Preface

In the past thirty years, (two-dimensional) conformal field theory has been developed into a deep, rich and beautiful mathematical theory and the study of conformal field theories and their applications in mathematics and physics has become an exciting area of mathematics. It has led to new ideas, surprising results and beautiful solutions of various problems in different branches in mathematics and physics, including, but not limited to, algebra, number theory, combinatorics, topology, geometry, critical phenomena, quantum Hall systems, disorder systems, quantum computing and string theory, and is expected to lead to many more.

During June 13 to June 17, 2011, a workshop "Conformal field theories and tensor categories" was held at Beijing International Center for Mathematical Research, Peking University, Beijing, China. This workshop was one of the main activities of a one-semester program on quantum algebra from February to July, 2011 at Beijing International Center for Mathematical Research. It was the aim of the workshop to bring together experts from several different areas of mathematics and physics who are involved in the new developments in conformal field theories, tensor categories and related research directions. Correspondingly the areas covered by the workshop were broad, including conformal field theories, tensor categories, quantum groups and Hopf algebras, representation theory of vertex operator algebras, nets of von Neumann algebras, topological order and lattice models and other related topics. Each of these fields was represented by leading experts.

The simplest class of conformal field theories are rational conformal field theories. In 1988, Moore and Seiberg obtained polynomial equations for the fusing, braiding and modular transformations in rational conformal field theory. They observed that some of these equations are analogous to some properties of tensor categories. Later, a notion of modular tensor category was formulated mathematically and examples of modular tensor categories were constructed from representations of quantum groups. The work of Moore and Seiberg can be interpreted as deriving a modular tensor category structure from a rational conformal field theory. Since then, the theory of various tensor categories has been greatly developed and has been applied to different areas of mathematics and physics. Now the theory of tensor categories not only provides a unifying language for various parts of mathematics and

applications of mathematics in physics, but also gives deep results and fundamental structures in different branches of mathematics and physics.

On the other hand, though a number of examples of modular tensor categories were constructed at the time that the notion of modular tensor category was introduced, it took many years and a lot of efforts for mathematicians to directly construct the modular tensor categories conjectured to appear in rational conformal field theories. Many mathematicians, including in particular Kazhdan-Lusztig, Beilinson-Feigin-Mazur, Finkelberg, Huang-Lepowsky, and Bakalov-Kirillov, contributed in 1990's and early 2000's to the construction of the particular class of examples of the modular tensor categories associated with the Wess-Zumino-Novikov-Witten models (the rational conformal field theories associated with suitable representations of affine Lie algebras). However, the construction of even this particular class of examples was not complete until 2005 when Huang gave a general construction of all the modular tensor categories conjectured to be associated with rational conformal field theories. The corresponding chiral rational conformal field theories are thereby largely under control. Indeed, many problems in rational conformal field theories have meanwhile been solved.

In the workshop, new developments beyond rational conformal field theories and modular tensor categories and new applications in mathematics and physics were presented by top experts. Here we would like to mention especially the following:

1. Construction of interesting tensor categories from representation categories of Hopf algebras, as reviewed by Andruskiewitsch in his overview talk and also in the contribution by Andruskiewitsch, Angiono, García Iglesias, Torrecillas and Vay in this volume.
2. New categorical techniques and structures in tensor categories, as reviewed by Ostrik in his overview talk. One also should include here the Witt group as discussed by Nikshych and Davydov and Hopf-monadic techniques as explained by Virelizier. In a sense, Semikhatov's contribution in this volume using Hopf-algebraic structures in representation categories interpolates between this point and the preceding point.
3. Applications to topological phases and gapped systems as reviewed by Wen in his overview talk and also in the contribution by Wen and Wang in this volume. The study of the Levin-Wen model as discussed by Wu is an important example of such applications.
4. Realization of the tensor-categorical structures in lattice models as in Fendley's overview and Gainutdinov's talk.
5. New developments in the representation theory of vertex operator algebras, especially the nonsemisimple theory corresponding to logarithmic conformal field theory, as reviewed by Lepowsky's overview talk and in the contribution by Huang, Lepowsky and Zhang in this volume. Recent results on representations and the structure of the representation category were reported by Adamovic, Arike, Milas and Miyamoto and also in the contributions by Adamovic and Milas and by Miyamoto in this volume. To some extent, Tsuchiya's talk also went in this direction. Connections to logarithmic conformal field theory were discussed

by Runkel and Semikhatov and also in the contributions by Runkel, Gaberdiel and Wood and by Semikhatov in this volume.

In the workshop, there were 21 invited talks by mathematicians and physicists from Argentina, China, Croatia, France, Germany, Japan, Russia and USA. Some of the invited talks were given by young researchers. The participants benefited a lot from communicating results between the various disciplines and from the attempt to understand them in the framework of conformal field theories and tensor categories. These attempts gave rise to further questions during and after the talks, and, maybe even more importantly, also resulted in numerous and lively private discussions among the participants.

The workshop also had important training impact on students. A number of undergraduate and beginning graduate students in the Enhanced Program for Graduate Study in Beijing International Center for Mathematical Research participated in the workshop. They benefited greatly from the talks, especially the five overview talks, and from discussions with active researchers in the workshop.

The present volume is a collection of seven papers that are either based on the talks presented in the workshop or are extensions of the material presented in the talks in the workshop. We believe that the papers in this volume will be useful to everyone who is interested in conformal field theories, tensor categories and related topics. We hope that these papers will also inspire more research activities in these directions.

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