2

On theoretical modeling of multiple-domain processes

The temporal behavior of multiple-domain systems, such as internal combustion engines, drivetrains and auxiliary units, can be described with the help of system theory according to uniform methods. For this, however, mathematical models have to exist for the static and dynamic behavior of the system components or the processes.

The derivation of mathematical models can take place in a theoretical (physical) or experimental way. Therefore, it is called theoretical modeling or experimental modeling or identification.

For combustion engines, mathematical models for different physical areas have to be set up and combined, e.g. for mechanics, combustion, thermodynamics, and electricity. The procedure during theoretical modeling is in principle known for the individual areas, and there also exist analogies for models between different areas. However, a basic, generally applicable methodology for theoretical modeling with an interdisciplinary view has several advantages, especially for applying computer-aided modeling. Therefore, a unified representation for modeling in different physical domains is briefly summarized in this chapter.

2.1 Theoretical and experimental modeling

The derivation of mathematical models of processes can be performed in a theoretical or experimental way. For engines and also vehicles mathematical models for different physical domains have to be set up and combined. This holds especially for mechatronic components and engines in the drivetrain. Therefore, this chapter gives an introduction into a systematic way to model multi-domain systems, also as a basis for computer-aided modeling.

The principles of theoretical modeling can follow a basic methodology, see Karnopp et al (1990), Gawthrop and Smith (1996), Isermann (2005). Fundamental equations are:

1. Balance equations for stored masses, energies, and momentum
2. Constitutive equations of special elements

3. Phenomenological equations, if irreversible processes take place
4. Entropy balance equations, if several irreversible processes are involved
5. Connection equations.

In stating these equations one has to distinguish between processes with distributed and lumped parameters. For *distributed parameters* the dependency on space and time has to be considered. This usually leads to partial differential equations. If the space dependency is negligible, the process can be considered with *lumped parameters* which leads to ordinary differential equations as a function of time. For combustion engines or drivetrains both types appear. However, one can frequently operate with lumped parameters.

By summarizing the basic equations of all process elements, one receives a *theoretical* or *physical process model* with a certain structure and certain parameters if it can be solved explicitly. Frequently, this model is extensive and complicated, so it must be simplified for further applications. The simplifications are made by linearization, reduction of the model order or approximation of systems with distributed parameters by lumped parameters when limiting on fixed locations. But also if the set of equations cannot be solved explicitly, the individual equations supply important hints for the model structure. So, e.g. balance equations are always linear and some phenomenological equations are linear in wide areas. The constitutive equations often introduce nonlinear relations.

During experimental modeling, which is also called *process identification*, one obtains the mathematical model of a process from measurements. Here, one always proceeds from *a priori* knowledge, which was gained, e.g. from the theoretical analysis or from preceding measurements. Then, input and output signals are measured and evaluated by means of identification methods in such a way that the relation between the input and output signal is expressed in a mathematical model. The result of the identification then is an *experimental model*, see Chap. 3.

Theoretical and experimental modeling mutually complete themselves. The theoretical model contains the functional description between the physical data of the process and its parameters. Therefore, one will use this model, e.g. if the process is to be favorably designed with regard to the dynamical behavior or if the process behavior has to be simulated before construction. The experimental model, on the other hand, contains parameters as numerical values whose functional relation with the physical basic data of the process remains unknown. In many cases, the real dynamic behavior can be described more exactly or it can be determined at smaller expenditure by experimentally obtained models which, e.g. is better suited for control design, the prediction of signals or for fault detection.

The following methodology for theoretical (physical) modeling is a strongly shorted version of Chap. 2 in Isermann (2005), which can be applied generally for technical systems and thus holds also for engines and vehicles. This is treated first because then the modeling of parts of the engines can be based on these general laws of modeling.

2.2 Process elements from different domains

In the following, the terms energy, matter, and information are called *quantity*. If one considers processes with *lumped parameters*, then the elements of technical processes can be classified according to the following idealized types, see Karnopp et al (1990), MacFarlane (1967), MacFarlane (1970), Isermann (2005), compare Fig. 2.2.1:

- *sources*: deliver an output quantity from a large supply, without or with losses
- *storages*: take up a quantity and deliver it in the same form
- *transformers*: take up a quantity and deliver it in the same form, without storing it
- *converters*: take up a quantity in certain form and deliver it after conversion into another form, without storing it
- *sinks*: take up an input quantity and consume it in the same or another form. Since mainly losses occur they are dissipative processes.

The first four elements are *ideal* if no losses are generated. However, *real* elements have losses. In the case of real sinks the input quantity is not always completely consumed.

The connecting lines in Fig. 2.2.1 represent the flows between the elements in the form of [quantity/time]. The arrows indicate the direction of the flows.

A further distinction of the process elements can be made with regard to their controllability with an additional auxiliary energy.

- *passive elements*: the transferred quantity is not controllable by an additional auxiliary energy. Examples are passive storages, e.g. capacitances, passive transformers as, e.g. fixed gear transmissions, or passive converters, e.g. fans with constant speed
- *active elements*: a quantity is controlled by an actuator. Thereby, an electrical or mechanical auxiliary energy usually has to supply the actuator. Examples are controllable sources as, e.g. controllable transformers as, e.g. tank with fuel pump, automatic transmissions, combustion chamber with variable valve train and EGR valves.

Active process elements are represented in Fig. 2.2.2. The storages basically show a dynamic, usually an integral, behavior. Sources, transformers, converters, and sinks can have both a mainly static transfer behavior as well as a distinct dynamic transfer behavior.

For processes with *distributed parameters*, process elements such as storages, transformers, converters, and sinks are distributed over space. By partitioning into infinitesimally small elements, one can determine process elements with lumped parameters, whose state variables are location-dependent from element to element.

Figure 2.2.3 depicts a simplified representation of a combustion engine using the introduced process elements and symbols. The inputs are a fuel mass flow and an air mass flow. A first converter generates by combustion a net heat release flow, which

generates displacement power by a thermodynamic operation cycle. A second converter converts it to mechanical rotational power. A part of the combustion heat flow is delivered through the cylinder walls to the cooling system and by heat exchange to the environment. The exhaust gas components generated during the combustion pass the exhaust after treatment (catalysts, filter) and then vanish to the environment. Main storages with regard to control are the intake system, the turbocharger (not included in Fig. 2.2.3), the crank shaft rotational mass, and the cooling circuit.
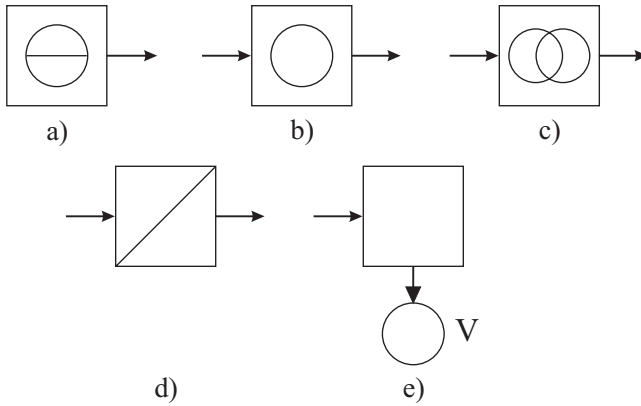
Fig. 2.2.1. Symbols of passive process elements. **a** source. **b** storage. **c** transformer. **d** converter. **e** sink (V: losses):
→ energy, matter or information flow.
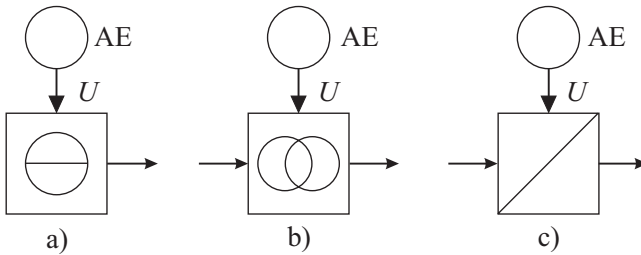
Fig. 2.2.2. Symbols of active process elements (the process element is controlled by an actuator, which is supplied by an auxiliary energy. AE: auxiliary energy; U: manipulated variable). **a** source with auxiliary energy. **b** transformer with auxiliary energy. **c** converter with auxiliary energy.
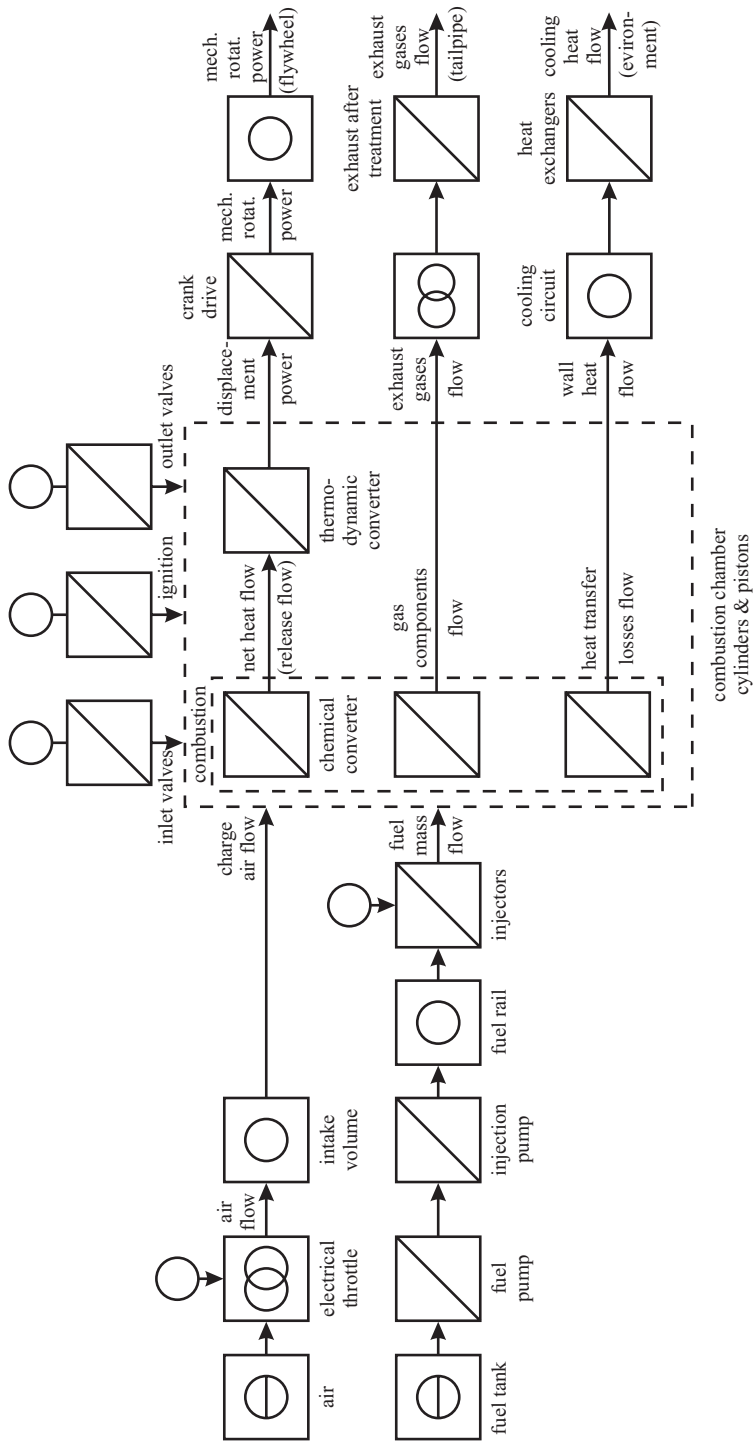
Fig. 2.2.3. Simplified schematic representation of a gasoline combustion engine with mass and energy flows and the symbols of Fig. 2.2.1 (not all losses considered)

2.3 Basic equations

After the definition of suitable cuts, the process is divided into elements such as sources, storages, transformers, converters, and sinks for energies and matters in order to set up the basic equations. These basic equations are for lumped parameter processes:

- *balance equations* (general storages, junction points)
- *constitutive equations* or *physical state equations* (sources, transformers, converters, special storages)
- *phenomenological equations* (sinks, dissipative elements).

This grouping of basic equations then applies both to processes with energy flows and matter flows.

2.3.1 Balance equations

Since the laws for conservation of mass, energy, and momentum are fundamental, they are regarded as the first type of equations. The balance equations, which are derived from these conservation laws, basically apply independently of the construction of the processes. They describe the global behavior. The mass balance applies to processes with moved matter, the energy balance to processes with all types of energy and the momentum balance to processes with moved masses.

If Q_ν describes a mass m_ν or energy E_ν, the principle of the conservation of mass or energy applied to a bounded area, where no mass or energy leaves, leads to

$$\sum_{\nu=1}^{n} Q_\nu = \text{const.} \tag{2.3.1}$$

compare Fig. 2.3.1a). If a mass or energy $\Delta Q_i(t)$ enters through the boundaries of an arbitrary control area and a mass or energy $\Delta Q_o(t)$ escapes through the boundaries in a time Δt, it holds with the conservation laws

$$\Delta Q_i(t) - \Delta Q_o(t) = \Delta Q_s(t) \tag{2.3.2}$$

where $\Delta Q_s(t)$ is the stored quantity. If (2.3.2) is divided by Δt

$$\lim_{\Delta t \to 0} \Delta Q / \Delta t = dQ/dt$$

a *generalized balance equation* for flows follows

$$\underset{\text{inflow}}{\dot{Q}_i(t)} - \underset{\text{outflow}}{\dot{Q}_o} = \underset{\text{stored flow}}{\tfrac{d}{dt} Q_s(t)} \tag{2.3.3}$$

see also Fig. 2.3.1b). Its signal flow is depicted in Fig. 2.3.2.

Balance equations for mass and energy stores thus lead to a linear integrating transfer element and cause a dynamically delayed behavior. Balance equations have to be set up for each storage. They are always linear.

The balance equations also describe the flows at the interconnection points of process elements if the storage capacity is set to zero. Balance equations are also called *continuity equations*.
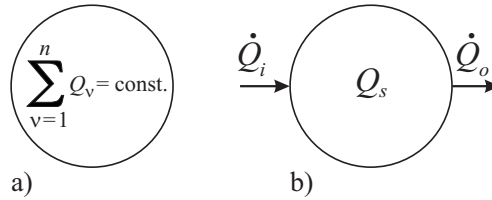
Fig. 2.3.1. General balance for a storage of energy and mass. **a** bounded area. **b** storage.
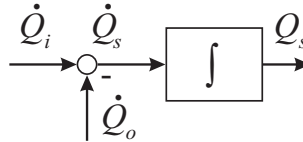
Fig. 2.3.2. Block diagram of an energy or mass storage (balance equation).

2.3.2 Constitutive equations

The coherence between input and output variables of the process elements in the form of sources, transformers, converters, sinks and also storage elements can be expressed by special physical laws in analytical form or by characteristic curves from experiments. The equations mentioned are called *constitutive equations* or *physical state equations*.

Many different physical laws that apply to the individual process elements exist. However, regarding the input/output behavior, several similarities exist.

a) Processes with energy flows

For technical processes which primarily transfer energy at their interconnections, it follows from the energy balance equation (2.3.3) without storage that the transferred energy per time interval or the power

$$P(t) = \frac{dE(t)}{dt} \tag{2.3.4}$$

is always equal at the interconnections between the process elements. Therefore, it is appropriate to determine the state variables in such a way that at the interfaces between process elements or subprocesses they describe a power. If one determines the interfaces in analogy to electrical transfer elements as a terminal pair, then the different process elements can be described as one-port systems (two-pole systems), two-port systems (four-pole systems) or generally multi-port systems (multi-pole systems), see Fig. 2.3.3. One can always distinguish two variables for a terminal pair at the transfer elements, Karnopp and Rosenberg (1975), Karnopp et al (1990), Takahashi et al (1972):

1. potential difference $e(t)$: variables such as electrical voltage, force, pressure difference appear as a difference between two terminals. They are called "effort"

2. flow $f(t)$: variables such as electrical current, velocity, volume flow enter in one of the terminals. They are called "flow".

The product of both terms is the transferred power

$$
\underset{\text{power}}{P(t)} = \underset{\text{flow}}{f(t)} \quad \underset{\substack{\text{potential} \\ \text{difference}}}{e(t).}
$$
(2.3.5)

Here, $f(t)$ and $e(t)$ are assigned covariables, which are also called *generalized power variables*. These variables are discussed for important technical systems with energy flows in Isermann (2005). The formulation of constitutive equations with potentials and flows is especially advantageous for mechanical and hydraulic components in connection with electrical ones, i.e. for mechatronic systems.
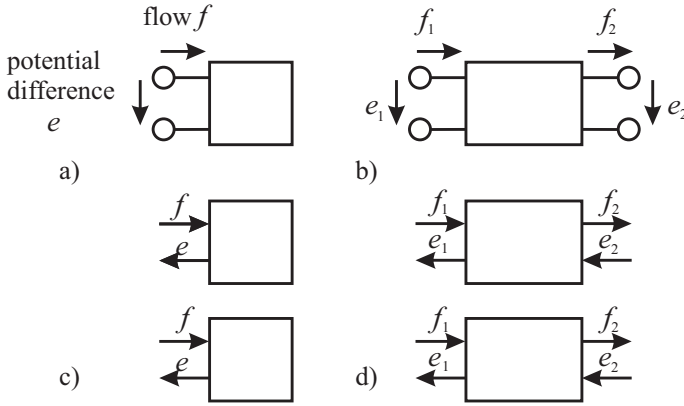
Fig. 2.3.3. Representation of one-port and two-port systems for process elements with power variables. **a** one-port system in terminal representation. **b** two-port system in terminal representation. **c** one-port system with input and output signals. **d** two-port system with input and output signals (2 of 4 possibilities).

b) Processes with different flows

The previous reflections were limited to processes that transfer exclusively energy flows. For *matter flows* (solids, liquids, steams, gases), it is practical to use the mass flow as flow $f(t)$, since the mass balance equation is a substantial basic equation. If one uses the usual variables for the potential difference $e(t)$, the product $e(t)f(t)$ is not always power $P(t)$ according to (2.3.5).

At the interconnection points between process elements, power $P(t)$ (energy flows), as well as the mass flows

$$
\dot{m}(t) = \frac{dm(t)}{dt}
$$
(2.3.6)

have to be equal between the process elements at any time. Therefore, one uses the mass flow as a further state variable at the interfaces. For incompressible matter one can also take the volume flow rate

$$\dot{V}(t) = \frac{1}{\rho} \dot{m}(t) \tag{2.3.7}$$

since the density ρ is constant.

For *heat flows* which are transferred by convection and therefore are coupled to a mass with heat capacity, the product according to Table 2.3.1 is power (in the case of convection by fluids it has to be multiplied by the specific heat). This does not apply to thermal conduction in matter if one selects the heat flow as flow f, since the heat flow itself represents power, see Table 2.3.1, last row.

For powers, it has to be determined if it is the power needed for the transportation of a mass flow, e.g. hydraulic power or the power that is transported by the mass flow, e.g. thermal energy flow.

Table 2.3.1. Variables for flow and potential difference for matter flows and heat flows

System	Flow f		Potential difference e			ef	Power
hydraulic	mass flow $\dot{m}$	$\frac{\text{kg}}{\text{s}}$	pressure difference	p		$\dot{m}p$	$\frac{1}{\rho}\dot{m}p$
thermal - convection through liquids	mass flow $\dot{m}$	$\frac{kg}{s}$	temperature difference T		K	$\dot{m}T$	$\dot{m}c_pT$
- convection through gas, steam	mass flow $\dot{m}$	$\frac{\text{kg}}{\text{s}}$	enthalpy difference	h	$\frac{\text{J}}{\text{kg}}$	$\dot{m}h$	$\dot{m}h$
chemical	mass flow $\dot{m}_i$	$\frac{\text{kg}}{\text{s}}$	concentration	c_i	$\frac{\text{mol}}{\text{kg}}$	$\dot{m}_ic_i$	$--$
thermal - heat conduction	heat flow $\dot{q}$	W	temperature difference T		K	$\dot{q}T$	$\dot{q}$

These constitutive equations are required for modeling many parts of *combustion engines* because of the energy flows for chemical energy through combustion, thermodynamic energy, mechanical energy and thermal energy. Additionally, liquid and gas flows have to be modeled which transport air, fuel or oil or transport combustion and thermodynamic energy.

2.3.3 Phenomenological equations

In the case of sinks and losses in some transformers and converters, the internal process runs only in one direction and is not reversible (without additional energy supply). Examples are thermal conduction, diffusion or chemical reaction. The appropriate processes are characterized by irreversible equalization processes with increasing entropy. The reasons for the entropy increase are the dissipation of mechanical and electrical power, the exchange of mass and heat and chemical reactions, Ahrendts (1989). Thus, dissipative systems can be represented as sinks. The irreversible transients are described by phenomenological equations. One example is:

a) Fourier's law of heat conduction

$$\dot{q}_z = -\lambda \frac{\partial T}{\partial z} = -\lambda \,\mathrm{grad}_z T \qquad (2.3.8)$$

($\dot{q}_z$ heat flow density, λ thermal conductivity, T temperature, z space coordinate).

Other examples are Fick's law of diffusion and Ohm's law of electrical current. These laws can be represented in a general form

$$\text{flow density} = -\frac{1}{\text{specific resistance}} \text{ potential gradient} \qquad (2.3.9)$$

and are linear in a wide range.

Another type of phenomenological equation is the

b) Chemical reaction law

A first-order reaction $A_i \rightarrow B_i$ with c_{Ai} and c_{Bi} for the concentrations yields

$$r_z = -kc_{Ai} \left[\frac{\mathrm{kmol}}{m^3 s} \right] \qquad (2.3.10)$$

$$k = k_\infty e^{-\frac{E}{RT}} \qquad \text{Arrhenius law} \qquad (2.3.11)$$

(r_z reaction rate, k reaction rate coefficient, c_{Ai} concentration of component A_i, E activation energy, k_∞ frequency factor).

c) General sinks, dissipative converters

Phenomenological equations show partially linear behavior, e.g. in the case of viscous friction or Ohm's resistances, and partially strong nonlinear behavior as in the case of throttles or friction. In the linear case, when using the one-port representation, e.g. the resistance equation applies

$$f_1 = \frac{1}{R} e_1 \qquad (2.3.12)$$

where R is the resistance coefficient (The negative sign in (2.3.9) is necessary because the gradient is used with regard to the location).

Dry friction has a direction-dependent discontinuous characteristic curve

$$\begin{aligned} F &= F_{G0} & \dot{z} &> 0 \\ F &= -F_{G0} & \dot{z} &< 0. \end{aligned} \qquad (2.3.13)$$

This leads to force-displacement characteristics in the form of rectangular hysteresis characteristics, whose range depends on the amplitude z_0, Isermann (2005).

In the case of combustion engines, the considered examples of phenomenological equations are required for modeling, for example, heat transfer to the cooling

system, combustion, mass flows through the intake and exhaust system and all kind of frictions in the mechanical parts.

This section described the basic balance equations and constitutive equations in a general form and some phenomenological equations. The balance equations resulted from the conservation laws of physics and have, in principle, the form of (2.3.3). However, the statement of the energy balance equations for mechanical and thermodynamic processes require the consideration of some special features as treated in related textbooks and also in Karnopp et al (1990) and Isermann (2005).

2.3.4 Summary

By following the approach indicated briefly in this chapter a consistent procedure for the modeling of processes with energy and mass flows from different physical domains results. Using all particular equations, an equation system for the considered process or process part follows. From this, a signal flow diagram may be systematically composed and the state space equations as well as the differential equations for the input and output variables may be derived. If desired, a linearization around the operating point may be carried out.

The treated systematic approach and unified procedure allows not only the recognition of many similarities but is also a prerequisite for computer-aided modeling with modern software tools. It can also be applied to modeling of processes with distributed parameters with finite elements.

2.4 Time-dependent and rotation-angle-dependent models

The behavior of internal combustion engines is dominated by the reciprocating working cycle and therefore dependent on the rotation angles of the crankshaft and the camshaft. As the camshaft rotates usually with half of the speed of the crankshaft, if suffices to consider the crankshaft speed only, assuming four-stroke engines. The models considered in this book are oriented to the design of control functions in the ECU and should therefore, if possible, be applicable for real-time computations. Therefore, one has to make simplifying assumptions and only the dominant dynamic effects can be taken into account with regard to important variables of the engine, for example, the manifold pressure, torque, exhaust gas pressure and temperature, fuel consumption and emissions. Engine part processes which directly influence the single combustion like combustion pressure and temperature, air charge, fuel injection, ignition, valve phasing have usually to be described by *crank-angle-dependent models* $f(\varphi)$ (also called discrete-event models, Guzzella and Onder (2010)), Fig. 2.4.1. On the other side some engine part processes outside of the cylinders can be described by *time-dependent models* $f(t)$. The crank-angle-dependent behavior can be neglected frequently as the reciprocating behavior of the cylinder is damped because of gas or thermal storages, especially for multi-cylinder engines. Examples are the air flow, manifold pressure, turbo charger torque and speed, and emissions. If the working stroke induced fluctuations are neglected, the resulting models are *mean-value models*.
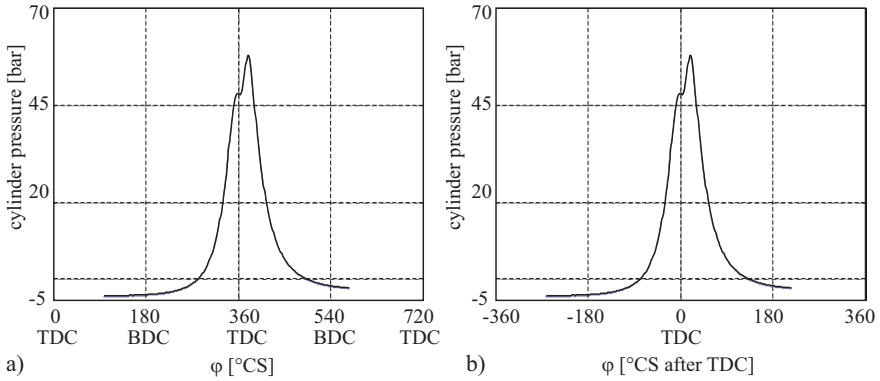
Fig. 2.4.1. Scaling of the crank angle for four-stroke engines. **a** absolute scaling: $0°$CS after expansion. **b** relative scaling: $0°$CS at top dead center (TDC).

The crank angle and the time are related by the engine speed

$$\varphi(t) = \omega_{\text{eng}}(t)t \tag{2.4.1}$$

and for infinitesimal small elements by

$$d\varphi = \omega_{\text{eng}}dt + td\omega_{\text{eng}}. \tag{2.4.2}$$

For constant engine speed this simplifies to

$$\varphi(t) = \omega_{\text{eng}}t \tag{2.4.3}$$
$$d\varphi = \omega_{\text{eng}}dt.$$

As the mean value of the crankshaft speed during one cycle does not vary significantly, one can assume a constant engine speed for some crank-angle-dependent models.

a) Continuous models

As known from system and control theory the treatment of dynamic systems with Laplace transformation has many advantages. The Laplace transform of a continuous-time function $x(t)$ is

$$x(s) = \int_0^\infty x(t)e^{-st}dt \tag{2.4.4}$$

with the Laplace variable

$$s = \delta + i\omega \qquad [1/s]. \tag{2.4.5}$$

The functions or signals $x(t)$ have to fulfill certain conditions as $x(t) = 0$ for $t < 0$ and must lead to a convergence of $x(s)$ (integrability). A continuous-time Laplace transformation for the input signal $u(t)$ and output signal $y(t)$ leads to a transfer-function

$$G(s) = \frac{y(s)}{u(s)} \tag{2.4.6}$$

and the frequency response for $s = i\omega$

$$G(i\omega) = \lim_{s \to i\omega} G(s). \tag{2.4.7}$$

Correspondingly, a continuous angle function $x(\varphi)$ can be Laplace-transformed, Schmitt (1995)

$$x(\sigma) = \int_0^\infty x(\varphi)e^{-\sigma\varphi}d\varphi \tag{2.4.8}$$

with the Laplace variable

$$\sigma = \Delta + i\Omega \qquad [1/\text{deg}]. \tag{2.4.9}$$

Hence, an angle-dependent damping $\Delta[1/\text{deg}]$ with the angle frequency $\Omega[1/\text{deg}]$ can be defined. The function $x(\varphi)$ also has to fulfill certain convergence conditions, like $x(\varphi) = 0$ for $\varphi < 0$ and integrability.

Example 2.1 (Continuous models).
A continuous-time first-order process with dead time T_t follows the differential equation in time

$$\frac{dy(t)}{dt} + ay(t) = bu(t - T_t)$$

and the s-transfer function becomes

$$G(s) = \frac{y(s)}{u(s)} = \frac{b}{s+a}e^{-T_t s} = \frac{K}{1+Ts}e^{-T_t s}$$

with $K = b/a$, $T = 1/a$. A continuous angle first-order process with dead angle φ_t is described by a differential equation in angle

$$\frac{dy(\varphi)}{d\varphi} + a'y(\varphi) = b'u(\varphi - \varphi_t)$$

and the σ-transfer function is

$$G(\sigma) = \frac{y(\sigma)}{u(\sigma)} = \frac{b'}{\sigma+a'}e^{-\varphi_t \sigma}.$$

If the dead time T_t results from a certain dead angle φ_t, then it follows for a rotation with constant speed ω_{eng}

$$\varphi_t = \omega_{\text{eng}}T_t$$

$$T_t = \frac{1}{\omega_{\text{eng}}}\varphi_t.$$

Under the assumption of a constant dead angle φ_t the dead time changes inversely proportional to the speed and the transfer function $G(s)$ has to consider this variable dead time $T_t(\omega_{\text{eng}})$. This is not the case for $G(\sigma)$, as the dead angle is independent of the speed.

□

b) Discrete models

As the ECU samples the continuous-time sensor signal with a sampling time T_0 the theory of sampled-data systems can be used leading to *discrete-time models* see, for example Isermann (1989).

An expansion of the Laplace transform by approximation of the sampled (trapezoidal) pulses by δ-impulses results in the z-transfer function of the sampled function $x(kT_0)$ with the discrete time $k = t/T_0 = 0, 1, 2, ...$

$$x(z) = \sum_{k=0}^{\infty} x(kT_0)z^{-k} \tag{2.4.10}$$

$$z = e^{T_0 s}. \tag{2.4.11}$$

A discrete-time transfer function is then

$$G(z) = \frac{y(z)}{u(z)}. \tag{2.4.12}$$

If the crank-angle-dependent function $x(\varphi)$ is sampled with the sampling angle φ_0 resulting in the discrete angle $\kappa = \varphi/\varphi_0 = 0, 1, 2, ...$, this leads, corresponding to (2.4.10), to the ζ-transform.

$$x(\zeta) = \sum_{k=0}^{\infty} x(\kappa\varphi_0)\,\zeta^{-k} \tag{2.4.13}$$

$$\zeta = e^{\varphi_0 \sigma}. \tag{2.4.14}$$

A discrete-angle transfer function follows

$$G(\zeta) = \frac{y(\zeta)}{u(\zeta)}. \tag{2.4.15}$$

For this ζ-transformation the same rules can be applied as for the z-transformation. For example, to avoid side band effects in sampling a continuous-time function with angular frequency $\omega_{\max}$ the sampling frequency ω_0 or sampling time T_0 has to be chosen according to Shannon's sampling theorem

$$\omega_0 = \frac{2\pi}{T_0} > 2\omega_{\max} \quad \text{or} \quad T_0 < \pi/\omega_{\max}. \tag{2.4.16}$$

This corresponds to a continuous angle function with angle frequency Ω_{max} to

$$\Omega_{\max} = \frac{2\pi}{\varphi_0} > 2\omega_{\max} \quad \text{or} \quad \varphi_0 < \pi/\Omega_{\max}. \tag{2.4.17}$$

Example 2.2 (Discrete models).
The continuous-time first-order process with dead time of Example 2.1 leads with a
zero-order hold to the z-transfer function

$$HG(z) = \frac{y(z)}{u(z)} = \frac{b_1 z^{-1}}{1 - a_1 z^{-1}} z^{-d}$$

$$a_1 = e^{-aT_0}$$

$$b_1 = \frac{1}{a}(1 - a_1)$$

$$d = T_t/T_0 = 0, 1, 2, \ldots$$

and with the shifting theorem to the difference equation

$$y(k) - a_1 y(k-1) = b_1 u(k-1-d).$$

Corresponding to this procedure one obtains for the discrete-angle first-order process
with dead angle and a zero-order hold

$$HG(\zeta) = \frac{y(\zeta)}{u(\zeta)} = \frac{b_1' \zeta^{-1}}{1 - a_1' \zeta^{-1}} \zeta^{-d}$$

$$a_1' = e^{-a'\varphi_0}$$

$$b_1' = \frac{1}{a'}(1 - a_1')$$

$$d' = \varphi_t/\varphi_0 = 0, 1, 2, \ldots$$

and the difference equation for angles follows

$$y(\kappa) - a_1' y(\kappa - 1) = b_1' u(\kappa - 1 - d').$$

It describes the crank-angle-dependent variable $y(\kappa)$ for a crank-angle-dependent
input variable $u(\kappa)$. Also here for a constant dead angle φ_t and different speeds ω_{eng}
the angle shift d' does not change, contrast to the discrete dead time d.

□

    The application of this crank-angle-dependent modeling was developed and ap-
plied in Schmidt (1995) to the torque models of a 4-cylinder diesel engine. Sampling
angle was selected such that the second harmonic of the ignition angle frequency
could be taken into account, resulting in $\varphi_0 = 30$ deg. Then, e.g. the development
of the combustion pressure and the resulting torque can be described for each sam-
pled crank angle $\varphi(\kappa\varphi_0)$. For constant speed this corresponds to the discrete times
$t_\kappa = \varphi(\kappa\varphi_0)/\omega_{\mathrm{eng}}$.
    Figure 2.4.1 depicts the scaling of the crank angle as mostly used in this book.

2.5 Semi-physical models

In general theoretical end experimental modeling complement each other. The theoretical model contains the functional description between the physical/chemical variables and includes their parameters. A considerable advantage is that the dependency of the parameters on construction data is usually explicitly known. Therefore this model is preferred for simulation before construction.

The experimental model on the other hand contains parameters as numerical values where functional relationships with physical basic process data remains unknown. However, the real stationary and dynamic behavior may be described more exactly or it can be determined at smaller expenditure by identification methods, which is, e.g. better suited for the calibration of control systems or fault detection methods.

Theoretical models are also called *"white-box models"* and experimental models *"black-box models"*. In many practical applications one has to use a suitable combination of both ways, compare Fig. 2.5.1.
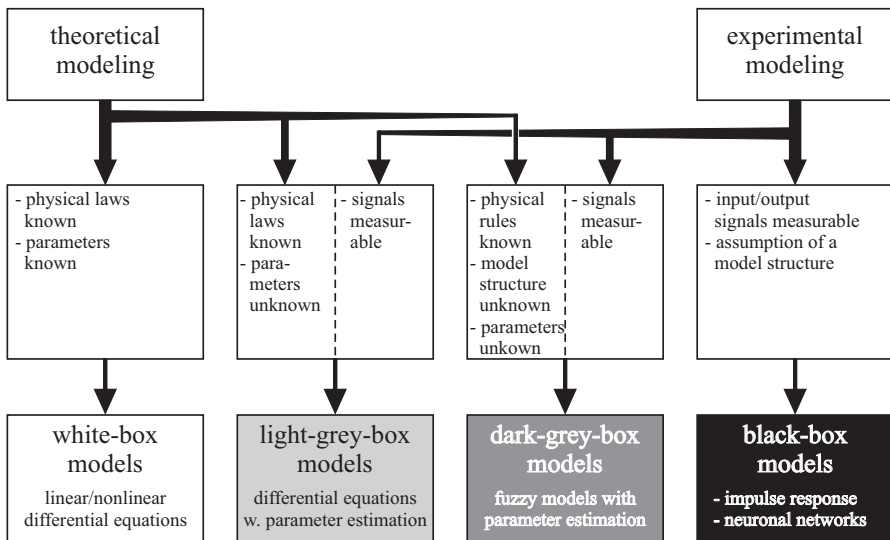

Fig. 2.5.1. Different kinds of mathematical process models.

If the physical laws are known, but the parameters not at all or not precisely enough, the parameters have to be determined experimentally, e.g. by parameter estimation methods. The resulting models can be called *"light-gray models"*. If only physical oriented if-then-rules are known, the model structure and the parameters have to be determined by experiments, leading to *"dark-gray models"*, for example by fuzzy if-then-rules and parameter adjustment. Both gray models can also be called *semi-physical models*.

These type of combined theoretical and experimental models are frequently the result in modeling combustion engines. For example the cylinder pressure is approximated by a special function, the Vibe function, and the parameters are estimated based on pressure measurements. In the case of exhaust gas emission formation only some rules are known and purely experimental approximations are mainly used.

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