
Contents

1	Sets, Relations, and Preferences	1
1.1	Elements of Set Theory	1
1.1.1	A Set Theory	2
1.1.2	A Propositional Calculus	4
1.1.3	Partitions and Covers	6
1.1.4	The Universal and Existential Quantifiers	7
1.2	Relations, Functions and Operations	7
1.2.1	Relations	7
1.2.2	Mappings	8
1.2.3	Functions	11
1.3	Groups and Morphisms	12
1.4	Preferences and Choices	24
1.4.1	Preference Relations	24
1.4.2	Rationality	25
1.4.3	Choices	27
1.5	Social Choice and Arrow's Impossibility Theorem	32
1.5.1	Oligarchies and Filters	33
1.5.2	Acyclicity and the Collegium	35
	Further Reading	38
2	Linear Spaces and Transformations	39
2.1	Vector Spaces	39
2.2	Linear Transformations	45
2.2.1	Matrices	46
2.2.2	The Dimension Theorem	49
2.2.3	The General Linear Group	53
2.2.4	Change of Basis	55
2.2.5	Examples	59
2.3	Canonical Representation	62
2.3.1	Eigenvectors and Eigenvalues	63
2.3.2	Examples	66
2.3.3	Symmetric Matrices and Quadratic Forms	67
2.3.4	Examples	71
2.4	Geometric Interpretation of a Linear Transformation	73

3	Topology and Convex Optimisation	77
3.1	A Topological Space	77
3.1.1	Scalar Product and Norms	77
3.1.2	A Topology on a Set	82
3.2	Continuity	88
3.3	Compactness	93
3.4	Convexity	99
3.4.1	A Convex Set	99
3.4.2	Examples	100
3.4.3	Separation Properties of Convex Sets	104
3.5	Optimisation on Convex Sets	110
3.5.1	Optimisation of a Convex Preference Correspondence	110
3.6	Kuhn-Tucker Theorem	115
3.7	Choice on Compact Sets	118
3.8	Political and Economic Choice	125
	Further Reading	132
4	Differential Calculus and Smooth Optimisation	135
4.1	Differential of a Function	135
4.2	C^r -Differentiable Functions	142
4.2.1	The Hessian	142
4.2.2	Taylor's Theorem	145
4.2.3	Critical Points of a Function	149
4.3	Constrained Optimisation	155
4.3.1	Concave and Quasi-concave Functions	155
4.3.2	Economic Optimisation with Exogenous Prices	162
4.4	The Pareto Set and Price Equilibria	171
4.4.1	The Welfare and Core Theorems	171
4.4.2	Equilibria in an Exchange Economy	180
	Further Reading	187
5	Singularity Theory and General Equilibrium	189
5.1	Singularity Theory	189
5.1.1	Regular Points: The Inverse and Implicit Function Theorem	189
5.1.2	Singular Points and Morse Functions	196
5.2	Transversality	200
5.3	Generic Existence of Regular Economies	203
5.4	Economic Adjustment and Excess Demand	207
5.5	Structural Stability of a Vector Field	210
5.6	Speculations on Chaos	221
	Further Reading	227
6	Topology and Social Choice	231
6.1	Existence of a Choice	231
6.2	Dynamical Choice Functions	233

6.3	Stochastic Choice	238
6.3.1	The Model Without Activist Valence Functions	244
	References	248
7	Review Exercises	251
7.1	Exercises to Chap. 1	251
7.2	Exercises to Chap. 2	251
7.3	Exercises to Chap. 3	253
7.4	Exercises to Chap. 4	255
7.5	Exercises to Chap. 5	255
	Subject Index	257
	Author Index	261



<http://www.springer.com/978-3-642-39817-9>

Mathematical Methods in Economics and Social Choice

Schofield, N.

2014, XV, 262 p. 100 illus., Hardcover

ISBN: 978-3-642-39817-9