)$  where  $A^- = \{a | X_{\alpha, \beta}^{11}(a) \neq \emptyset\}$ .

Type XII optimal alternative  $a^{12*}(\alpha, \beta)$ : If  $\forall a X_{\alpha, \beta}^{12}(a) \neq \emptyset$ , then  $a^{12*}(\alpha, \beta) = \arg \min_{a \in A} \max_{x^{12*}(a) \in X_{\alpha, \beta}^{12}(a)} r(x^{12*}(a), a)$ ; if  $\forall a X_{\alpha, \beta}^{12}(a) = \emptyset$ , then  $a^{12*}(\alpha, \beta) \in \emptyset$ ; else  $a^{12*}(\alpha, \beta) \in \{a | X_{\alpha, \beta}^{12}(a) = \emptyset\}$ .

### Comments:

This research extends the results of the paper [16] in two aspects. The first aspect is that instead of the satisfaction level we utilize the regret level to seek focus points because regret is a common emotion in one-shot decision problems. The second aspect is introducing the step for checking the validity of Type IX, X, XI and XII regret focus points. It should be noted that such a step is also applicable to the focus points with satisfaction levels. We also can define dissatisfaction function and use possibility and dissatisfaction to find out focus points. It is especially appropriate for emergency management problems where the upper and lower bounds of losses correspond to the dissatisfaction levels 1 and 0, respectively.

## 4 Numerical Example: The Newsvendor Problem

In this study, we consider the newsvendor problem for a new product with a short life cycle. As the product is new, there is no data available for forecasting the upcoming demand via statistical analysis. As the life cycle of the product is short, determining optimal order quantity is a typical one-shot decision problem.

The newsvendor problem is described as follows. The retailer orders  $q$  units before the season at the unit wholesale price  $W$ . When the demand  $x$  is observed, the retailer sells goods (limited by the supply  $q$  and the demand  $x$ ) at the unit revenue  $R$  with  $R > W$ . Any excess units can be salvaged at the unit salvage price  $S_o$  with  $W > S_o$ . If there is a shortage, the lost chance price is  $S_u$ . The profit function of the retailer is

$$r(x, q) = \begin{cases} Rx + S_o(q - x) - Wq; & \text{if } x < q \\ (R - W)q - S_u(x - q); & \text{if } x \geq q. \end{cases} \quad (18)$$



**Table 5** Profits obtained for each order quantity

|        |    | Demand |     |     |     |     |     |
|--------|----|--------|-----|-----|-----|-----|-----|
|        |    | 5      | 6   | 7   | 8   | 9   | 10  |
| Orders | 5  | 200    | 180 | 160 | 140 | 120 | 100 |
|        | 6  | 150    | 240 | 220 | 200 | 180 | 160 |
|        | 7  | 100    | 190 | 280 | 260 | 240 | 220 |
|        | 8  | 50     | 140 | 230 | 320 | 300 | 280 |
|        | 9  | 0      | 90  | 180 | 270 | 360 | 340 |
|        | 10 | −50    | 40  | 130 | 220 | 310 | 400 |

**Table 6** Regret levels for each order quantity

|        |    | Demands |      |       |       |       |     |
|--------|----|---------|------|-------|-------|-------|-----|
|        |    | 5       | 6    | 7     | 8     | 9     | 10  |
| Orders | 5  | 0       | 0.3  | 0.8   | 1     | 1     | 1   |
|        | 6  | 0.2     | 0    | 0.4   | 0.667 | 0.75  | 0.8 |
|        | 7  | 0.4     | 0.25 | 0     | 0.333 | 0.5   | 0.6 |
|        | 8  | 0.6     | 0.5  | 0.333 | 0     | 0.25  | 0.4 |
|        | 9  | 0.8     | 0.75 | 0.667 | 0.278 | 0     | 0.2 |
|        | 10 | 1       | 1    | 1     | 0.556 | 0.208 | 0   |

**Table 7** Possibility degrees of demands

| Demands             | 5   | 6   | 7   | 8 | 9   | 10  |
|---------------------|-----|-----|-----|---|-----|-----|
| Possibility degrees | 0.2 | 0.5 | 0.7 | 1 | 0.8 | 0.6 |

The unit wholesale price  $W$ , the unit revenue  $R$ , the unit salvage price  $S_o$ , and the lost chance price  $S_u$  are set, e.g. as 60 \$, 100 \$, 10 \$ and 20 \$, respectively. Following (18), we calculate the profits (see Table 5). Using (1), we obtain the regret quantile for each order and demand. In this example we set  $r(w) = w$ , that is, the regret quantile is the same as the regret level. The regret levels for each order and demand are listed in Table 6.

Let us analyze this one-shot decision problem in the form of  $(A, S, \pi, u)$ . The set of alternatives is the set of order quantities  $A = \{5, 6, 7, 8, 9, 10\}$ . The set of states of nature is the set of demands  $S = \{5, 6, 7, 8, 9, 10\}$ . The regret levels are shown in Table 6. We assume that the possibility degrees of the demands 8, 9, 7, 10, 6, and 5 are 1, 0.8, 0.7, 0.6, 0.5, and 0.2, respectively (shown in Table 7).

The thresholds of possibility degrees and satisfaction levels,  $\alpha$  and  $\beta$ , are set, e.g. as 0.55 and 0.52, respectively. In Step 1, all regret focus points are obtained and listed in Table 8. For avoiding unnecessary repetition, only some results are explained below. Amongst the high possible demands  $\{7, 8, 9, 10\}$ , 8, 9 or 10 makes order 5 most regretful. In other words, any other order will be better than them if demand 5 comes true. As a result, demands 8, 9 and 10 are Type I regret focus point. Demand

**Table 8** Regret focus points of order quantities

|           | Order quantities |    |       |    |    |      |
|-----------|------------------|----|-------|----|----|------|
|           | 5                | 6  | 7     | 8  | 9  | 10   |
| Type I    | 8, 9, 10         | 10 | 10    | 10 | 7  | 7    |
| Type II   | 7                | 7  | 7     | 8  | 9  | 10   |
| Type III  | 6                | 5  | 5     | 5  | 5  | 5, 6 |
| Type IV   | 5                | 6  | 6     | 6  | 6  | 5, 6 |
| Type V    | 8                | 8  | 10    | 5  | 7  | 8    |
| Type VI   | 10               | 10 | 10    | 5  | 5  | 5    |
| Type VII  | 6                | 7  | 8     | 8  | 8  | 9    |
| Type VIII | 5                | 5  | 5     | 6  | 10 | 10   |
| Type IX   | 8                | 9  | 10    | 6  | 7  | 7    |
| Type X    | 5                | 5  | 5     | 6  | 10 | 10   |
| Type XI   | 6                | 7  | 7     | 8  | 9  | 9    |
| Type XII  | 10               | 10 | 5, 10 | 5  | 5  | 5    |

7 can lead to the least regret for order 5 amongst high possible demands so that it is Type II regret focus point. A decision maker might think about the scenarios which have the low possibility to occur. They correspond to Type III and IV regret focus points. Amongst the demands with low possibilities, that is  $\{5, 6\}$ , demand 6 makes order 5 more regrettable than demand 5. Thus, demands 6 and 5 are regarded as Type III and IV regret focus points, respectively. Amongst the demands  $\{7, 8, 9, 10\}$  which can generate the high regret for order 5, demand 8 is Type V regret focus point due to its highest possibility whereas demand 10 is Type VI regret focus point due to its lowest possibility. Amongst the demands  $\{6, 7, 8, 9, 10\}$  which can bring about low regret for order 8, demand 8 is identified as Type VII regret focus point because of its highest possibility whereas demand 6 is chosen as Type VIII regret focus points because of its lowest possibility. Type IX, X, XI and XII regret focus points are obtained according to (6), (7), (8) and (9). The regret levels brought by twelve types of regret focus points for each order quantity are listed in Table 9. In Step 2, let us examine the validity of the obtained Type IX, X, XI and XII regret focus points. Since  $x^{9*}(8) \notin X^{\geq\alpha} \cap X^{\geq\beta}(8)$ ,  $x^{10*}(9) \notin X^{\leq\alpha} \cap X^{\leq\beta}(9)$ ,  $x^{10*}(10) \notin X^{\leq\alpha} \cap X^{\leq\beta}(10)$ ,  $x^{11*}(5) \notin X^{\geq\alpha} \cap X^{\leq\beta}(5)$ ,  $x^{12*}(5) \notin X^{\leq\alpha} \cap X^{\geq\beta}(5)$ ,  $x^{12*}(6) \notin X^{\leq\alpha} \cap X^{\geq\beta}(6)$  and  $x^{12*}(7) \notin X^{\leq\alpha} \cap X^{\geq\beta}(7)$  hold,  $x^{9*}(8)$ ,  $x^{10*}(9)$ ,  $x^{10*}(10)$ ,  $x^{11*}(5)$ ,  $x^{12*}(5)$ ,  $x^{12*}(6)$  and  $x^{12*}(7)$  are not acceptable for  $\alpha = 0.55$  and  $\beta = 0.52$ . The regret levels brought by twelve types of valid regret focus points for each order quantity are listed in Table 10.

In Step 3, the optimal order quantities are selected based on the regret levels of valid regret focus points. The optimal orders are 8,  $\{7, 8, 9, 10\}$ , 6,  $\{5, 6\}$ , 10,  $\{7, 8\}$ , 8,  $\{5, 10\}$ , 8, 5,  $\{7, 8, 9\}$  and  $\{5, 6, 7\}$  which corresponds to Types I to XII regret focus points, respectively. As the retailer sells seasonal goods, there is one and only one chance for him/her to decide how many should be ordered. Hence, considering a reasonable level of demand before determining how many products should be ordered is appropriate for such one-shot decision problems.

**Table 9** Regret levels for regret focus points

|           | Order quantities |       |         |     |       |       |
|-----------|------------------|-------|---------|-----|-------|-------|
|           | 5                | 6     | 7       | 8   | 9     | 10    |
| Type I    | 1,1,1            | 0.8   | 0.6     | 0.4 | 0.667 | 1     |
| Type II   | 0.8              | 0.4   | 0       | 0   | 0     | 0     |
| Type III  | 0.3              | 0.2   | 0.4     | 0.6 | 0.8   | 1,1   |
| Type IV   | 0                | 0     | 0.25    | 0.5 | 0.75  | 1,1   |
| Type V    | 1                | 0.667 | 0.6     | 0.6 | 0.667 | 0.556 |
| Type VI   | 1                | 0.8   | 0.6     | 0.6 | 0.8   | 1     |
| Type VII  | 0.3              | 0.4   | 0.334   | 0   | 0.278 | 0.208 |
| Type VIII | 0                | 0.2   | 0.4     | 0.5 | 0.2   | 0     |
| Type IX   | 1                | 0.75  | 0.6     | 0.5 | 0.667 | 1     |
| Type X    | 0                | 0.2   | 0.4     | 0.5 | 0.2   | 0     |
| Type XI   | 0.3              | 0.4   | 0       | 0   | 0     | 0.208 |
| Type XII  | 1                | 0.8   | 0.4,0.6 | 0.6 | 0.8   | 1     |

**Table 10** Regret levels for valid regret focus points

|          | Order quantities |       |       |     |       |       |
|----------|------------------|-------|-------|-----|-------|-------|
|          | 5                | 6     | 7     | 8   | 9     | 10    |
| TypeI    | 1,1,1            | 0.8   | 0.6   | 0.4 | 0.667 | 1     |
| TypeII   | 0.8              | 0.4   | 0     | 0   | 0     | 0     |
| TypeIII  | 0.3              | 0.2   | 0.4   | 0.6 | 0.8   | 1,1   |
| TypeIV   | 0                | 0     | 0.25  | 0.5 | 0.75  | 1,1   |
| TypeV    | 1                | 0.667 | 0.6   | 0.6 | 0.667 | 0.556 |
| TypeVI   | 1                | 0.8   | 0.6   | 0.6 | 0.8   | 1     |
| TypeVII  | 0.3              | 0.4   | 0.334 | 0   | 0.278 | 0.208 |
| TypeVIII | 0                | 0.2   | 0.4   | 0.5 | 0.2   | 0     |
| TypeIX   | 1                | 0.75  | 0.6   | *   | 0.667 | 1     |
| TypeX    | 0                | 0.2   | 0.4   | 0.5 | *     | *     |
| TypeXI   | *                | 0.4   | 0     | 0   | 0     | 0.208 |
| TypeXII  | *                | *     | *     | 0.6 | 0.8   | 1     |

## 5 Conclusions

The difference between OSDT and the decision based on optimistic and pessimistic utilities have been comprehensively addressed in the paper [14]. It is especially worthy making a detailed comparison between OSDT and SEU as follows:

**Comparison 1:** In SEU, there are two steps:

**Step 1:** Evaluating each alternative by using the weighted average utility of all outcomes;

**Step2:** Selecting the alternative with the maximum average.

In OSDT, there are two steps:

**Step 1:** Scenario (focus point) seeking for each alternative;

**Step 2:** Choosing the alternative with the maximal satisfaction level or minimal regret level of the focus point.

**Comparison 2:** In SEU the utility function is used whereas in OSDT the satisfaction function or regret function is used. Utility function is associated with risky situations. If a person is a risk avoider, the utility function is concave. If a person is a risk taker, the utility function is convex. If a person is risk neutral, the utility function is linear. Satisfaction function or regret function has no relation with risk situations, which just represents the relative position of payoff or regret. In OSDT, taking into account which kind of focus point reflects the attitude of the individual to uncertainty.

**Comparison 3:** SEU uses subjective probability to characterize uncertainty whereas OSDT applies possibility distribution.

**Comparison 4:** SEU amounts to the expected payoff based on the distorted probabilities as follows:

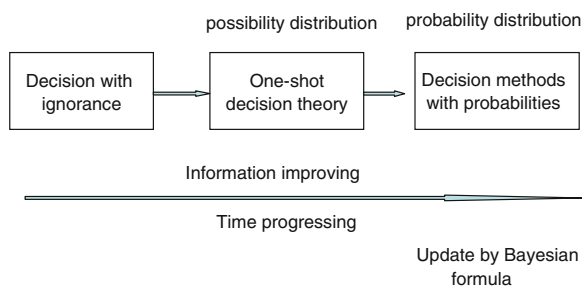
$$EU = \sum p_i u(x_i) = k \sum p'_i x_i,$$

where  $k$  is a positive constant and  $p'_i$  is a distorted probability. The conventional explanation of the optimal decision with SEU is that it can lead to the maximal average utility when the decision is repeated infinite time in the sense of strong law of large numbers. Hence, it is lack of consistency for the one-shot decision cases because the expected value will never appear. On the other hand, OSDT give a clear answer to why the decision maker makes such a decision in the face of uncertainty and why the decision might not generate a satisfactory result after the uncertainty resolving.

In conclusion, OSDT provides a scenario-based choice instead of the lottery-based choices as in other decision theories under uncertainty. Therefore, it is a scenario-based decision theory. OSDT is a fundamental alternative theory for decision under uncertainty with greater appeal to intuition, simplicity of application and explicability. Because it is very close to the human way of thinking, the decision with OSDT is of human-centric decision making. OSDT also provides one of the basic theories for behavioral operations research.

It is pointless to dispute which decision theory is better. There is no simple theory which is appropriate for any decision situation and in this respect the one-shot decision theory is no exception. It is true that different theories play different roles for different decision situations.

The one-shot decision theory is mainly utilized in the situation where a decision is experienced only once and the probability distribution is unavailable due to lack of enough information. However it might play an indispensable role of a bridge in linking decision under ignorance and decision with probabilities (shown in Fig. 2). For a repeatable decision problem, at the beginning, a decision maker has to make a decision under ignorance because the decision situation is completely new for him/her and therefore he/she has no ability to tell the difference between the states of the nature. After the first decision is made based on maximin or maximax or minmax regret or Hurwicz criterion, he/she would has some knowledge about the state of nature so that it is possible to construct an initial possibility distribution of states



**Fig. 2** The role of bridge between decision with ignorance and decision with probabilities

of nature. He/she could make a one-shot decision and repeat such decision with the updated possibility distributions. As time progresses, the information improves. The possibility distribution will switch into a probability distribution when the data is rich enough. The switching criterion is the hypothesis test for the probability distribution. After that decision methods with probability distributions should be utilized with the probabilities updated using Bayesian formula.

Finally, let us give some comments on the case of one-shot decision under risk. In such a case, for example, a game of tossing an ordinary coin, the objective probabilities are exactly known. When making a one-shot decision under risk, we can obtain the possibility distribution by normalizing a probability mass function (for a discrete random variable) or a probability density function (for a continuous random variable) and make a decision with OSDT.

The research on one-shot decision under uncertainty is in its early stages. There is potential for research on theoretical and applied aspects. As a direct extension of this research, multistage one-shot decision problems can be studied. One-shot game theory can be developed and the case studies of international conflict resolutions can be done. Newsvendor problems and supply chain management for innovative products are other interesting and important applications of OSDT. Use of OSDT in behavioral finance problems is another interesting research area. Other decision problems, such as mergers and acquisitions (M&A), emergency management for irregular events such as earthquakes, or nuclear power plant accidents, social policy decision making for environment, energy, social insurance and infrastructure can also be analyzed using OSDT. It may be especially interesting to test the hypotheses—the aggregation result of individual decision making with OSDT can be approximated by the decision result with SEU by empirical studies.

## References

1. Allais, M.: Le comportement de l'homme rationnel devant le risque: Critique des postulats et axiomes de l'école américaine. *Econometrica* **21**(4), 503–546 (1953)

2. Baillon, A.: Eliciting subjective probabilities through exchangeable events: an advantage and a limit. *Decis. Anal.* **5**(2), 76–87 (2008)
3. Bell, D.E.: Risk premiums for decision regret. *Manag. Sci.* **29**(10), 1156–1166 (1983)
4. Bertrand, M., Schoar, A.: Managing with style: the effect of managers on firm policies. *Q. J. Econ.* **118**(4), 1169–1208 (2003)
5. Bewley, T.: Knightian decision theory, part I. *Decis. Econ. Finan.* **25**(2), 79–110 (2002)
6. Brandstatter, E., Gigerenzer, G., Hertwig, R.: The priority heuristic: making choices without trade-offs. *Psychol. Rev.* **113**(2), 409–432 (2006)
7. Clemen, R.T., Kwit, R.C.: The value of decision analysis at Eastman Kodak company, 1990–1999. *Interfaces* **31**(5), 74–92 (2001)
8. Daston, L.J.: *Classical probability in the enlightenment*. Princeton University Press, Princeton, NJ (1988)
9. Dubois, D., Prade, H., Sabbadin, R.: Decision-theoretic foundations of possibility theory. *Eur. J. Oper. Res.* **128**(3), 459–478 (2001)
10. Ellsberg, D.: Risk, ambiguity and savage axioms. *Q. J. Econ.* **75**(4), 643–669 (1961)
11. Fine, C.H.: *Clock Speed: Winning Industry Control in the Age of Temporary Advantage*. Perseus Books, New York (1998)
12. Giang, P.H., Shenoy, P.: Two axiomatic approaches to decision making using possibility theory. *Eur. J. Oper. Res.* **162**(2), 450–467 (2005)
13. Gilboa, I.: Expected utility with purely subjective nonadditive probabilities. *J. Math. Econ.* **16**(1), 65–88 (1987)
14. Guo, P.: One-shot decision approach and its application to Duopoly market. *Int. J. Inf. Decis. Sci.* **2**(3), 213–232 (2010a)
15. Guo, P.: Private real estate investment analysis within one-shot decision framework. *Int. Real Estate Rev.* **13**(3), 238–260 (2010b)
16. Guo, P.: One-shot decision theory. *IEEE Trans. Syst. Man Cybern. Part A Syst. Hum.* **41**(5), 917–926 (2011)
17. Guo, P., Tanaka, H.: Decision analysis based on fused double exponential possibility distributions. *Eur. J. Oper. Res.* **148**(3), 467–479 (2003)
18. Guo, P., Yan, R., Wang, J.: Duopoly market analysis within one-shot decision framework with asymmetric possibilistic information. *J. Comput. Intell. Syst.* **3**(6), 786–796 (2010)
19. Kaha, B.E., Sarin, R.K.: Modelling ambiguity in decision under uncertainty. *J. Consum. Res.* **15**(2), 265–272 (1988)
20. Kahneman, D., Tversky, A.: Prospect theory: an analysis of decision under risk. *Econometrica* **47**(2), 263–291 (1979)
21. Kahneman, D., Tversky, A. (eds.): *Choices, Values, and Frames*. Cambridge University Press, Cambridge (2000)
22. Katsikopoulos, K.V., Gigerenzer, G.: One-reason decision-making: modeling violations of expected utility theory. *J. Risk Uncertainty* **37**(1), 35–56 (2008)
23. King, W.R.: Offshoring decision time is at hand. *Inf. Syst. Manag.* **23**(1), 102–103 (2006)
24. Knight, F.H.: *Risk, Uncertainty and Profit*. Houghton Mifflin, Boston (1921)
25. List, J.A., Mason, C.F.: Are CEOs expected utility maximizers? *J. Econometrics* **162**(1), 114–123 (2011)
26. Loomes, G., Sugden, R.: Regret theory: an alternative theory of rational choice under uncertainty. *Econ. J.* **92**(368), 805–824 (1982)
27. Savage, L.J.: *The Foundations of Statistics*. Wiley, New York (1954)
28. Schmeidler, D.: Subjective probability and expected utility without additivity. *Econometrica* **57**(3), 571–587 (1989)
29. Segal, U.: The Ellsberg paradox and risk aversion: an anticipated utility approach. *Int. Econ. Rev.* **28**(1), 175–202 (1987)
30. Shafer, G.: Savage revisited. *Stat. Sci.* **1**(4), 463–501 (1986)
31. Shimanoff, S.B.: Commonly named emotions in everyday conversations. *Percept. Mot. Skills* **58**(2), 514 (1984)
32. Sugden, R.: Regret, recrimination and rationality. *Theor. Decis.* **19**(1), 77–99 (1985)

33. Von Neumann, J., Morgenstern, O.: *Theory of Games and Economic Behavior*. Princeton University Press, Princeton (1944)
34. Whalen, T.: Decision making under uncertainty with various assumptions about available information. *IEEE Trans. Syst. Man Cybern.* **14**(6), 888–900 (1984)
35. Yager, R.R.: Possibilistic decision making. *IEEE Trans. Syst. Man Cybern.* **9**, 388–392 (1979)

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