

# Contents

<b>The Abel Prize—The Missing Nobel in Mathematics?</b> . . . . .	1
Kim G. Helsvig	
1 Science Prizes in Historical Perspective . . . . .	2
2 A National Icon . . . . .	4
3 The Initiative . . . . .	6
4 Mobilization . . . . .	9
5 The Abel Prize “Working Group” . . . . .	10
6 Scientific Legitimization and Support . . . . .	12
7 Political Lobbying . . . . .	14
8 Breakthrough . . . . .	17
9 High Expectations . . . . .	18
10 Nobel Level? . . . . .	22
11 Conclusion—And the Need for Future Adjustments? . . . . .	25
References . . . . .	27
<b>2008 John G. Thompson and Jacques Tits</b>	
<b>Some Reflections</b> . . . . .	31
John G. Thompson	
<b>A Biography of Jacques Tits</b> . . . . .	35
Francis Buekenhout	
1 1930–1944 . . . . .	35
1.1 A Belgian Mathematician . . . . .	35
1.2 Ancestors . . . . .	36
1.3 Parents . . . . .	36
1.4 Grandparents . . . . .	37
1.5 Child Prodigy—Always Ahead of His Age and of His Time . . . . .	38
1.6 Charles Nootens and Petit Jacques . . . . .	39
2 1945–1949 . . . . .	40

2.1	At the Age of Fourteen, Tits Entered University . . . . .	40
2.2	Jean-Claude Piret, a Friend for Life . . . . .	41
2.3	Lectures of Libois in 1945–1946 . . . . .	42
2.4	Research . . . . .	43
2.5	First Degree in 1948 . . . . .	43
2.6	Paris and Emil Artin . . . . .	43
3	1950–1963 . . . . .	44
3.1	Docteur ès Sciences Mathématiques . . . . .	44
3.2	To Heinz Hopf in Zürich in 1950, 1951, and 1953 . . . . .	44
3.3	Institute for Advanced Study, Princeton and H.C. Wang (1951–1952) . . . . .	45
3.4	The Cremona Plane . . . . .	45
3.5	The Thèse d’Agrégation (1955) . . . . .	45
3.6	Memoir for the Prix Louis Empain (1955) . . . . .	47
3.7	Prehistory of Buildings (1955–1961) . . . . .	47
3.8	Birth of the General Theory of Coxeter Groups (1961) . . . . .	47
3.9	Denied Access to the US from 1953 to 1963 . . . . .	48
3.10	International Congress of Mathematicians (1954–1994) . . . . .	49
4	1964–1975 . . . . .	50
4.1	Professor at the Universität Bonn (1964–1974) . . . . .	50
4.2	Buildings Coming of Age (1974) . . . . .	50
4.3	Collège de France (1975) . . . . .	50
5	1976–2000 . . . . .	50
5.1	Professor at the Collège de France (1973–2000) . . . . .	50
5.2	No Pension in Belgium (1994) . . . . .	51
6	2001–2012 . . . . .	52
6.1	The Book with Weiss . . . . .	52
6.2	Editor of Mathematical Journals . . . . .	52
7	Postscript . . . . .	52

**The Work of John Griggs Thompson: A Survey . . . . . 55**

Richard Lyons and Robert M. Guralnick		
1	Thompson’s Thesis, and Local Analysis . . . . .	55
2	The Thompson $J$ -Subgroup and Weak Closure Arguments . . . . .	58
3	Groups of Odd Order Are Solvable . . . . .	60
4	$N$ -Groups and Minimal Simple Groups . . . . .	64
5	The $B$ -Conjecture and the Grand Conjecture . . . . .	66
6	Factorizations, Quadratic Action, and Quadratic Pairs . . . . .	68
7	The Ree Groups . . . . .	70
8	The Finite Sporadic Simple Thompson Group $Th$ , also Known as $F_3$ . . . . .	72
9	“Elementary” Group-Theoretic Results . . . . .	73
10	The Inverse Galois Problem . . . . .	74
11	The Genus of a Permutation Group . . . . .	76
12	Representation Theory . . . . .	77

13	Projective Planes . . . . .	79
14	Cosets . . . . .	79
15	Divisor Matrix . . . . .	80
16	Other Work . . . . .	80
	References . . . . .	80
<b>A Report on the Scientific Contributions of Jacques Tits . . . . .</b>		<b>87</b>
	Francis Buekenhout	
1	Introduction . . . . .	87
2	The Projective Line . . . . .	88
3	The Cremona Plane Made Invariant Under the Cremona Group . . . . .	89
4	Lie Groups and the Riemann–Helmholtz–Lie Problem . . . . .	89
5	Doubly Homogeneous Spaces, and Homogeneous and Isotropic Spaces . . . . .	91
6	Geometric Interpretation of the Five Exceptional Simple Lie Groups and the Magic Square . . . . .	92
7	A World of Incidence Geometries . . . . .	92
8	Generalized Polygons . . . . .	93
9	Moufang Polygons . . . . .	93
10	General Theory of Coxeter Groups . . . . .	94
11	Theory of Buildings: Birth 1961 . . . . .	94
12	Applications of Buildings . . . . .	96
13	Affine Buildings . . . . .	96
14	Diagram Geometries and Sporadic Groups . . . . .	97
15	The Local Approach to Buildings . . . . .	97
16	Free Constructions . . . . .	97
17	Algebraic Groups . . . . .	98
18	Kac–Moody Groups and Twin Buildings . . . . .	98
19	Moufang Polygons: Thirty Years Later . . . . .	98
	References . . . . .	99
<b>List of Publications for John Griggs Thompson . . . . .</b>		<b>101</b>
<b>List of Publications for Jacques Tits . . . . .</b>		<b>109</b>
<b>Curriculum Vitae for John Griggs Thompson . . . . .</b>		<b>123</b>
<b>Curriculum Vitae for Jacques Tits . . . . .</b>		<b>125</b>
<b>2009 Mikhail Gromov</b>		
<b>A Few Recollections . . . . .</b>		<b>129</b>
	Mikhail Gromov	
<b>A Few Snapshots from the Work of Mikhail Gromov . . . . .</b>		<b>139</b>
	D. Burago, Y. Eliashberg, M. Bestvina, F. Forstnerič, L. Guth, A. Nabutovsky, A. Phillips, J. Roe, and A. Vershik	
1	Introduction. Conceptual Thinking (by Dima Burago) . . . . .	140

2	Gromov’s Geometry (by Anatoly Vershik) . . . . .	143
3	The Gromomorphism $SU \rightarrow US$ (by Tony Phillips) . . . . .	148
4	The $h$ -Principle (by Yasha Eliashberg) . . . . .	149
	4.1 Holonomic Approximation . . . . .	150
	4.2 Removal of Singularities . . . . .	156
	4.3 Convex Integration . . . . .	157
5	The Homotopy Principle in Complex Analysis (by Franc Forstnerič) . . . . .	161
	5.1 The Oka–Grauert Principle . . . . .	161
	5.2 Gromov’s Oka Principle . . . . .	162
	5.3 From Elliptic Manifolds to Oka Manifolds and Oka Maps . . . . .	164
6	Soft and Hard Symplectic Geometry (by Yasha Eliashberg) . . . . .	165
	6.1 Gromov’s Alternative . . . . .	166
	6.2 Proof of the Arnold Fixed Point Conjecture for the $2n$ -Torus . . . . .	169
	6.3 Advent of Holomorphic Curves . . . . .	170
	6.4 Flexible Side of Symplectic Geometry is Still Alive . . . . .	180
7	The Waist Inequality in Gromov’s Work (by Larry Guth) . . . . .	181
	7.1 Why is the Waist Inequality Hard? . . . . .	182
	7.2 A Quick History of the Waist Inequality, Part 1 . . . . .	183
	7.3 Combinatorial Analogues of the Waist Inequality . . . . .	184
	7.4 Topological Analogues of the Waist Inequality . . . . .	185
	7.5 A Quick History of the Waist Inequality, Part 2 . . . . .	187
	7.6 Quantitative Topology . . . . .	188
	7.7 Gromov’s Short Proof of the Waist Inequality . . . . .	190
	7.8 Gromov’s Proof of Point Selection . . . . .	193
8	Quantitative Topology and Quantitative Geometric Calculus of Variations (by Alex Nabutovsky) . . . . .	196
	8.1 Quantitative Topology . . . . .	196
	8.2 Quantitative Geometric Calculus of Variations . . . . .	201
	8.3 Gromov’s Filling Technique . . . . .	204
	8.4 Slicing Riemannian Manifolds . . . . .	205
	8.5 Filling Riemannian Manifolds . . . . .	207
9	Geometric Group Theory (by Mladen Bestvina) . . . . .	208
	9.1 Groups of Polynomial Growth . . . . .	208
	9.2 Gromov–Hausdorff Limits . . . . .	210
	9.3 Groups as Metric Spaces and Quasi-isometries . . . . .	212
	9.4 $CAT(-1)$ and $CAT(0)$ Spaces . . . . .	212
	9.5 Hyperbolization of Polyhedra . . . . .	213
	9.6 Hyperbolic Groups . . . . .	214
	9.7 Isoperimetric Functions . . . . .	215
	9.8 $L_2$ -Cohomology . . . . .	216
	9.9 Random Groups . . . . .	218
10	Gromov’s Work on Manifolds of Positive Scalar Curvature (by John Roe) . . . . .	221

10.1	Introduction . . . . .	221
10.2	Simply-Connected Compact Manifolds . . . . .	222
10.3	Beyond Simple Connectivity . . . . .	223
10.4	Macroscopic Dimension and $K$ -Area . . . . .	225
	References . . . . .	227
<b>List of Publications for Mikhail Leonidovich Gromov . . . . .</b>		<b>235</b>
<b>Curriculum Vitae for Mikhail Leonidovich Gromov . . . . .</b>		<b>245</b>
<b>2010 John Torrence Tate</b>		
<b>Autobiography . . . . .</b>		<b>249</b>
John Tate		
<b>The Work of John Tate . . . . .</b>		<b>259</b>
J.S. Milne		
Notations . . . . .		259
1	Hecke $L$ -Series and the Cohomology of Number Fields . . . . .	260
	1.1 Background . . . . .	260
	1.2 Tate’s Thesis and the Local Constants . . . . .	262
	1.3 The Cohomology of Number Fields . . . . .	265
	1.4 The Cohomology of Profinite Groups . . . . .	269
	1.5 Duality Theorems . . . . .	270
	1.6 Expositions . . . . .	273
2	Abelian Varieties and Curves . . . . .	273
	2.1 The Riemann Hypothesis for Curves . . . . .	273
	2.2 Heights on Abelian Varieties . . . . .	274
	2.3 The Cohomology of Abelian Varieties . . . . .	277
	2.4 Serre-Tate Liftings of Abelian Varieties . . . . .	280
	2.5 Mumford-Tate Groups and the Mumford-Tate Conjecture . . . . .	281
	2.6 Abelian Varieties over Finite Fields (Weil, Tate, Honda Theory) . . . . .	283
	2.7 Good Reduction of Abelian Varieties . . . . .	284
	2.8 CM Abelian Varieties and Hilbert’s Twelfth Problem . . . . .	285
3	Rigid Analytic Spaces . . . . .	286
	3.1 The Tate Curve . . . . .	287
	3.2 Rigid Analytic Spaces . . . . .	288
4	The Tate Conjecture . . . . .	290
	4.1 Beginnings . . . . .	291
	4.2 Statement of the Tate Conjecture . . . . .	292
	4.3 Homomorphisms of Abelian Varieties . . . . .	293
	4.4 Relation to the Conjectures of Birch and Swinnerton-Dyer . . . . .	295
	4.5 Poles of Zeta Functions . . . . .	296
	4.6 Relation to the Hodge Conjecture . . . . .	298
5	Lubin-Tate Theory and Barsotti-Tate Group Schemes . . . . .	299
	5.1 Formal Group Laws and Applications . . . . .	299

- 5.2 Finite Flat Group Schemes . . . . . 302
- 5.3 Barsotti-Tate Groups ( $p$ -Divisible Groups) . . . . . 303
- 5.4 Hodge-Tate Decompositions . . . . . 305
- 6 Elliptic Curves . . . . . 306
  - 6.1 Ranks of Elliptic Curves over Global Fields . . . . . 306
  - 6.2 Torsion Points on Elliptic Curves over  $\mathbb{Q}$  . . . . . 307
  - 6.3 Explicit Formulas and Algorithms . . . . . 307
  - 6.4 Analogues at  $p$  of the Conjecture of Birch and Swinnerton-Dyer . . . . . 308
  - 6.5 Jacobians of Curves of Genus One . . . . . 309
  - 6.6 Expositions . . . . . 310
- 7 The  $K$ -Theory of Number Fields . . . . . 310
  - 7.1  $K$ -Groups and Symbols . . . . . 310
  - 7.2 The Group  $K_2F$  for  $F$  a Global Field . . . . . 312
  - 7.3 The Milnor  $K$ -Groups . . . . . 314
  - 7.4 Other Results on  $K_2F$  . . . . . 315
- 8 The Stark Conjectures . . . . . 315
- 9 Noncommutative Ring Theory . . . . . 319
  - 9.1 Regular Algebras . . . . . 319
  - 9.2 Quantum Groups . . . . . 321
  - 9.3 Sklyanin Algebras . . . . . 321
- 10 Miscellaneous Articles . . . . . 322
- Appendix Bibliography of Tate’s Articles . . . . . 328
- References . . . . . 334

**List of Publications for John Torrence Tate . . . . . 341**

**Curriculum Vitae for John Torrence Tate Jr. . . . . 349**

**2011 John W. Milnor**

**Autobiography . . . . . 353**  
John Milnor

**Milnor’s Work in Algebra and Its Ramifications . . . . . 361**  
Hyman Bass

- 1 Introduction . . . . . 361
- 2 Hopf Algebras . . . . . 362
- 3 Growth of Groups . . . . . 363
- 4 The Congruence Subgroup Problem . . . . . 364
- 5 Algebraic K-Theory and Quadratic Forms . . . . . 368
- References . . . . . 372

**John Milnor’s Work in Dynamics . . . . . 375**

- Mikhail Lyubich
- 1 Preface . . . . . 375
- 2 Selected Themes . . . . . 376

2.1	Kneading Theory . . . . .	376
2.2	Milnor’s Attractors . . . . .	377
2.3	Self-similarity and Hairiness of the Mandelbrot Set . . . . .	379
2.4	Beyond the Quadratic Family . . . . .	381
2.5	Two-Dimensional Dynamics . . . . .	385
2.6	Art Gallery . . . . .	388
	References . . . . .	389
<b>John W. Milnor’s Work on the Classification of Differentiable Manifolds</b>		<b>393</b>
L.C. Siebenmann		
1	Some Preliminaries . . . . .	393
2	The Discovery of Exotic 7-Spheres . . . . .	395
2.1	Synopsis . . . . .	395
2.2	1956: Why the Surprise? Some History . . . . .	396
2.3	Milnor’s Incendiary 1956 Article Appears . . . . .	398
2.4	From Thom’s Cobordism to Diffeomorphism? . . . . .	398
2.5	Milnor’s Test Manifolds . . . . .	399
2.6	Towards an Easy ‘Endoscopic’ Classification of these 8-Manifolds . . . . .	400
2.7	Towards a Classification of the 7-Manifolds $M(a, b)$ . . . . .	401
2.8	Milnor’s $SO(4)$ Bundle Notations . . . . .	402
2.9	The First Pontrjagin Class . . . . .	403
2.10	Exotic Homotopy 7-Spheres Appear . . . . .	406
2.11	Milnor’s Invariant $\lambda$ and Its Refinement $\mu$ . . . . .	407
2.12	Weak Equivalences Among the $SO(4)$ Disk Bundles . . . . .	409
2.13	Twisted Spheres Appear . . . . .	411
2.14	Conjecturally Nonsmoothable Manifolds Appear . . . . .	412
2.15	Comments on Motivation and Strategy . . . . .	413
2.16	Smale’s Dramatic Explanation of Milnor’s ‘deus ex machina’ . . . . .	414
3	The Early Achievements of Surgery . . . . .	415
3.1	A Rough Description of Surgery . . . . .	415
3.2	The Springtime of Surgery . . . . .	416
3.3	The First Flowering of Surgery . . . . .	417
3.4	An Exact Sequence Entrapping $\Theta_n$ , for $n \geq 5$ . . . . .	420
3.5	Analysis of the Subgroup $bP$ of $\Theta_n$ . . . . .	421
3.6	Complements Concerning Boundaries of Parallelizable Manifolds . . . . .	423
4	A Metamorphosis . . . . .	425
4.1	Milnor’s Microbundles . . . . .	425
4.2	Surgery for Classical Smooth Manifolds . . . . .	426
4.3	Further Extensions of Surgery . . . . .	426
4.4	Conjectures . . . . .	427
	References . . . . .	427
<b>List of Publications for John Willard Milnor</b>		<b>435</b>

<b>Curriculum Vitae for John Willard Milnor</b> . . . . .	447
<b>2012 Endre Szemerédi</b>	
<b>Autobiography</b> . . . . .	451
Endre Szemerédi	
<b>The Mathematics of Endre Szemerédi</b> . . . . .	459
W.T. Gowers	
1 Introduction . . . . .	459
2 Szemerédi’s Theorem . . . . .	460
2.1 Sketch Proof of Szemerédi’s Theorem when $k = 3$ . . . .	461
2.2 What Happens when the Progressions Are Longer? . . . .	463
3 Szemerédi’s Regularity Lemma . . . . .	464
3.1 Quasirandom Graphs and the Counting Lemma . . . . .	465
3.2 Statement of the Regularity Lemma . . . . .	465
3.3 Sketch Proof of the Regularity Lemma . . . . .	466
3.4 The Regularity Lemma and Szemerédi’s Theorem . . . . .	468
4 The Triangle Removal Lemma . . . . .	470
4.1 Sketch Proof of the Triangle Removal Lemma . . . . .	470
4.2 Applications of the Triangle Removal Lemma . . . . .	471
5 A Sharp Upper Bound for the Ramsey Number $R(3, k)$ . . . . .	473
5.1 Choosing an Independent Set More Carefully . . . . .	474
6 A Counterexample to Heilbronn’s Triangle Conjecture . . . . .	477
7 An Optimal Parallel Sorting Network . . . . .	480
8 A Theorem on Point-Line Incidences . . . . .	484
8.1 Székely’s Proof of the Szemerédi–Trotter Theorem . . . . .	485
8.2 An Application of the Szemerédi–Trotter Theorem . . . . .	486
8.3 What Are the Extremal Sets in the Szemerédi–Trotter Theorem? . . . . .	487
9 The Probability that a Random $\pm 1$ Matrix is Singular . . . . .	487
9.1 The Need to Consider Dependences . . . . .	488
9.2 The Main Idea . . . . .	490
9.3 Subsequent Improvements . . . . .	491
10 Conclusion . . . . .	491
References . . . . .	492
<b>List of Publications for Endre Szemerédi</b> . . . . .	495
<b>Curriculum Vitae for Endre Szemerédi</b> . . . . .	507
<b>A Letter from Niels Henrik Abel to August Leopold Crelle</b>	
<b>Abel and the Theory of Algebraic Equations</b> . . . . .	517
Christian Skau	
1 Historical Context . . . . .	517
2 Correspondence with Legendre . . . . .	519

3	The Addition Theorem . . . . .	520
4	Algebraic Equations—Primitive Elements . . . . .	522
5	Irreducibility Principle . . . . .	525
6	The Galois Group . . . . .	526
7	The Fundamental Theorem and Solvability Criterion . . . . .	529
8	Elliptic Functions and Algebraic Equations . . . . .	530
9	Transformation Theory and Teilungsgleichungen . . . . .	532
10	Posthumous Article . . . . .	536
11	Kronecker’s Reaction . . . . .	541
12	Galois’ Legacy . . . . .	546
13	Twists of Fate—Poetic Justice . . . . .	547
14	The Abel–Galois Linkage . . . . .	548
	References . . . . .	550
	<b>The Abel Committee . . . . .</b>	<b>553</b>
	<b>The Niels Henrik Abel Board . . . . .</b>	<b>555</b>
	<b>The Abel Lectures 2003–2012 . . . . .</b>	<b>557</b>
	<b>The Abel Laureate Presenters 2003–2012 . . . . .</b>	<b>561</b>
	<b>The Interviews with the Abel Laureates . . . . .</b>	<b>563</b>
	<b>Addenda, Errata, and Updates . . . . .</b>	<b>565</b>
	2003 Jean-Pierre Serre . . . . .	565
	2004 Sir Michael Atiyah and Isadore M. Singer . . . . .	566
	2005 Peter D. Lax . . . . .	568
	2006 Lennart Carleson . . . . .	569
	2007 S.R. Srinivasa Varadhan . . . . .	570



<http://www.springer.com/978-3-642-39448-5>

The Abel Prize 2008-2012

Holden, H.; Piene, R. (Eds.)

2014, XVII, 571 p., Hardcover

ISBN: 978-3-642-39448-5