Preface to the Second Edition

For the second edition, I have added a new chapter six. This chapter continues with the model presented in Chap. 3 by developing the idea of dynamical social choice. In particular the chapter considers the possibility of cycles enveloping the set of social alternatives.

A theorem of Saari (1997) shows that for any non-collegial set, $\mathbb{D}$, of decisive or winning coalitions, if the dimension of the policy space is sufficiently large, then the choice is empty under $\mathbb{D}$ for all smooth profiles in a residual subspace of $C^r(W, \Re^n)$. In other words the choice is generically empty.

However, we can define a social solution concept, known as the heart. When regarded as a correspondence, the heart is lower hemi-continuous. In general the heart is centrally located with respect to the distribution of voter preferences, and is guaranteed to be non-empty. Two examples are given to show how the heart is determined by the symmetry of the voter distribution.

Finally, to be able to use survey data of voter preferences, the chapter introduces the idea of stochastic social choice. In situations where voter choice is given by a probability vector, we can model the choice by assuming that candidates choose policies to maximise their vote shares. In general the equilibrium vote maximising positions can be shown to be at the electoral mean. The necessary and sufficient condition for this is given by the negative definiteness of the candidate vote Hessians. In an empirical example, a multinomial logit model of the 2008 Presidential election is presented, based on the American National Election Survey, and the parameters of this model used to calculate the Hessians of the vote functions for both candidates. According to this example both candidates should have converged to the electoral mean.

Saint Louis, Missouri, USA — Norman Schofield
June 13, 2013

Preface to the First Edition

In recent years, the optimisation techniques, which have proved so useful in microeconomic theory, have been extended to incorporate more powerful topological and differential methods. These methods have led to new results on the qualitative behaviour of general economic and political systems. However, these developments have also led to an increase in the degree of formalism in published work. This formalism can often deter graduate students. My hope is that the progression of ideas presented in these lecture notes will familiarise the student with the geometric concepts underlying these topological methods, and, as a result, make mathematical economics, general equilibrium theory, and social choice theory more accessible.

The first chapter of the book introduces the general idea of mathematical structure and representation, while the second chapter analyses linear systems and the representation of transformations of linear systems by matrices. In the third chapter, topological ideas and continuity are introduced and used to solve convex optimisation problems. These techniques are also used to examine existence of a "social equilibrium." Chapter four then goes on to study calculus techniques using a linear approximation, the differential, of a function to study its "local" behaviour.

The book is not intended to cover the full extent of mathematical economics or general equilibrium theory. However, in the last sections of the third and fourth chapters I have introduced some of the standard tools of economic theory, namely the Kuhn Tucker Theorem, together with some elements of convex analysis and procedures using the Lagrangian. Chapter four provides examples of consumer and producer optimisation. The final section of the chapter also discusses, in a heuristic fashion, the smooth or critical Pareto set and the idea of a regular economy. The fifth and final chapter is somewhat more advanced, and extends the differential calculus of a real valued function to the analysis of a smooth function between "local" vector spaces, or manifolds. Modem singularity theory is the study and classification of all such smooth functions, and the purpose of the final chapter to use this perspective to obtain a generic or typical picture of the Pareto set and the set of Walrasian equilibria of an exchange economy.

Since the underlying mathematics of this final section are rather difficult, I have not attempted rigorous proofs, but rather have sought to lay out the natural path of development from elementary differential calculus to the powerful tools of singularity theory. In the text I have referred to work of Debreu, Balasko, Smale, and Saari, among others who, in the last few years, have used the tools of singularity theory to

develop a deeper insight into the geometric structure of both the economy and the polity. These ideas are at the heart of recent notions of "chaos." Some speculations on this profound way of thinking about the world are offered in Sect. 5.6. Review exercises are provided at the end of the book.

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I am also indebted to my graduate students for the pertinent questions they asked during the courses on mathematical methods in economics and social choice, which I have given at Essex University, the California Institute of Technology, and Washington University in St. Louis.

In particular, while I was at the California Institute of Technology I had the privilege of working with Richard McKelvey and of discussing ideas in social choice theory with Jeff Banks. It is a great loss that they have both passed away. This book is dedicated to their memory.

Saint Louis, Missouri, USA Norman Schofield

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