Chapter 67
Rough Approximation Based Decentralized Bi-level Model for the Supply Chain Distribution Problem

Zhimiao Tao, Yuan Wang, Zhibin Wu and Jinwei Hu

Abstract This paper considers a core enterprise-dominant supply chain distribution modeling problem under a fuzzy environment. In particular, the common benefit and mutual interact of the upstream and downstream enterprise in the supply chain is considered. Thus, a decentralized bi-level programming model is constructed. To deal with the fuzzy parameters in the objective functions, an expected value operation based on Me is employed. As to the feasible region with fuzzy coefficient, a similarity relation based on the fuzzy measure Pos is defined, based on which, the rough approximation method is adopted. Then, two rough approximation models (UAM and LAM) is developed. To solve the two models, a rough simulation is developed, after which the fuzzy interactive programming and genetic algorithm can be adopted to find the solutions.

Keywords Rough approximation · Decentralized bi-level programming · Supply chain · Rough simulation · Fuzzy environment

67.1 Introduction

With the increasing competition, the effectiveness of supply chain management is more and more important. A well organized supply chain distribution system can simplify the sales section, decrease the selling cost, increase the satisfactory degree of the supply chain members, maximize the corporate profit and improve the final performance [8]. According to former reaches, the core enterprise in a supply chain is the key role in leading function and promoting performance. Hence, how the

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core enterprise organize the resource effectively, handle the information rapidly and consider the interaction of upstream and downstream enterprise is vital to increase the member satisfactory degree and improve the supply chain stability and virtuous circle.

In the former researches about distribution system, some scholars from the qualitative point of view, such as Ji [4] and Guo [2]. Some scholars also from quantitative point of view. Hamedi [3] constructed a mixed integer non-linear programming model for a single-objective, multi-period, and single-product problem. Moreover, Bilge Bilgen [1] constructed a fuzzy model taking into account the fuzziness. Also, Selim [7] constructed a multi-objective linear programming mode into which decision makersimprecise aspiration levels for the goals are incorporated using fuzzy goal programming approach. However, these researches neglected the interaction of upstream and downstream enterprise. So, a bi-level programming model is constructed in this paper. Meanwhile, as the consumer demand is influenced by many factors, such as product price, quality, consumer preference, even season, it is with a high level of uncertainty. Considering lacking of data, the uncertainty is regarded as fuzzyness as the uncertain parameters can be estimated by some experts. The usual method to handle the fuzzyness is the expected value. When the constraints also contain fuzzy parameters, the expected operator may lose some information even a optimal solution. Rough set theory, which was introduced by Pawlak [5] gives us an other direction to handle this problem. Rough approximation method has been applied to some models successfully [9, 10]. In this paper, it is adopted to handle the constraints featured by fuzzy parameter to make relatively a expanding and a shrinking feasible region.

Above all, the aim of this paper is to develop a decentralized bi-level model considering the interaction of upstream and downstream enterprise and common benefit. Meanwhile, considering the uncertain phenomenon, fuzzy variables are recommended. To handle the fuzzy variables, expected value based on Me and rough approximation are adopted, through which the model is transformed into two approximation models, i.e. the upper approximation model (UAM) and the lower approximation model (LAM). Finally, to solve the models, rough simulation is developed. After the simulation, the models can be solved by fuzzy interactive programming and genetic algorithm.

67.2 Key Problem Statement

This paper considers a supply chain consisting of a manufacturer, which is the core enterprise, and N distributors. The N distributors have different retail prices and quality levels because of their different wholesale prices and sale costs. The quality level here includes the service level and business reputation which the customs perceive. They competed with each other. As the manufacturer is the core enterprise, the objective of optimizing the supply chain is maximizing its total income, to which is leaded by a high sales volume. To stimulate the distributors to order more products, the manufacturer makes a buy back contract with each distributor, which allows the distributors sell the remaining products back to the manufacturer at a fixed price. In this supply chain, all the distributors and the manufacturer want the maximizing income. For the manufacturer, he/she should set appropriate wholesale prices, buy back prices and order quantities of each distributor to maximize its total income. For the distributors, they should make appropriate price and quality level according to the decisions of the manufacturer to make the income maximized. In the process of making a contract, there exist negotiations between them. However, as the manufacturer is the core enterprise, he/she is in a strong standing throughout the negotiation, which results in the distributors' signing contracts meeting the manufacturer's demand. Hence, the relationship between them can be regarded as hierarchical. The manufacturer is the leader, while the distributors are the followers. The manufacturer makes decisions first. Then, the distributors make their decisions accordingly. So, the problem considered in this paper can be regarded as a decentralized bi-level programming problem.

67.3 Modelling

67.3.1 Assumptions

To construct a model for the problem mentioned above, the following assumptions are adopted.

- N distributors are in the same market, they compete with each other based on retail price and quality level (business reputation and service level, etc.).
- The product in this supply chain is perishable, which can't be stored.
- The remaining products have residual value.
- The manufacturer is the upper-level DM, while the distributors are the lower-level DMs.

67.3.2 Objective Functions

(1) Objective function of the upper-level model

In this bi-level programming problem, the manufacturer is the leader. The objective of he/she is maximizing the total income through deciding the appropriate wholesale prices, buy back prices and the order quantities of each distributor. The function of it can be formulated as:

$$
\begin{aligned}
\widetilde{I} = & \sum_{n=1}^{N}(f_n-c)d_n-\sum_{n=1}^{N}(h_n-v)(d_n-\tilde{q}_n)\\
= & \sum_{n=1}^{N}(f_n-c)d_n-\sum_{n=1}^{N}(h_n-v)\Bigg(d_n-\Bigg(\tilde{a}-\sum_{j\neq n}\tilde{a}_j\Bigg)+\tilde{b}_n p_n\\
& -\sum_{j\neq n}\tilde{c}_j p_j-\tilde{\alpha}_n m_n+\sum_{j\neq n}\tilde{\beta}_j m_j\Bigg),
\end{aligned}
\tag{67.1}
$$

where f_n, d_n, h_n represent the wholesale price, buy back price and order quantity of the n-th distributor, respectively. c represents the unit cost of product. v represents the unit residual value of the remaining product.

(2) Objective function of the lower-level model

According to the decisions that manufacturer makes, i.e. the wholesale price, buy back price and order quantity of each distributor, the distributors decide the retail price and service level to maximize their income. The function can be formulated as:

$$
\begin{aligned}
\tilde{i}_n = & (p_n-c_n)\tilde{q}_n-f_n d_n+h_2(d_n-\tilde{q}_n)-\tfrac{l_n m_n^2}{2}\\
= & (p_n-c_n)\Bigg(\Bigg(\tilde{a}-\sum_{j\neq n}\tilde{a}_j\Bigg)-\tilde{b}_n p_n+\sum_{j\neq n}\tilde{c}_j p_j+\tilde{\alpha}_n m_n-\sum_{j\neq n}\tilde{\beta}_j m_j\Bigg)-f_n d_n\\
& +h_2\Bigg(d_n-\Bigg(\Bigg(\tilde{a}-\sum_{j\neq n}\tilde{a}_j\Bigg)-\tilde{b}_n p_n+\sum_{j\neq n}\tilde{c}_j p_j+\tilde{\alpha}_n m_n-\sum_{j\neq n}\tilde{\beta}_j m_j\Bigg)\Bigg)-\tfrac{l_n m_n^2}{2},
\end{aligned}
\tag{67.2}
$$

where p_n, m_n are decision variables, representing the retail price and quality level of the n-th distributor. c_n represents the selling cost of the n-th distributor. $\frac{l_n m_n^2}{2}$ represents the cost of quality level m_n. l_n represents the cost coefficient of quality level.

67.3.3 Constraints

According to the problem, it is easy to know that the constraints of the upper-level model is:

$$
\begin{cases}
d_n > 0, & n = 1,2,\cdots,N,\\
f_n > c, & n = 1,2,\cdots,N,\\
h_n > v, & n = 1,2,\cdots,N.
\end{cases}
\tag{67.3}
$$

Then consider the lower-level model. In the real world, government or the industry association may set limitation values of price and service level in some industry. Hence, the retail price should be less than the limitation value and the quality level should be more than the limitation value. Moreover, From a long-term point of view, the distributor should increase their sales volume gradually to get attention of the manufacturer, which will lead to lower wholesale price resulting in more profit. So, there should be a lower limit for the sales volume. Whole the constraints can be represented as:

$$\begin{cases} \left(\tilde{a} - \sum_{j \neq n} \tilde{a}_j\right) - \tilde{b}_n p_n + \tilde{\alpha}_n m_n \geq S_{\min}, \\ p_n \leq p_{\max}, \\ m_n \geq m_{\min}. \end{cases} \tag{67.4}$$

The first formula represents the lower limit of sales volume, which is the minimal sales volume without considering the other competitors.

67.3.4 Global Model

From the discuss above, the global model can be represented as:

$$\begin{cases} \max \widetilde{I} = \sum_{n=1}^{N} (f_n - c) d_n - \sum_{n=1}^{N} (h_n - v) \left(d_n - \left(\tilde{a} - \sum_{j \neq n} \tilde{a}_j\right) + \tilde{b}_n p_n \right. \\ \qquad \left. - \sum_{j \neq n} \tilde{c}_j p_j - \tilde{\alpha}_n m_n + \sum_{j \neq n} \tilde{\beta}_j m_j \right), \\ \text{s.t.} \begin{cases} d_n > 0,\ n = 1, 2, \cdots, N, \\ f_n > c,\ n = 1, 2, \cdots, N, \\ h_n > v,\ n = 1, 2, \cdots, N, \end{cases} \\ \text{where } \ p_n, m_n (n = 1, 2, \cdots, N) \text{ solve} \\ \max \tilde{i}_n = (p_n - c_n) \left(\left(\tilde{a} - \sum_{j \neq n} \tilde{a}_j\right) - \tilde{b}_n p_n + \sum_{j \neq n} \tilde{c}_j p_j + \tilde{\alpha}_n m_n \right. \\ \qquad \left. - \sum_{j \neq n} \tilde{\beta}_j m_j \right) - f_n d_n + h_2 \left(d_n - \left(\left(\tilde{a} - \sum_{j \neq n} \tilde{a}_j\right) - \tilde{b}_n p_n \right. \right. \\ \qquad \left. \left. + \sum_{j \neq n} \tilde{c}_j p_j + \tilde{\alpha}_n m_n - \sum_{j \neq n} \tilde{\beta}_j m_j \right) \right) - \frac{l_n m_n^2}{2}, \\ \text{s.t.} \begin{cases} \left(\tilde{a} - \sum_{j \neq n} \tilde{a}_j\right) - \tilde{b}_n p_n + \tilde{\alpha}_n m_n \geq S_{\min}, \\ p_n \leq p_{\max}, \\ m_n \geq m_{\min}, \end{cases} \\ n = 1, 2, \cdots, N. \end{cases} \tag{67.5}$$

67.4 Model Analysis

Noticing that both the objectives and constraints of model (67.5) contains fuzzy parameters. The objective functions with fuzzy parameters represent a vague goal, which can't be maximized. Moreover, the constraints with fuzzy parameters define a uncertain feasible region. Hence, it should be converted into a crisp model before solving it.

67.4.1 Expected Value Based on Me

To convert the model (67.5) to be a crisp one, the expected value based on *Me* is adopted. There are many definitions for expected value in fuzzy theory. There are definitions based on *Pos*, Nec and Cr. The measure Pos represents an absolutely optimistic attitude while Nec represents an absolutely pessimistic attitude. The measure Cr represents a composite attitude which combines half optimistic and half pessimistic. In a realistic decision problem, however, the attitudes of the different decision makers are different and therefore may be not absolutely optimistic or pessimistic, or half optimistic and half pessimistic, so, the expected value based on Me is adopted, which is defined as: $\text{Me} = \lambda\text{Pos} + (1-\lambda)\text{Nec}$, where λ is the optimistic-pessimistic parameter to determine the combined attitude of the decision makers. The expected value based on Me [11] of a fuzzy variable ξ is defined as:

$$E[\xi] = \int_0^{+\infty} Me\{\xi \geq r\}dr - \int_{-\infty}^0 Me\{\xi \leq r\}dr. \tag{67.6}$$

Using the definition of expected value based on Me, the objectives can be clearly formulated. The objective of the upper-level model is:

$$\begin{aligned}
\max E(\widetilde{I}) = \ & E\Bigg(\sum_{n=1}^{N}(f_n - c)d_n - \sum_{n=1}^{N}(h_n - v)\Bigg(d_n - \Bigg(\tilde{a} - \sum_{j\neq n}\tilde{a}_j\Bigg) + \tilde{b}_n p_n, \\
& - \sum_{j\neq n}\tilde{c}_j p_j - \tilde{\alpha}_n m_n + \sum_{j\neq n}\tilde{\beta}_j m_j\Bigg)\Bigg), \qquad (67.7)\\
\max E(\tilde{i}_n) = \ & E\Bigg((p_n - c_n)\Bigg(\Bigg(\tilde{a} - \sum_{j\neq n}\tilde{a}_j\Bigg) - \tilde{b}_n p_n + \sum_{j\neq n}\tilde{c}_j p_j + \tilde{\alpha}_n m_n - \sum_{j\neq n}\tilde{\beta}_j m_j\Bigg) \\
& - f_n d_n + h_2\Bigg(d_n - \Bigg(\Bigg(\tilde{a} - \sum_{j\neq n}\tilde{a}_j\Bigg) - \tilde{b}_n p_n \\
& + \sum_{j\neq n}\tilde{c}_j p_j + \tilde{\alpha}_n m_n - \sum_{j\neq n}\tilde{\beta}_j m_j\Bigg)\Bigg) - \frac{l_n m_n^2}{2}\Bigg). \qquad (67.8)
\end{aligned}$$

As the upper and lower boundary and the value with the most possibility of the fuzzy parameters in Equations (67.7) and (67.8) can be estimated by the experts with specialized knowledge and experience, they can be regarded as triangle fuzzy numbers. If $\tilde{a} = (a_1, a_2, a_3)$ (assume that other fuzzy numbers also have the similar form), the expected value of it can be represented as below:

$$E^{Me}[\tilde{a}] = \frac{(1-\lambda)a_1 + a_2 + \lambda a_3}{2}, \tag{67.9}$$

where λ is the optimistic-pessimistic index to determine the combined attitude of a decision maker. The expected value of other fuzzy numbers can also be represented as similar form. Then, the expected value of the consumer demand function can be formulated as:

$$
\begin{aligned}
E(\tilde{q}) = {} & \frac{(1-\lambda)a_1 + a_2 + \lambda a_3}{2} - \sum_{j\neq n} \frac{(1-\lambda)a_{j1} + a_{j2} + \lambda a_j 3}{2} \\
& - \frac{(1-\lambda)b_{n1} + b_{n2} + \lambda b_{n3}}{2} p_n + \sum_{j\neq n} \frac{(1-\lambda)\alpha_{n1} + \alpha_{n2} + \lambda \alpha_{n3}}{2} m_n \\
& - \sum_{j\neq n} \frac{(1-\lambda)\beta_{j1} + \beta_{j2} + \lambda \beta_{j3}}{2} m_j. \qquad (67.10)
\end{aligned}
$$

Then, Equations (67.7), (67.8) can be transformed into:

$$\max E(\widetilde{I}) = \sum_{n=1}^{N} (f_n - c)d_n - \sum_{n=1}^{N} (h_n - v)(d_n - E(\tilde{q})), \qquad (67.11)$$

$$\max E(\tilde{i}_n) = (p_n - c_n)E(\tilde{q}) - f_n d_n + h_n(d_n - E(\tilde{q})) - \frac{l_n m_n^2}{2}, \qquad (67.12)$$

where $E(\tilde{q})$ represents the value of Equation (67.10).

67.4.2 Rough Approximation

As mentioned above, the feasible region of model (67.5) is uncertain, the feasibility of a solution can't be judged. Using the expected value or fuzzy measure to handle it may lose many useful information. In this paper, rough approximation is adopted.

Assume $x_1 = (f_1^1, h_1^1, d_1^1, \cdots, f_n^1, h_n^1, d_n^1, p_1^1, m_1^1, \cdots, p_n^1, m_n^1)$ and $x_2 = (f_1^2, h_1^2, d_1^2, \cdots, f_n^2, h_n^2, d_n^2, p_1^2, m_1^2, \cdots, p_n^2, m_n^2)$ are two solutions of the model. Based on the definition of the *Pos* measure, the following similarity relationship R_h^δ can be defined for the constraints:

$$
\begin{aligned}
& R_h^\delta(x^1, x^2) : Pos\left\{ \left(\tilde{a} - \sum_{j\neq n} \tilde{a}_j \right) - \tilde{b}_n(p_n^1 - p_n^2) + \tilde{\alpha}_n(m_n^1 - m_n^2) \leq h \right\} \geq \delta, \\
& \quad (n = 1, 2, \cdots, N), \qquad (67.13)
\end{aligned}
$$

where δ is the confidence level, and h is the deviation which the DM permits. Here, the measure Pos is used as it determines a relatively expanding feasible region where more information and a possible solution may be combined.

For the relationship, there are:

$$
\begin{aligned}
& Pos\left\{ \left(\tilde{a} - \sum_{j\neq n} \tilde{a}_j \right) - \tilde{b}_n(p_n^1 - p_n^1) + \tilde{\alpha}_n(m_n^1 - m_n^1) \leq h \right\} \\
& = Pos\left\{ \left(\tilde{a} - \sum_{j\neq n} \tilde{a}_j \right) \right\} > 1 \geq \delta \qquad (67.14)
\end{aligned}
$$

and

$$Pos\{(\tilde{a} - \sum_{j\neq n} \tilde{a}_j) - \tilde{b}_n(p_n^1 - p_n^2) + \tilde{\alpha}_n(m_n^1 - m_n^2) \leq h\} \geq \delta$$
$$\Longleftrightarrow Pos\left\{\left(\tilde{a} - \sum_{j\neq n} \tilde{a}_j\right) - \tilde{b}_n(p_n^2 - p_n^1) + \tilde{\alpha}_n(m_n^2 - m_n^1) \leq h\right\} \geq \delta, \quad (67.15)$$

That is, the relationship has reflexivity and symmetry. Then R_h^δ is a similarity relationship.

With the similarity relationship, the constraints of the contractor-level model can be formulated as:

$$\begin{cases} Pos\left\{\left(\tilde{a} - \sum_{j\neq n} \tilde{a}_j\right) - \tilde{b}_n p_n + \tilde{\alpha}_n m_n \geq S_{\min} - h^*\right\} \geq \delta, \\ h^* = \max\left\{h | \frac{|\underline{X_h}|}{|\overline{X_h}|} \geq \rho\right\}, \\ |\underline{X_h}| = \int\int_{\underline{X_h}} 1 dp_n dm_n, \\ |\overline{X_h}| = \int\int_{\overline{X_h}} 1 dp_n dm_n, \\ p_n \leq p_{\max}, \\ m_n \geq M_{\min} \end{cases} \quad (67.16)$$

or

$$\begin{cases} Pos\left\{\left(\tilde{a} - \sum_{j\neq n} \tilde{a}_j\right) - \tilde{b}_n p_n + \tilde{\alpha}_n m_n \geq S_{\min} + h^*\right\} \geq \delta, \\ h^* = \max\left\{h | \frac{|\underline{X_h}|}{|\overline{X_h}|} \geq \rho\right\}, \\ |\underline{X_h}| = \int\int_{\underline{X_h}} 1 dp_n dm_n, \\ |\overline{X_h}| = \int\int_{\overline{X_h}} 1 dp_n dm_n, \\ p_n \leq p_{\max}, \\ m_n \geq M_{\min}, \end{cases} \quad (67.17)$$

where the region represented by Equation (67.16) is the R-upper approximation of the feasible region for the contractor. While the one at Equation (67.17) is the R-lower approximation. Here, the second equation guarantees the required accuracy is achieved. The third and the fourth equations express the cardinal numbers of $\underline{X_h}$ and $\overline{X_h}$, respectively as they are a finite and discrete set. $\underline{x_{ij}}$ and $\overline{x_{ij}}$ represent the following regions, respectively.

$$\begin{cases} Pos\left\{\left(\tilde{a} - \sum_{j\neq n} \tilde{a}_j\right) - \tilde{b}_n p_n + \tilde{\alpha}_n m_n \geq S_{\min} - h\right\} \geq \delta, \\ h^* = \max\left\{h | \frac{|\underline{X_h}|}{|\overline{X_h}|} \geq \rho\right\}, \\ p_n \leq p_{\max}, \\ m_n \geq M_{\min} \end{cases} \quad (67.18)$$

and

$$\begin{cases} Pos\left\{\left(\tilde{a} - \sum\limits_{j\neq n} \tilde{a}_j\right) - \tilde{b}_n p_n + \tilde{\alpha}_n m_n \geq S_{\min} + h\right\} \geq \delta, \\ h^* = \max\left\{h | \frac{|\underline{X_h}|}{|\overline{X_h}|} \geq \rho\right\}, \\ p_n \leq p_{\max}, \\ m_n \geq M_{\min}. \end{cases} \tag{67.19}$$

Also, from the relevant theorems [11], the following conclusions can be derived:

$$\begin{aligned} &Pos\{(\tilde{a} - \sum_{j\neq n} \tilde{a}_j) - \tilde{b}_n p_n + \tilde{\alpha}_n m_n \geq S_{min} - h^*\} \geq \delta \\ &\Longleftrightarrow (a_2 - \sum_{j\neq n} a_{j2}) - b_{n2} p_n + \tilde{\alpha}_{n2} m_n + (1 - \delta(((a_3 - a_2) - \sum_{j\neq n} (a_{j3} - a_{j2}) \\ &\quad -(b_{n3} - b_{n2}) p_n + (\alpha_{n3} - \alpha_{n2}) m_n) \geq S_{min} - h^*, \end{aligned} \tag{67.20}$$

$$\begin{aligned} &Pos\{(\tilde{a} - \sum_{j\neq n} \tilde{a}_j) - \tilde{b}_n p_n + \tilde{\alpha}_n m_n \geq S_{min} + h^*\} \geq \delta \\ &\Longleftrightarrow (a_2 - \sum_{j\neq n} a_{j2}) - b_{n2} p_n + \tilde{\alpha}_{n2} m_n + (1 - \delta(((a_3 - a_2) - \sum_{j\neq n} (a_{j3} - a_{j2}) \\ &\quad -(b_{n3} - b_{n2}) p_n + (\alpha_{n3} - \alpha_{n2}) m_n) \geq S_{min} + h^*. \end{aligned} \tag{67.21}$$

67.4.3 Equivalent Crisp Global Model

After the handling above, model (67.5) can be transformed into two crisp models (67.22) and (67.23), which are the upper-approximation model (UAM) and the lower-approximation model (LAM). These two models, which are with a relative expanding and shrinking feasible region, respectively, provide the DMs different solutions. For the completely optimistic DMs, they can choose the solution of the UAM, while the DMs with completely pessimistic attitude can choose the solution of the LAM. Other DMs can choose the linear combination of the two solutions to find a solution that fits the attitude of them.

$$
(\mathrm{UAM})\begin{cases}
\max E(\widetilde{I}) = \sum\limits_{n=1}^{N}(f_n - c)d_n - \sum\limits_{n=1}^{N}(h_n - v)(d_n - E(\tilde{q})),\\
\text{s.t.}\begin{cases} d_n > 0,\ n = 1,2,\cdots,N,\\ f_n > c,\ n = 1,2,\cdots,N,\\ h_n > v,\ n = 1,2,\cdots,N,\end{cases}\\
\text{where } \ p_n, m_n (n = 1,2,\cdots,N) \text{ solve}\\
\max E(\tilde{i}_n) = (p_n - c_n)E(\tilde{q}) - f_n d_n + h_n(d_n - E(\tilde{q})) - \frac{l_n m_n^2}{2},\\
\text{s.t.}\begin{cases}
\left(a_2 - \sum\limits_{j\neq n} a_{j2}\right) - b_{n2}p_n + \tilde{\alpha}_{n2}m_n + (1 - \delta(((a_3 - a_2)\\
\qquad - \sum\limits_{j\neq n}(a_{j3} - a_{j2}) - (b_{n3} - b_{n2})p_n\\
\qquad + (\alpha_{n3} - \alpha_{n2})m_n) \geq S_{\min} - h^*,\\
h^* = \max\left\{h \middle| \frac{|\underline{X_h}|}{|\overline{X_h}|} \geq \rho\right\},\\
|\underline{X_h}| = \int\int_{\underline{X_h}} 1 d p_n d m_n,\\
|\overline{X_h}| = \int\int_{\overline{X_h}} 1 d p_n d m_n,\\
p_n \leq p_{\max},\ m_n \geq M_{\min}, n = 1,2,\cdots,N
\end{cases}
\end{cases}
\tag{67.22}
$$

and

$$
(\mathrm{LAM})\begin{cases}
\max E(\widetilde{I}) = \sum\limits_{n=1}^{N}(f_n - c)d_n - \sum\limits_{n=1}^{N}(h_n - v)(d_n - E(\tilde{q})),\\
\text{s.t.}\begin{cases} d_n > 0,\ n = 1,2,\cdots,N,\\ f_n > c,\ n = 1,2,\cdots,N,\\ h_n > v,\ n = 1,2,\cdots,N,\end{cases}\\
\text{where } \ p_n, m_n (n = 1,2,\cdots,N) \text{ solves}\\
\max E(\tilde{i}_n) = (p_n - c_n)E(\tilde{q}) - f_n d_n + h_n(d_n - E(\tilde{q})) - \frac{l_n m_n^2}{2},\\
\text{s.t.}\begin{cases}
\left(a_2 - \sum\limits_{j\neq n} a_{j2}\right) - b_{n2}p_n + \tilde{\alpha}_{n2}m_n + (1 - \delta(((a_3 - a_2)\\
\qquad - \sum\limits_{j\neq n}(a_{j3} - a_{j2}) - (b_{n3} - b_{n2})p_n\\
\qquad + (\alpha_{n3} - \alpha_{n2})m_n) \geq S_{\min} + h^*,\\
h^* = \max\left\{h \middle| \frac{|\underline{X_h}|}{|\overline{X_h}|} \geq \rho\right\},\\
|\underline{X_h}| = \int\int_{\underline{X_h}} 1 d p_n d m_n,\\
|\overline{X_h}| = \int\int_{\overline{X_h}} 1 d p_n d m_n,\\
p_n \leq p_{\max},\ m_n \geq M_{\min},\ n = 1,2,\cdots,N.
\end{cases}
\end{cases}
\tag{67.23}
$$

67.5 Solution Approach

To solve models (67.22) and (67.23), we design a so-called rough simulation-based hybrid genetic algorithm (RS-based hGA), which is a combination of rough simulation, fuzzy interactive programming [6] and genetic algorithm.

67.5.1 Rough Simulation

Consider the following constraint:

$$h^* = \max\left\{ h | \frac{|\underline{X}_{h_1}|}{|\overline{X}_{h_1}|} \geq \rho \right\}. \tag{67.24}$$

Step 1. Randomly generate two numbers $h_1$and h_2 such that according to the relationship $R^{\delta}_{h_1}$ and $R^{\delta}_{h_2}$,

$$\frac{|\underline{X}_{h_1}|}{|\overline{X}_{h_1}|} \geq \rho,\ \frac{|\underline{X}_{h_2}|}{|\overline{X}_{h_2}|} < \rho. \tag{67.25}$$

Step 2. Let $h = (h_1 + h_2)/2$.
Step 3. If $\frac{|\underline{X}_h|}{|\overline{X}_h|} \geq \rho$, let $h_1 = h$; Otherwise, let $h_2 = h$.
Step 4. If $|h_1 - h_2| \geq \varepsilon$(a given small positive number), go to the step 2; otherwise, let $h^* = h_1$. Then, h^* is the maximal h.

67.5.2 Hybrid Genetic Algorithm

Since bilevel model has special structure, simple genetic algorithm can not balance the relationship between the UAM and LAM. We embed the fuzzy interactive programming method in to genetic algorithm. The key part is the setting of fitness function. Consider the satisfaction levels of the upper DM and lower DM in fuzzy interactive algorithm, let X be the feasible regions and

$$\begin{aligned} I_{\min} &= I(x_1^0) = \min_{x \in X} I(x),\ i^n_{\min} = i^n(x_n) = \min_{x \in X} i^n(x)\ (n = 1, 2, \cdots, N), \\ I_m &= \max\{I(x_1^0), I(x_1), \cdots, I(x_N)\},\ i^n_m = \max\{i_n(x_1^0), i_n(x_1), \cdots, i_n(x_N)\} \\ &\quad (n = 1, 2, \cdots, N). \end{aligned}$$

Then, the membership function of each objective is formulated as:

$$\mu(I(x)) = \begin{cases} 1, & \text{if } C(x) \geq C_m, \\ \frac{C(x) - C_m}{C_{min} - C_m}, & \text{if } C_{\min} \leq C(x) \leq C_m, \\ 0, & \text{if } C(x) \leq C_{\min}, \end{cases}$$

$$\mu_n(i_n(x)) = \begin{cases} 1, & \text{if } i_n(x) \geq i^n_m, \\ \frac{i_n(x) - i^n_m}{i^n_{\min} - i^n_m}, & \text{if } i^n_{\min} \leq i_n(x) \leq i^n_m, \\ 0, & \text{if } i_n(x) \leq i^n_m. \end{cases}$$

To guarantee the equity between the upper DM and the lower DM, the following ratios of satisfactory degrees is introduced to balance the satisfactory degrees between the two levels:

$$\Delta = \frac{\min\{\mu_1(i_1(x)), \mu_2(i_2(x)), \cdots, \mu_N(i_N(x))\}}{\mu(I(x))}, \tag{67.26}$$

$$\Delta_n = \frac{\mu_n(i_n(x))}{\mu(I(x))}, \tag{67.27}$$

where Equation (67.26) balances the satisfactory degrees between the upper level and lower level; Equation (67.26) balances the satisfactory degrees among the lower level DMs. The upper DM can set satisfactory ration of maximal satisfactory degree and minimal satisfactory degree such that $\Delta \in [\Delta_{\min}, \Delta_{\max}]$ and $\Delta_n \in [\Delta^0_{\min}, \Delta^0_{\max}]$.

From the formulation of satisfactory degrees, denote a chromosome and its corresponding solution by s and x. Then adopt the following satisfactory degree:

$$f_s = \max\{\mu(I(x)), \zeta(x)\}, \tag{67.28}$$

where

$$\zeta(x) = \begin{cases} 1, & \text{if } i_n(x) \geq i^n_m, \\ \frac{i_n(x)-i^n_m}{i^n_{\min}-i^n_m}, & \text{if } i^n_{\min} \leq i_n(x) \leq i^n_m, \\ 0, & \text{if } i_n(x) \leq i^n_m. \end{cases} \tag{67.29}$$

As the above fitness function is adopted, the solution is not only balance the the satisfactory degrees between upper level and lower level but also balance the the satisfactory degrees among lower level DMs. The lower level DM can adjust satisfactory ratio of maximum and minimum until he/she obtain satisfactory solutions.

We state the rough simulation-based genetic algorithm procedure as follows:

Step 1. Input the parameters $N_{pop-size}, P_c$ and P_m.
Step 2. Initialize $N_{pop-size}$ chromosomes whose feasibility may be checked by rough simulation.
Step 3. Update the chromosomes by crossover and mutation operations and rough simulation is used to check the feasibility of offspring.
Step 4. Compute the fitness of each chromosome.
Step 5. Select the chromosomes by spinning the roulette wheel.
Step 6. Repeat Step 3 to Step 5 for a given number of cycles.
Step 7. Return the best chromosome as the optimal solution.

67.6 Numerical Example

To demonstrate the feasibility of the proposed decentralized bi-level programming model and RS-based hGA, consider the following supply chain distribution decision-making problem.

Assume that there are only two supplier. The parameters values are $\tilde{a} = (9, 10, 11)$, $\tilde{a}_1 = (1, 2, 3), \tilde{a}_2 = (2, 3, 4), \tilde{b}_1 = (1, 2, 3), \tilde{b}_2 = (1, 2, 3), \tilde{c}_1 = (2, 3, 4), \tilde{c}_2 = (1, 2, 3)$, $\tilde{\alpha}_1 = (2, 3, 4), \tilde{\alpha}_2 = (3, 4, 5), \tilde{\beta}_1 = (0.8, 1, 0.2), \tilde{\beta}_1 = (0.9, 1.1, 1.2), c = 20, v = 8, c_1 =$

$10, c_2 = 9, l_1 = 7, l_2 = 5, S_{\min} = 9, p_{\max} = 6, m_{\min} = 2$. Take these value into model (67.5), we obtain the global model for this supply chain distribution decision-making problem. For solving model, fuzzy parameters are tackled firstly. By substituting values into Equations (67.11), (67.12), (67.16) and (67.17), and it follows from Equations (67.20) and (67.21), we obtain UAM and LAM as Equations (67.21) and (67.22), respectively. These two models are RS-based hGA and ran on MATLAB. Let $\rho = 0.8$ and obtain $h* = 4.8172$ by rough simulation. Let $\lambda = 0.7$ and use hGA to solve the two models. Essential parameters are: $N_p = 50$, $P_c = 0.5$, $P_m = 0.05$, $N_g = 200$, $[\Delta_{\min}, \Delta_{\max}] = [0.75, 1]$, $[\delta^0_{\min}, \delta^0_{\max}] = [0.65, 1]$. The results of UAM are: $d_1 = 11.3812$, $d_2 = 14.5173$, $f_1 = 24.2269$, $f_2 = 23.5671$, $h_1 = 9.0173$, $h_2 = 8.8524$, $p_1 = 38.5024$, $p_2 = 37.6083$, $m_1 = 2.0244$, $m_2 = 2.1208$. The results of LAM are: $d_1 = 10.5876$, $d_2 = 13.2731$, $f_1 = 23.5824$, $f_2 = 22.6723$, $h_1 = 8.2347$, $h_2 = 8.7236$, $p_1 = 37.6238$, $p_2 = 36.8234$, $m_1 = 2.0239$, $m_2 = 2.0989$. The difference between the results of UAM and LAM reflects the DM's different risk attitudes. The results can provide flexible choices for DM. If the core enterprise are not satisfied for the results, he/she can adjust the values of $\Delta_{\min}$, $\Delta_{\max}$, $\Delta^0_{\min}$, $\Delta^0_{\max}$.

By running the above algorithm to solve UAM, we find the variance is small (0.5824), which shows the stability of the algorithm.

67.7 Conclusions

In this paper, a distribution system modelling problem in which the manufacturer is the core enterprise is considered. After the problem analysis, a decentralized bi-level programming model with fuzzy parameters is constructed.

Through the model proposed in this paper, the benefit of each enterprise in the supply chain distribution system is well balanced, and the uncertain phenomenon is well handled. Future reach may focus on the different uncertain environment (e.g. random) in the supply chain distribution system.

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