

# U.S. Business Cycles, Monetary Policy and the External Finance Premium

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**Abstract** I investigate a model of the U.S. economy with nominal rigidities and a financial accelerator mechanism à la Bernanke et al. (Handbook of Macroeconomics, Vol. 1, Elsevier Sci., Chap. 21, pp. 1341–1393, 1999). I calculate total factor productivity (TFP) and monetary policy deviations for the U.S. and quantitatively explore the ability of the model to account for the cyclical patterns of U.S. per capita real private GDP (excluding government), U.S. per capita real private investment, U.S. per capita real private consumption, the share of hours worked per capita, (year-over-year) inflation and the quarterly interest rate spread between the Baa corporate bond yield and the 20-year Treasury bill rate during the Great Moderation. I show that the magnitude and cyclicity of the external finance premium depend nonlinearly on the degree of price stickiness (or lack thereof) in the Bernanke et al. (Handbook of Macroeconomics, Vol. 1, Elsevier Sci., Chap. 21, pp. 1341–1393, 1999) model and on the specification of both the target Taylor (Carnegie-Rochester Conf. Ser. Pub. Policy 39:195–214, 1993) rate for policy and the exogenous monetary shock process. The strong countercyclicality of the external finance premium (the interest rate spread) induces substitution away from consumption and into investment in periods where output grows above its long-run trend as the premium tends to fall below its steady state and financing investment becomes temporarily “cheaper”. The less frequently prices change in this environment, the more accentuated the fluctuations of the external finance premium are and the more dominant they become on the dynamics of investment, hours worked and output. However, these features—the countercyclicality and large volatility of the spread—are counterfactual and appear to be a key impediment limiting the ability of the model to account for the U.S. data over the Great Moderation period.

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# 1 Introduction

The 2007 recession has led to renewed concern about the role of the financial system among researchers and policymakers alike. The ‘credit crunch’ in the U.S. has focused attention back on the determinants of lending and the impact of financing conditions on the transmission mechanism for monetary policy. However, the standard variants of the New Keynesian framework that had become dominant for the analysis of monetary policy since the 1990s (see, e.g., Woodford 2003 and Galí 2008) typically abstract from financial frictions. Evidence from past banking crises and the 2007 downturn suggests—or, at least, has re-invigorated the view—that the role of the financial channel may be important in the propagation and amplification of shocks.

The role of monetary policy rules and their interaction with financial frictions has become also an issue of first-order importance in academic and policy circles. Indeed, the monetary authorities’ reaction—both in the U.S. and other major industrialized countries—has been unusual during the current episode and very aggressive relative to their prior experience over the past 25 years of the so-called Great Moderation. In this context, the role of monetary policy is once again being hotly contested. A heated debate about the scope of monetary policy and the contribution to business cycles of deviations from well-established policy rules such as Taylor (1993)’s rule has ensued, and it is likely to continue for a long time.

To provide a quantitative analysis of the issues raised by these ongoing policy debates, I focus my attention on the nexus between monetary policy and financial frictions. In particular, I ask how one can evaluate the macroeconomic performance of monetary policy in an environment where policymakers understand that the nominal short-term interest rate they control—net of inflation—is not equal to the marginal lending rates that determine the cost of borrowing for economic agents—in other words, in economic environments where there is a non-trivial spread between the actual cost of borrowing and the real risk-free rate.

In a conventional New Keynesian model with no financial frictions, the transmission mechanism for monetary policy is rather stylized. Borrowing and lending has no impact on the monetary transmission mechanism and, consequently, no real effects. In a world with financial frictions, the implications of the Modigliani-Miller theorem no longer hold and the capital structure of firms and other economic agents becomes important, so the financial-side of the model can no longer be ignored.<sup>1</sup>

To investigate the economic consequences of financial frictions, I draw on the well-known financial accelerator model of Bernanke et al. (1999) where interest rate spreads are tied to the aggregate characteristics of the borrowers (more precisely, to the borrowers’ leverage ratio). This model offers a tractable framework for integrating financial frictions into an otherwise standard New Keynesian general equilibrium model with nominal rigidities. Moreover, the model has the appealing feature

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<sup>1</sup>The Modigliani-Miller theorem, derived from the seminal work of Modigliani and Miller (1958), is also known as the capital structure irrelevance principle. The theorem indicates that, lacking some specific frictions or taxes, the value of the firm does not depend on whether the firm is financed by issuing equity (from their net worth) or debt (or simply taking on loans).

relative to other models of financial frictions that: (a) defaults and spreads (the external finance premium) occur endogenously in equilibrium, and (b) asset prices (the price of capital) feed into the spreads linking the two together endogenously.

I find that the economy has a stronger financial mechanism when the model incorporates standard New Keynesian features such as monopolistic competition and price stickiness. I emphasize that the financial accelerator by itself has only mild effects unless it interacts with frictions such as the type of nominal rigidities favored in the New Keynesian literature. I also illustrate that the financial accelerator model can have a significant amplification effect when it interacts with different specifications of the policy rule and with the addition of monetary policy shocks. However, these results are very sensitive to: (a) the degree of price stickiness assumed under Calvo price-setting, (b) the specification of the systematic part of the monetary policy rule, and (c) the interpretation one assigns to the exogenous and discretionary component of monetary policy.

Furthermore, I also show that a stronger financial accelerator mechanism does not necessarily mean that the model of Bernanke et al. (1999) is better suited to explain the path of endogenous variables like real per capita private output (excluding government), real per capita investment, real per capita consumption, the share of hours worked per capita, the year-over-year inflation rate or even the quarterly interest rate spread between the Baa corporate bond yield and the 20-year Treasury bill rate since the onset of the Great Moderation. In fact, a plain vanilla Real Business Cycle (RBC) model parameterized in a way consistent with that of the financial accelerator model—or a variant of it augmented with the financial friction, but no nominal rigidities—produce simulations of the endogenous variables that correlate more strongly with the actual data than the full-fledge financial accelerator model does.

I have several additions to the literature. First, I consistently and thoroughly examine the U.S. data and provide a coherent mapping between the data and the model. I also explicitly consider the possibility that there was a level shift in the data after the 2007 recession in establishing the mapping of the data into the model. The consistency between the way in which the model is laid down to account for the business cycle fluctuations and how the data itself is measured and detrended (or expressed in deviations from a long-run mean or target) is crucial in helping evaluate the strength and weaknesses of the model.

Second, I quantitatively investigate the ability of the financial accelerator model of Bernanke et al. (1999) to explain the cyclical fluctuations in the U.S. data. Although this is not the first paper to investigate the financial accelerator model's performance (see, e.g., the estimation in Meier and Müller 2006), it is the first paper to my knowledge that does it by the simulation method taking as given the realizations of the detrended Solow residual and the monetary policy deviations straight from the data—rather than estimating them based on imposing *ex ante* the structure of the model on the observable variables. While both approaches are complementary, I argue that the exercise I conduct in this paper is useful for the purpose of evaluating the model and accounting for the cyclical features of the data without having to worry (among other things) that misspecification may be biasing the estimates of

the structural parameters. Moreover, it is also quite useful as a tool to inspect the financial accelerator mechanism and understand how it operates.

Third, I also aim to provide insight about the first-order effects of the interaction between financial frictions and nominal rigidities in the model of Bernanke et al. (1999). To do so, I adopt a simple first-order perturbation method to characterize the short-run dynamics of the financial accelerator model as Bernanke et al. (1999) did too. First-order approximations to the equilibrium conditions can be very useful to track fluctuations around the steady state arising from small perturbations of exogenous shocks, but might be quite inaccurate when the shocks are fairly large or the economy is far away from its long-run steady state. When I take account of the non-stationarity in the U.S. data and calculate the realization of the TFP and monetary shocks driving the business cycle, it is reassuring that I do not see a strong case to back the idea that fluctuations have been unusually pronounced during most of the period since the mid-1980s—although in the case of the monetary shocks the question may be far less settled.

While the short-run dynamics of the model are indeed linear in the variables under the first-order approximation that I have adopted, the coefficients are highly nonlinear functions of the structural parameters of the model. I contend that these nonlinearities in the coefficients are important to understand the interaction between nominal rigidities and financial frictions. This nonlinear interaction, in turn, can have large effects on the path the endogenous variables take in response to a given realization of the shocks—I find the degree of price stickiness, in particular, to be crucial for the amplification of fluctuations in the external finance premium and on investment.

My paper proceeds as follows: Section 2 outlines the Bernanke et al. (1999) financial accelerator and several nested variants that abstract from all frictions (the RBC model), that abstract from nominal rigidities (the FA model), and that eliminate the financial friction (the DNK model). I continue in Section 3 with a discussion of the parameterization of the model and the derivation of the shock realizations, and then I present the quantitative findings in Section 4. Section 5 provides some discussion and concludes.

## 2 The Financial Accelerator Model

One framework incorporating a financial accelerator in general equilibrium that has been extensively used in the literature is Bernanke et al. (1999)'s model of financial intermediation with 'costly state verification'. Costly monitoring of the realized return on capital of the defaulting borrowers and an endogenous probability of default result in increased borrowing costs on loans over the risk-free rate and introduce time-variation on the loan rates over the business cycle. The *external finance premium*—the spread of the loan rate over the risk-free rate—makes investment and capital accumulation more expensive. This, in turn, intensifies the impact and can even alter the propagation of a given shock. The model of Bernanke et al. (1999),

however, includes other distortions—in particular, it includes standard New Keynesian frictions such as monopolistic competition and nominal price rigidities.

I adopt the model of Bernanke et al. (1999) for its tractability and intuitive economic appeal. Also, because financial intermediation plays a key role in funding investment—a connection that I want to explore further in light of the investment collapse observed in the U.S. data during the 2007 recession.<sup>2</sup> The model shares an important characteristic with the framework of collateral borrowing constraints articulated by Kiyotaki and Moore (1997) in that asset price movements serve to reinforce credit market imperfections. Fluctuations on the value of capital contribute directly to volatility in the leverage of the borrowers. This feature is missing in the Carlstrom and Fuerst (1997) framework which also builds on the idea of ‘costly state verification’, as noted by Gomes et al. (2003). Another difference between the Carlstrom and Fuerst (1997) and Bernanke et al. (1999) environments is that financial intermediation is intratemporal in the former and intertemporal in the latter.<sup>3</sup>

The model of Bernanke et al. (1999) is populated by households and entrepreneurs, a variety of firm types (capital producers, wholesale producers, and retailers) as well as financial intermediaries (banks) and a central bank entrusted with the conduct of monetary policy. Households own all capital producing firms, retailers and banks. Capital producers determine a relative price for investment goods, and are subject to technological constraints in how they can transform final output into productive capital that can be used to produce wholesale output.

Retailers are separated from wholesale producers in order to introduce differentiation in the wholesale goods, and add nominal rigidities into the model. Wholesale producers are formed and operated by entrepreneurs. The capital returns they generate tomorrow with today’s allocation of capital are paid net of borrowing costs as dividends back to the entrepreneurs if there is no default. Capital returns on wholesalers are subject to idiosyncratic shocks that affect the revenue stream for the entrepreneurs who own them. Therefore, entrepreneurs are exposed to bankruptcy risk on the wholesale firms which occurs whenever capital returns fall short of the required loan repayment. In that case, the entrepreneurs lose the capital returns and the undepreciated stock of capital on the defaulting wholesalers.

The financial system intermediates between the households and the entrepreneurs. Banks are risk-neutral firms facilitating loans to the risk-neutral entrepreneurs who borrow to fund the stock of capital they need for wholesale production. Entrepreneurs are more impatient than households, dying out at an exogenous rate, and that motivates them to borrow. Entrepreneurs’ deaths also prevents them from accumulating enough net worth (internal funds) to be able to self-finance their capital holdings every period.

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<sup>2</sup>The literature has investigated other roles of financial intermediation: for instance, funding the wage bill instead of the capital bill (see, e.g., Carlstrom and Fuerst 2001). The financial accelerator model of Bernanke et al. (1999) has the potential to amplify the effects of a shock, but by constraining capital accumulation, it can affect the propagation of shocks as well.

<sup>3</sup>Faia and Monacelli (2007) and Walentin (2005) provide a comparative analysis of the Bernanke et al. (1999) and Carlstrom and Fuerst (1997) models.

Capital returns are determined by the marginal product of capital and the capital gains on the value of the assets (the capital), but also by the realization of an idiosyncratic shock which is observable to the entrepreneurs but not to the financial intermediaries. Banks can only determine the realization of the idiosyncratic shock and, therefore, the true returns to capital after paying a non-zero monitoring or verification cost. Loan contracts cannot be made conditional on the realization of the idiosyncratic shock because they are unobserved by the banks. However, the design of the loans is meant to reduce the costs associated with this asymmetry of information between the entrepreneurs who own the wholesale firms and the banks.

Financial intermediaries offer one-period deposits available to households promising the real risk-free rate and use the funds they are able to raise to make one-period loans available to the entrepreneurs. The implied loan rate charges a spread over the real risk-free rate—the external finance premium—for banks to cover the costs of monitoring the defaulting entrepreneurs and any shortfall on loan repayment that may occur. All entrepreneurs face the same borrowing costs. Ex post there is always a fraction of wholesale producers with low draws from the idiosyncratic shock that do not generate enough revenue from their capital for the entrepreneurs to meet the loan repayment, causing them to default.

Ex ante the banks know the distribution of the idiosyncratic shock and can determine the probability of default and its associated costs under the terms of the loan—even if banks do not know which entrepreneurs will end up defaulting next period, they know how many defaults to expect. Banks are perfectly competitive so they structure their loans to cover solely the costs of default (as they face no other costs), and make no profits on the loans. The expected default rates priced into the loan rates are always confirmed ex post in equilibrium. Banks supply whatever loan amount is desired by the entrepreneurs under the terms of the offered loan, and accept any amount that households wish to deposit at the prevailing real risk-free rate. As a result, ex post banks always break even and distribute zero-profits in every period to the households who own them.

Finally, a central bank is added which sets monetary policy in terms of a nominal short-term interest rate. Monetary policy is non-neutral in the short run, irrespective of the capital structure of the entrepreneurs or the functioning of the loan market. Monetary policy non-neutrality arises as in the standard New Keynesian framework simply because of nominal rigidities on prices. I modify the model of Bernanke et al. (1999) to include a more standard monetary policy rule à la Taylor (1993) to characterize the perceived monetary policy regime over the Great Moderation period. The model is, otherwise, the same one derived in Bernanke et al. (1999) in log-linear form with only minor simplifications in the timing of pricing decisions and the role of entrepreneurs' consumption and government consumption shocks.

The contribution of this paper is not predicated on any theoretical improvement upon what is already a well-established framework for understanding financial distortions, but it is primarily a quantitative one. For a conventional parameterization of the model, I provide a careful quantitative evaluation of the ability (of lack thereof) of this financial accelerator channel to answer questions on the role of monetary policy over the U.S. business cycle, on the cyclical factors behind the Great Moderation, and on the financial aspects of the 2007 recession.

### Log-linearized Equilibrium Conditions of the Financial Accelerator Model

Since the model of Bernanke et al. (1999) is quite well-known, I refrain from a detailed discussion of its first principles. This section describes the log-linearized equilibrium conditions of the model that I use and a frictionless variant—the RBC counterpart—to make the presentation as compact as possible. As a notational convention, all variables identified with lower-case letters and a caret on top are expressed in logs and as deviations relative to their steady state values. Since the model abstracts from population growth and accounts only for stationary cyclical fluctuations, the endogenous variables are matched whenever appropriate to do so with observed time series expressed in per capita terms and detrended (or demeaned). Further discussion on the mapping between the data and the model can be found in Appendices A and B.

On the demand-side, households are infinitely-lived and maximize their lifetime discounted utility, which is additively separable in consumption and leisure in each period. Aggregate consumption evolves according to a standard Euler equation,

$$\hat{c}_t \approx \mathbb{E}_t[\hat{c}_{t+1}] - \sigma \hat{r}_{t+1}, \quad (1)$$

where  $\hat{c}_t$  denotes real aggregate consumption, and  $\hat{r}_{t+1}$  is the Fisherian real interest rate. This consumption Euler equation is fairly standard and implies that the financial frictions do not directly affect the consumption-savings decision of the households. Financial intermediaries pay the same real risk-free rate on deposits. The intertemporal elasticity of substitution,  $\sigma > 0$ , regulates the sensitivity of the consumption-savings decision to the Fisherian real interest rate.

The Fisherian real interest rate is defined as the one-period nominal (risk-free) interest rate minus the expected inflation over the next quarter, i.e.,

$$\hat{r}_{t+1} \equiv \hat{i}_{t+1} - \mathbb{E}_t[\hat{\pi}_{t+1}], \quad (2)$$

where  $\hat{\pi}_t \equiv \hat{p}_t - \hat{p}_{t-1}$  is the inflation rate, and  $\hat{p}_t$  is the consumer price index (CPI). Nominal (uncontingent) one-period bonds are traded in zero net supply and guarantee a nominal risk-free rate of  $\hat{i}_{t+1}$  paid at time  $t + 1$  but set at time  $t$ . Here,  $\mathbb{E}_t[\cdot]$  denotes the expectations operator conditional on information available up to time  $t$ .

The first-order condition on labor supply from the households' problem can be expressed as follows,

$$\hat{w}_t - \hat{p}_t \approx \frac{1}{\sigma} \hat{c}_t + \frac{1}{\varphi} \hat{h}_t, \quad (3)$$

where  $\hat{h}_t$  represents aggregate household labor, and  $\hat{w}_t$  is the competitive nominal wage. The Frisch elasticity of labor supply,  $\varphi \equiv \eta(\frac{1-H}{H}) > 0$ , indicates the sensitivity of the supply of labor to changes in real wages, ceteris paribus. The parameter  $\eta$  corresponds to the inverse of the coefficient of relative risk aversion on leisure, and  $H$  defines the share of hours worked in steady state.<sup>4</sup>

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<sup>4</sup>Total hours worked  $H_t$  and hours spent in leisurely activities  $L_t$  are normalized to add up to one (i.e.,  $H_t + L_t = 1$ ). If consumption and leisure are additively separable as assumed by Bernanke

On the supply-side, there are retailers, capital producers, wholesale producers (owned and operated by the entrepreneurs), and financial intermediaries. I implicitly assume that the only input required in the production of retail varieties is the wholesale good. Retailers acquire wholesale output, costlessly differentiate the wholesale goods into retailer-specific varieties, and sell their varieties for either consumption or investment. Preferences are defined over all the retail varieties, but not directly over the wholesale goods which are only utilized as inputs in the production of retail varieties.

Each retailer has monopolistic power in its own variety and chooses its price to maximize the expected discounted value of its current and future profits, subject to a downward-sloping demand constraint. Price stickiness is modeled à la Calvo (1983), so in each period only a fraction  $0 < 1 - \alpha < 1$  of the retailers gets to re-optimize prices.<sup>5</sup> The CPI inflation dynamics resulting from aggregating over all retail prices are given by the following forward-looking Phillips curve,

$$\hat{\pi}_t \approx \beta \mathbb{E}_t[\hat{\pi}_{t+1}] + \left( \frac{(1 - \alpha\beta)(1 - \alpha)}{\alpha} \right) \widehat{mc}_t, \quad (4)$$

where I define the real marginal cost as  $\widehat{mc}_t \equiv (\hat{p}_t^w - \hat{p}_t)$  and denote the wholesale output price as  $\hat{p}_t^w$ . The intertemporal discount factor of the households is  $0 < \beta < 1$ . Under flexible prices, the retailers intermediate the exchanges in the market for wholesale goods charging a mark-up over marginal costs but have no discernible impact on the short-run dynamics (i.e.,  $\widehat{mc}_t = 0$ ) since the monopolistic competition mark-up is time-invariant. The mark-up, however, still distorts the steady state allocation relative to the case under perfect competition.

In keeping with the precedent of Bernanke and Woodford (1997), Bernanke et al. (1999) assume that prices are set prior to the realization of any aggregate time  $t$  shock. The timing in Bernanke et al. (1999) distorts the equilibrium beyond what the monopolistic competition mark-up and Calvo (1983) price stickiness already do. In turn, I adopt the convention that prices are set after observing the realized shocks at time  $t$  as in Woodford (2003). The model solution then approximates the case where prices equal a mark-up over marginal costs in the limit when only an arbitrarily small fraction of firms  $\alpha \rightarrow 0$  cannot re-optimize. This facilitates the comparison between the financial accelerator model and the frictionless model that I investigate in the paper.

Capital accumulation evolves according to a standard law of motion,

$$\hat{k}_{t+1} \approx (1 - \delta)\hat{k}_t + \delta\hat{x}_t, \quad (5)$$

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et al. (1999), and I define the per-period preferences over leisure generically as  $V(L_t)$ , then it follows that in steady state  $\eta^{-1} \equiv -\frac{LV''(L)}{V'(L)}$ .

<sup>5</sup>The retailers add a ‘brand’ name to the wholesale good which introduces differentiation across varieties and, consequently, retailers gain monopolistic power to charge a mark-up in their prices. The retailers are not price-takers under this market structure.



where  $\widehat{k}_t$  denotes the stock of capital available at time  $t$  and  $\widehat{x}_t$  stands for real investment in the same period. The depreciation rate of physical capital is given by  $0 < \delta < 1$ . The capital goods producers use the same aggregate of retail varieties that households consume in the production of new capital. To be consistent with the convention of Bernanke et al. (1999), I also assume that entrepreneurs buy all capital they need from the capital goods producers—the period before production takes place—and then sell the depreciated capital stock back to them after being used for the production of wholesale goods.

Capital goods producers face increasing marginal adjustment costs in the production of new capital, modelled in the form of an increasing and concave adjustment cost which is a function of the investment-to-capital ratio.<sup>6</sup> The technological constraint on capital goods producers implies that the investment-to-capital ratio ( $\widehat{x}_t - \widehat{k}_t$ ) is tied to the shadow value of an additional unit of capital (or Tobin's  $q$ ) in units of consumption,  $\widehat{q}_t$ , by the following relationship,

$$\widehat{q}_t \approx \chi (\widehat{x}_t - \widehat{k}_t). \quad (6)$$

The degree of concavity of the cost function around its steady state,  $\chi \geq 0$ , regulates the sensitivity of the investment-to-capital ratio to fluctuations in Tobin's  $q$ . Without adjustment costs (i.e., if  $\chi = 0$ ), Tobin's  $q$  becomes time-invariant, i.e.,

$$\widehat{q}_t \approx 0, \quad (7)$$

and the investment-to-capital ratio is unconstrained. However, without adjustment costs the financial accelerator mechanism in Bernanke et al. (1999) would lose the characteristic that asset price movements serve to reinforce loan market imperfections.

The wholesale firms employ homogenous labor supplied by both households and entrepreneurs as well as capital in order to produce wholesale output. Entrepreneurs' labor is differentiated from that of the households. All factor markets are perfectly competitive, and each wholesale producer relies on the same Cobb-Douglas technology in capital and in labor from households and entrepreneurs. Aggregate wholesale output can be expressed as follows,

$$\widehat{y}_t \approx \widehat{a}_t + \psi \widehat{k}_t + (1 - \psi - \varrho) \widehat{h}_t, \quad (8)$$

where  $\widehat{y}_t$  denotes wholesale output, and  $\widehat{a}_t$  is an aggregate productivity (TFP) shock. The capital share in the production function is  $0 < \psi < 1$ , while the entrepreneurs' labor share is  $0 \leq \varrho < 1$  and the households' labor share is  $0 < 1 - \psi - \varrho < 1$ .<sup>7</sup> Entrepreneurs' labor is assumed to be inelastically supplied and time-invariant, so

<sup>6</sup>As in Bernanke et al. (1999), profits of the capital goods producers are of second-order importance and, therefore, omitted. For more details, see footnote 13 in p. 1357.

<sup>7</sup>As in Bernanke et al. (1999), the entrepreneurs' labor share is chosen to be small enough that this modification of the standard production function does not have a significant direct effect on the aggregate dynamics of the model.

it drops out of the log-linearized production function in (8). The TFP shock follows an AR(1) process of the following form,

$$\widehat{a}_t = \rho_a \widehat{a}_{t-1} + \varepsilon_t^a, \quad \varepsilon_t^a \sim N(0, \sigma_a^2), \quad (9)$$

where  $\varepsilon_t^a$  is a zero mean, uncorrelated and normally-distributed innovation. The parameter  $-1 < \rho_a < 1$  determines the persistence of the TFP shock and  $\sigma_a$  its volatility.

The competitive real wage paid to households is equal to their marginal product, i.e.,

$$\widehat{w}_t - \widehat{p}_t \approx \widehat{m}\widehat{c}_t + (\widehat{y}_t - \widehat{h}_t). \quad (10)$$

Entrepreneurs' real wages—which differ from those of the households—are not needed to characterize the short-run dynamics of the model, though. Combining the labor supply equation for households in (3) with the households' labor demand in (10), I derive a households' labor market equilibrium condition in the following terms,

$$\widehat{m}\widehat{c}_t + (\widehat{y}_t - \widehat{h}_t) - \frac{1}{\sigma} \widehat{c}_t \approx \frac{1}{\varphi} \widehat{h}_t. \quad (11)$$

This condition suffices to describe the real marginal costs faced by the retailers, without having to keep track of any real wages explicitly.

Entrepreneurs operating the wholesale firms buy the capital stock every period from the capital goods producers at a price determined by Tobin's  $q$ , using both internal funds (that is, their own net worth) and external loans from the financial intermediaries. After production takes place the next period, the depreciated stock of capital is sold back to the capital goods producers. Accordingly,

$$\widehat{r}_t^k \approx (1 - \epsilon)(\widehat{m}\widehat{c}_t + (\widehat{y}_t - \widehat{k}_t)) + \epsilon \widehat{q}_t - \widehat{q}_{t-1}, \quad (12)$$

where the aggregate real return on capital,  $\widehat{r}_t^k$ , is equal to a weighted combination of the marginal product of capital,  $\widehat{m}\widehat{c}_t + (\widehat{y}_t - \widehat{k}_t)$ , and the re-sale value of the depreciated capital stock (as captured by Tobin's  $q$ ),  $\widehat{q}_t$ , minus the cost of acquiring the stock of capital from the capital goods producers in the previous period,  $\widehat{q}_{t-1}$ .

The composite coefficient in the definition of the returns to capital in (12) is characterized as  $\epsilon \equiv (\frac{1-\delta}{\nu(\gamma_n^{-1})\beta-1})$ . This composite depends on the gross steady state ratio between the cost of external funding for entrepreneurs and the real risk-free rate  $\nu(\gamma_n^{-1}) \equiv \frac{R^k}{R} \geq 1$ . Moreover,  $\nu(\gamma_n^{-1})$  is a function of the steady state gearing or leverage ratio of the entrepreneurs,  $\gamma_n^{-1} \equiv \frac{K}{N}$ , that is the ratio of total assets—the stock of capital  $K$ —over the total real net worth—equity  $N$ —of the entrepreneurs. Tobin's  $q$  is equal to 1 in steady state and, therefore,  $K$  corresponds to both the stock of capital as well as its value in units of consumption.

Following the logic of the 'costly state verification' framework embedded in Bernanke et al. (1999), the returns to capital of each wholesale producer are subject to idiosyncratic (independent and identically-distributed) shocks that are observable to the entrepreneurs but costly to monitor for the financial intermediaries. The

idiosyncratic shocks are realized only after capital is acquired for wholesale production and external loans for funding have been secured. Therefore, such idiosyncratic shocks have a direct impact on the capital returns that entrepreneurs obtain from allocating capital to wholesale production, but do not affect the allocation of capital itself to each wholesale producer.

Financial intermediaries raise funds from households by offering deposits that pay the real risk-free rate,  $\widehat{r}_{t+1}$ , and make loans in real terms to entrepreneurs to finance their capital stock. On one hand, the return on deposits for households is guaranteed and inflation-protected. On the other hand, entrepreneurs can default on their loan contract obligations, and financial intermediaries can find out about their true capital returns (that is, learn about the realization of the idiosyncratic shock) only after paying a monitoring or verification cost. The bank lenders solely monitor the entrepreneurs who default, pay the verification costs when default occurs, and seize all income revenues obtained from the allocation of capital and the remaining assets (capital) of the defaulting entrepreneurs.<sup>8</sup>

In equilibrium, the financial intermediaries—which are assumed to be risk-neutral—price into their loan contracts the probability and costs of default, so an endogenous spread arises between the cost at which banks fund themselves through deposits from households (the real risk-free rate) and the real cost of external financing through loans faced by the entrepreneurs. The entrepreneurs—who are also assumed to be risk-neutral—borrow up to the point where the expected real return to capital equals the real cost of external funding through loans, i.e.,

$$\mathbb{E}_t[\widehat{r}_{t+1}^k] \approx \widehat{r}_{t+1} + \vartheta(\widehat{q}_t + \widehat{k}_{t+1} - \widehat{n}_{t+1}). \quad (13)$$

As shown in Bernanke et al. (1999), the external financing premium or spread over the real risk-free rate demanded by the financial intermediaries,  $\widehat{s}p_t \equiv \mathbb{E}_t[\widehat{r}_{t+1}^k] - \widehat{r}_{t+1}$ , is a function of the leverage ratio of the entrepreneurs in any given period,  $\widehat{q}_t + \widehat{k}_{t+1} - \widehat{n}_{t+1}$ , where  $\widehat{n}_{t+1}$  denotes the net worth (or equity) of the entrepreneurs at the end of time  $t$  and  $\widehat{q}_t + \widehat{k}_{t+1}$  denotes the total value of their assets (the value of their outstanding stock of capital) also at the end of time  $t$ .

The composite coefficient in (13) is characterized as  $\vartheta \equiv (\frac{v'(\gamma_n^{-1})\gamma_n^{-1}}{v(\gamma_n^{-1})})$  where the parameter  $v'(\gamma_n^{-1}) \equiv \frac{\partial v(\gamma_n^{-1})}{\partial \gamma_n^{-1}} \geq 0$  is the first derivative of the external financing premium with respect to the entrepreneurs' leverage ratio  $\gamma_n^{-1}$  in steady state. Then, the composite coefficient  $\vartheta$  can be interpreted as the elasticity of the external financing premium with respect to the entrepreneurs' leverage ratio evaluated in steady state. The lower the entrepreneurs' leverage in steady state (i.e., the closer  $\gamma_n^{-1} \equiv \frac{K}{N}$  is to one), the lower the associated costs of default (and the smaller the elasticity  $\vartheta$ ) will be.

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<sup>8</sup>Loan contracts are enforced under limited liability, so the bank cannot appropriate more than the value of the collateral assets (capital) and earned capital income of the defaulting entrepreneurs. Default takes place before the entrepreneurs earn any labor income.

The balance sheet of the entrepreneurs requires the real value of the stock of capital to be equal to real net worth (equity) plus the real amount in borrowed external funds (loans), i.e.,

$$\widehat{q}_t + \widehat{k}_{t+1} \approx \gamma_n \widehat{n}_{t+1} + (1 - \gamma_n) \widehat{l}_{t+1}, \quad (14)$$

where  $\widehat{l}_{t+1}$  denotes the total loans in real terms provided by the financial intermediaries to fund the stock of capital,  $\widehat{k}_{t+1}$ , valued at  $\widehat{q}_t$  per unit of capital at time  $t$ . As a result, the leverage or gearing ratio of the entrepreneurs is simply proportional to the entrepreneurs' debt-to-equity ratio, i.e.,

$$\widehat{q}_t + \widehat{k}_{t+1} - \widehat{n}_{t+1} \approx (1 - \gamma_n)(\widehat{l}_{t+1} - \widehat{n}_{t+1}). \quad (15)$$

Hence, the more indebted the entrepreneurs become—or the least equity they have at stake—in any given period, the more leveraged they are and the costlier it gets for entrepreneurs to fund their desired stock of capital with bank loans given the capital demand in (13).

Banks are perfectly competitive and real deposits held by households must be equal to the total loanable funds in real terms supplied to the entrepreneurs in every period  $t$ , i.e.,

$$\widehat{q}_t \approx \widehat{d}_t, \quad (16)$$

where  $\widehat{d}_t$  represents the real value of the households' deposits. Given the simplicity of the balance sheet of the banks, it can be said that the model of Bernanke et al. (1999) is silent about the bank lending channel and in turn places all the emphasis on the borrowers-side. Hence, the external finance premium is unaffected by the characteristics of the lenders, and only depends on the characteristics of the borrowers (more specifically, on the leverage of the entrepreneurs). I leave for future research the extension of the model to incorporate an economically-relevant bank lending channel.

The aggregate real net worth of the entrepreneurs accumulates according to the following law of motion,

$$\begin{aligned} \widehat{n}_{t+1} &\approx (\zeta \beta^{-1} \gamma_n^{-1})(\widehat{r}_t^k - \widehat{r}_t) + \widehat{r}_t + \widehat{n}_t + \dots \\ &\quad (v(\gamma_n^{-1}) - 1) \gamma_n^{-1} (\widehat{r}_t^k + \widehat{q}_{t-1} + \widehat{k}_t) + \dots \\ &\quad \frac{\varrho}{\psi} (v(\gamma_n^{-1}) \beta^{-1} - (1 - \delta)) \gamma_n^{-1} \widehat{y}_t + \widehat{m} \widehat{c}_t, \end{aligned} \quad (17)$$

where  $0 < \zeta < 1$  is interpreted as a *survival rate* for entrepreneurs in the same spirit as Bernanke et al. (1999). Households' consumption and savings are governed by the standard consumption Euler equation described in (1), but the entrepreneurs' consumption  $\widehat{c}_t^e$  is simply proportional to their net worth  $\widehat{n}_{t+1}$ , i.e.,

$$\widehat{c}_t^e \approx \widehat{n}_{t+1}, \quad (18)$$

plus a term of second-order importance that drops out from the log-linearized model.

Equation (17) indicates that the real net worth (or equity) of the entrepreneurs,  $\hat{n}_{t+1}$ , accumulates over the previous period real net worth,  $\hat{n}_t$ , at the real risk-free rate,  $\hat{r}_t$ , plus a retained share of the capital returns net of borrowing costs which is proportional to the real capital return relative to the real risk-free rate,  $\hat{r}_t^k - \hat{r}_t$ . The fraction of net real capital returns retained is a function of the steady state gearing or leverage ratio  $\gamma_n^{-1}$ , the steady state real interest rate  $\beta^{-1}$ , and the survival rate of the entrepreneurs  $\zeta$ . The law of motion for net worth in (17) also includes a variety of additional terms of lesser importance under standard parameterizations—partly related to entrepreneurial labor income.

Entrepreneurs are risk-neutral and discount the future at the same rate  $\beta$  as households. The assumption that a fraction of entrepreneurs  $(1 - \zeta)$  dies out in every period and gets replaced by the same proportion of new entrepreneurs without any net worth of their own—but with some labor income—introduces entry and exit in the model. In that case, the effective discount rate for entrepreneurs  $\beta\zeta < \beta$  is lower than that of households. Entrepreneurs, who are more impatient as a result, borrow to fund the acquisition of capital; households save the loanable funds through riskless deposits with the risk-neutral financial intermediaries.

Entrepreneurs have an incentive to borrow, but also to delay consumption and accumulate net worth (equity) in order to retain more of the high returns on capital that can be obtained using internal funds. This is because the opportunity cost of internal funds is given by the risk-free rate  $\hat{r}_t$  which is lower than the implied loan rates from the financial intermediaries. The assumption that a fraction of entrepreneurs  $(1 - \zeta)$  dies out in every period, therefore, is also meant to preclude entrepreneurs from becoming fully self-financing over the long-run since that would eliminate the need for external finance through banks and kill the financial accelerator channel.

The resource constraint can be approximated as follows,

$$\hat{y}_t \approx \gamma_c \hat{c}_t + \gamma_x \hat{x}_t + \gamma_{ce} \hat{c}_t^e, \quad (19)$$

where  $0 < \gamma_c < 1$  denotes the households' consumption share in steady state,  $0 < \gamma_x < 1$  is the investment share, and  $0 < \gamma_{ce} < 1$  is the entrepreneurs' consumption share. By construction, it must be the case that  $\gamma_c \equiv 1 - \gamma_x - \gamma_{ce}$ . The investment share is a composite coefficient of the structural parameters of the model given by  $\gamma_x \equiv \delta \frac{K}{Y} = \delta \left( \frac{\psi}{\mu(v(\gamma_n^{-1})\beta^{-1} - (1-\delta))} \right)$  where  $\mu \equiv \frac{\theta}{\theta-1} > 1$  is the monopolistic competition mark-up and  $\theta > 1$  is the elasticity of substitution across retail varieties. Monopolistic competition distorts the dynamics of the model through the resource constraint in (19) because the mark-up lowers the long-run investment share and increase the share of consumption. Similarly, the investment share is also distorted by the gross steady state ratio between the cost of external funding for entrepreneurs and the real risk-free rate  $v(\gamma_n^{-1}) \equiv \frac{R^k}{R}$ . The higher the ratio between these two rates, the lower the investment share will be.

The entrepreneurs have been largely modeled as in Bernanke et al. (1999), but I depart from them in one respect: instead of assuming that dying entrepreneurs consume all their entire net worth and disappear, I assume that they consume only an arbitrarily small fraction as they exit the economy while the rest is transferred to the

households. This does not change the entrepreneurs' consumption relationship with net worth described in (18), but it affects the entrepreneurs' consumption share in steady state  $\gamma_{ce}$  and the resource constraint in (19). The steady state share  $\gamma_{ce}$  under this alternative assumption is chosen to be very small such that the entrepreneurs' consumption does not have a significant direct effect on the model dynamics.

As discussed in Christiano et al. (2003) and Meier and Müller (2006), this assumption suffices to ensure the objective function of the entrepreneurs is well-defined. It also has the desirable feature that entrepreneurs' consumption—which is an artifact of the heterogeneity across agents needed to introduce borrowing and lending—is almost negligible and, therefore, that total consumption is essentially pinned down by the households' consumption and governed by the standard Euler equation from the households' maximization problem.

The resource constraint in (19) abstracts from the consideration of the resources devoted to monitoring costs, as those ought to be negligible on the dynamics of the model under standard parameterizations according to Bernanke et al. (1999). It also equates final aggregate output of all varieties for consumption and investment purposes with the wholesale output that is used as the sole input in the production of each retail variety.

In Bernanke et al. (1999) government consumption is modeled as an exogenous shock which detracts resources from the resource constraint. I simplify the financial accelerator model by excluding government consumption entirely. I contend that eliminating government consumption shocks does not fundamentally alter the financial accelerator mechanism developed in Bernanke et al. (1999) or the dynamics of the model in response to monetary and TFP shocks since fiscal policy is not fleshed out beyond the exogenous impact of this government shock on aggregate demand. In turn, I focus my investigation primarily on the traditional main driver of the business cycle (aggregate TFP) and on the connection between lending and monetary policy.<sup>9</sup> I leave the investigation of the role of fiscal policy and its interplay with loan market imperfections for future research.

Another important departure from the original model set-up comes from replacing the monetary policy rule of Bernanke et al. (1999) with a more standard specification. In line with most of the current literature, I assume that the central bank follows a conventional Taylor (1993)-type reaction function under a dual mandate that adjusts the short-term nominal rate,  $\hat{i}_t$ , to respond to fluctuations in inflation,  $\tilde{\pi}_t$ , and some real economic activity measure of the business cycle,  $\tilde{y}_t$ . Thus, monetary policy is determined by the following general expression,

$$\hat{i}_{t+1}^{AR} = \rho_i \hat{i}_t^{AR} + (1 - \rho_i)[\phi_\pi \tilde{\pi}_t + \phi_y \tilde{y}_t] + \hat{m}_t, \quad (20)$$

where the policy parameters  $\phi_\pi \geq 1$  and  $\phi_y \geq 0$  regulate the sensitivity of the policy rule to inflation and output fluctuations, and  $0 \leq \rho_i < 1$  is the interest rate smoothing parameter. I use the annualized short-term interest rate as the relevant policy

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<sup>9</sup>To make the data consistent with the model, however, output is measured as private market output (excluding government compensation of employees).

instrument,  $\widehat{i}_t^{AR}$ , i.e.,

$$\widehat{i}_{t+1}^{AR} \approx 4\widehat{i}_{t+1}. \quad (21)$$

The monetary policy shock,  $\widehat{m}_t$ , follows an AR(1) process that can be represented as,

$$\widehat{m}_t = \rho_m \widehat{m}_{t-1} + \varepsilon_t^m, \quad \varepsilon_t^m \sim N(0, \sigma_m^2), \quad (22)$$

where  $\varepsilon_t^m$  is a zero mean, uncorrelated and normally-distributed innovation. The parameter  $-1 < \rho_m < 1$  determines the persistence of the policy shock and the parameter  $\sigma_m \geq 0$  its volatility. I assume that monetary and TFP shocks are uncorrelated.

In keeping with Taylor (1993)'s original prescription, I consider a specification where the inflation rate is measured over the previous four quarters,  $(\widehat{p}_t - \widehat{p}_{t-4})$ , and real economic activity over the business cycle is tracked with output in deviations from its steady state,  $\widehat{y}_t$ , i.e.,

$$\widetilde{y}_t \approx \widehat{y}_t, \quad (23)$$

$$\widetilde{\pi}_t \approx (\widehat{p}_t - \widehat{p}_{t-4}) \approx \widehat{\pi}_t + \widehat{\pi}_{t-1} + \widehat{\pi}_{t-2} + \widehat{\pi}_{t-3}, \quad (24)$$

I also experiment with an alternative specification of the policy rule in which  $(\widehat{p}_t - \widehat{p}_{t-4})$  is replaced with the annualized quarter-over-quarter rate,  $\widehat{\pi}_t^{AR}$ , i.e.,

$$\widetilde{\pi}_t \approx \widehat{\pi}_t^{AR} \approx 4\widehat{\pi}_t. \quad (25)$$

The inflation rate in (25) is consistent with how the Taylor rule is specified in most quantitative and theoretical models, but is not the preferred measure of inflation in Taylor (1993).<sup>10</sup> Another alternative conception of the monetary policy reaction function that I do consider here respond to deviations of output from its potential,  $\widehat{x}_t$ , i.e.,

$$\widetilde{y}_t \approx \widehat{x}_t, \quad (27)$$

rather than to deviations of output from its long-run steady state (i.e.,  $\widehat{y}_t$ ). The output gap  $\widehat{x}_t \equiv \widehat{y}_t - \widehat{y}_t^F$  measures the deviations of output  $\widehat{y}_t$  from potential  $\widehat{y}_t^F$  where the potential is defined as the output level that would prevail in the frictionless model (abstracting from monopolistic competition, nominal rigidities and the financial frictions under 'costly state verification').

**Nested Models Without Nominal Rigidities and/or Financial Frictions** The financial accelerator mechanism developed in Bernanke et al. (1999) is integrated

<sup>10</sup>The rule of Bernanke et al. (1999) characterizes monetary policy in the following form,

$$\widehat{i}_{t+1} = \rho_i \widehat{i}_t + (1 - \rho_i) \psi_\pi \widehat{\pi}_t + \widehat{m}_t. \quad (26)$$

This feedback rule assumes monetary policy is inertial and the inflation rate is quarter-over-quarter, but that policymakers do not respond to output at all (i.e.,  $\psi_y = 0$ ).

into an otherwise standard New Keynesian model that features nominal rigidities—that is, price stickiness and monopolistic competition—as well. The combination of both frictions constitutes my benchmark—which I refer to as the BGG model. In investigating the amplification and propagation effects of the financial accelerator mechanism over the business cycle, I need to establish a reference for what would have happened otherwise without these two frictions. The frictionless allocation abstracting from nominal rigidities and financial frictions—which reduces the BGG model to a standard Real Business Cycle (RBC) economy—offers a natural point of reference for the assessment.

Up to a first-order approximation, the dynamics of the RBC model without frictions differ from those of the financial accelerator model only in the specification of a small subset of the log-linearized equilibrium conditions described before. Hence, the RBC variant of the model can be easily nested within the framework of Bernanke et al. (1999).

Moreover, the financial accelerator also nests other economically-relevant variants that strip down either financial frictions or nominal frictions alone. Abstracting from each friction separately conveys useful information to quantitatively assess the contribution of each friction and the interaction between them in the set-up of Bernanke et al. (1999). The specification variant that eliminates solely the financial friction reduces the BGG model to a Dynamic New Keynesian (DNK) one. In turn, the specification that assumes flexible prices and perfect competition—without nominal rigidities—can be interpreted as an RBC model augmented with financial frictions. I refer to this latter variant of the BGG model as the Financial Accelerator (FA) model.

The Phillips curve equation in (4)—which emerges under Calvo price stickiness—and the resource constraint in (19) are two of the equilibrium conditions that need to be modified under the assumption of flexible prices and perfect competition. The allocation abstracting from nominal rigidities and monopolistic competition mark-ups can be obtained simply assuming that: (a) the Phillips curve in (4) is replaced with a formula that equates real marginal costs  $\widehat{mc}_t$  to zero since under flexible prices and perfect competition the price charged by retailers must be equal to its marginal costs; and (b) the monopolistic competition (gross) mark-up is set to 1 (i.e.,  $\mu = 1$ ) in the resource constraint in (19) given the assumption of perfect competition. The changes postulated in (a) and (b) are needed for the RBC and FA variants of the model, as they both abstract from nominal rigidities.

Equation (13), which determines the optimal capital allocation, is another one of the equilibrium conditions that needs to be changed whenever state-contingent loans can be used to diversify away all idiosyncratic risks under the additional assumption of perfect information among borrowers and lenders. In that case, the allocation abstracting from financial distortions and inefficiencies can be obtained assuming that: (c) the gross external finance premium in steady state is set to 1 (i.e.,  $v(\gamma_n^{-1}) = 1$ ) in Eqs. (12) and (13) which implies that the borrowing cost is equal to the opportunity cost (the cost of internal funds) given by the real risk-free rate; and (d) the elasticity of the external finance premium relative to the entrepreneurs' leverage ratio evaluated in steady state is set to 0 (i.e.,  $v'(\gamma_n^{-1}) = 0$  or  $\vartheta = 0$ ) which eliminates



the spread between real borrowing rates and the real risk-free rate in Eq. (13). The changes required under the terms of (c) and (d) are necessary to implement the frictionless allocation of the RBC model in addition to (a) and (b). Conditions (c) and (d) are also needed in the standard DNK model set-up.

Assumptions (a) and (b) eliminate the standard New Keynesian distortions, while assumptions (c) and (d) ensure that it becomes efficient and optimal to accumulate capital to the point where the expected real return on capital equals the real risk-free rate. However, the role of the entrepreneurs' must also be reconsidered in the frictionless RBC and in the DNK cases as it becomes negligible for the aggregate dynamics. Entrepreneurs' consumption and labor income are already negligible by construction.<sup>11</sup> Absent financial frictions, entrepreneurs' aggregate characteristics do not matter for the determination of the investment path either. The leverage of the entrepreneurs (the borrowers) and their net worth (equity)—which is a state variable given by Eq. (17)—become irrelevant to set the borrowing costs and, therefore, the demand for capital. Hence, entrepreneurs' can be dropped without much loss of generality whenever the financial friction is eliminated.

Why does the model of Bernanke et al. (1999) incorporate entrepreneurs anyway? The financial accelerator model distinguishes between two types of economic agents, households and entrepreneurs. Entrepreneurs are risk-neutral agents which decide on the capital to be accumulated for the purposes of wholesale production and on how to finance that stock of capital with a combination of internal funds (net worth or equity) and external borrowing. The households are savers originating the external funds that are intermediated by the banks and eventually borrowed by the entrepreneurs. These two types of agents characterize the borrowers and savers of the economy, respectively.

Absent any financial distortions, the funding costs between internal and external sources must be equalized and given by the real risk-free rate. The predictions of the Modigliani-Miller theorem in a sense are restored and how the capital stock is funded should not matter for the aggregate dynamics of the economy. Therefore, the distinction between savers and borrowers becomes irrelevant for the allocation when the capital structure is undetermined—after all, funding from internal or external sources costs basically the same and should not affect the capital demand or any other economic decision.

Given the negligible impact of the entrepreneurs, the frictionless allocation of the RBC model and the DNK set-up can be approximated under the additional simplifying assumption that: (e) entrepreneurs can be ignored entirely by imposing  $\varrho = 0$  and  $\gamma_{c^e} = 0$  in order to derive the first-best allocation in the RBC case or the standard DNK solution. The simplification introduced in (e), which abstracts from entrepreneurs altogether, is of little quantitative significance to describe the dynamics of either variant of the model, but it has the advantage of reducing the number of

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<sup>11</sup>The labor share of entrepreneurs in the production function is small by assumption (guarantees the entrepreneurs only a small income stream in every period). The steady state consumption share of the entrepreneurs is small by assumption too.

state variables since tracking the entrepreneurs' net worth as in Eq. (17) is no longer needed.

These modifications and simplifications of the financial accelerator model of Bernanke et al. (1999) suffice to characterize an approximation to the frictionless RBC allocation with flexible prices, perfect competition and no-financial accelerator. This approximation of the frictionless model defines the notion of potential for the economy as it abstracts from all frictions. Together with the DNK and FA variants, it also provides the basis on which to assess the contribution to account for the U.S. business cycle of the financial distortion and the New Keynesian frictions (monopolistic competition and nominal rigidities) embedded in the Bernanke et al. (1999) model.

### 3 Model Parameterization

#### 3.1 Structural Parameters

In this section I describe the choice of the parameter values summarized in Table 1. The values for the taste and technology parameters that I use are fairly standard in the literature, and keep the model comparable to that of Bernanke et al. (1999) also in its parameterization. I assume that the discount factor,  $\beta$ , equals 0.99, which implies an annualized real rate of return of 4 %. The intertemporal elasticity of substitution,  $\sigma$ , and the preference parameter on leisure,  $\eta$ , are both equal to 1. Given that the utility function is assumed to be additively separable in consumption and leisure, the parameterization of  $\sigma$  and  $\eta$  ensures that preferences on both consumption and leisure are logarithmic and, therefore, that the model is consistent with a balanced growth path. The Frisch elasticity of labor supply,  $\varphi \equiv \eta(\frac{1-H}{H})$ , is determined by the share of hours worked in steady state,  $H$ , and the preference parameter  $\eta$ . Given that  $\eta = 1$ , I fix the share of hours worked,  $H$ , to be 0.25 in order to match the Frisch elasticity of labor supply,  $\varphi$ , of 3 preferred by Bernanke et al. (1999).<sup>12</sup>

The capital share,  $\psi$ , is set to 0.35 and the share of entrepreneurial labor,  $\varrho$ , is kept small at 0.01 as in Bernanke et al. (1999). I maintain the capital share, but set the entrepreneurial labor share to 0 abstracting from the entrepreneurs altogether whenever financial frictions are excluded. As a result, the households' labor share,  $1 - \psi - \varrho$ , is 0.64 in the financial accelerator BGG model and 0.65 in the DNK and RBC cases. The quarterly depreciation rate,  $\delta$ , is set to 0.025, which implies an annualized depreciation rate of approximately 10 %. The elasticity of Tobin's  $q$  with respect to the investment-to-capital ratio, given by the coefficient  $\chi$ , is taken to be 0.25.

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<sup>12</sup>The share of hours worked is broadly consistent with the U.S. data. The average hours worked relative to hours available per quarter in the U.S. for the period between 1971:III and 2007:IV is 0.2664. The average for the Great Moderation between 1984:I and 2007:IV is similar at 0.2771. For more details on the dataset, see Appendices A and B.

Table 1 Benchmark model parameterization

| Parameters                                         | BGG model                       | FA model | DNK model      | RBC model      |
|----------------------------------------------------|---------------------------------|----------|----------------|----------------|
| Intertemporal discount factor                      | $\beta = 0.99$                  | 0.99     | 0.99           | 0.99           |
| Elasticity of intertemporal substitution           | $\sigma = 1$                    | 1        | 1              | 1              |
| Inverse coefficient of risk aversion on leisure    | $\eta = 1$                      | 1        | 1              | 1              |
| Steady state share of household hours worked       | $H = 0.25$                      | 0.25     | 0.25           | 0.25           |
| Capital income share                               | $\psi = 0.35$                   | 0.35     | 0.35           | 0.35           |
| Entrepreneurs' labor share                         | $\varrho = 0.01$                | 0.01     | 0 <sup>a</sup> | 0 <sup>a</sup> |
| Depreciation rate                                  | $\delta = 0.025$                | 0.025    | 0.025          | 0.025          |
| Adjustment cost parameter                          | $\chi = 0.25$                   | 0.25     | 0.25           | 0.25           |
| Calvo price stickiness                             | $\alpha = 0.75$                 | –        | 0.75           | –              |
| Elasticity of substitution across retail varieties | $\theta = 10$                   | –        | 10             | –              |
| Entrepreneurs' "Survival Rate"                     | $\zeta = 0.9728$                | 0.9728   | –              | –              |
| Entrepreneurs' (inverse) leverage ratio            | $\gamma_n = 0.5$                | 0.5      | –              | –              |
| Entrepreneurs' consumption share                   | $\gamma_{ce} = 0.01$            | 0.01     | 0 <sup>a</sup> | 0 <sup>a</sup> |
| Steady state external finance premium              | $\nu(\gamma_n^{-1}) = 1.003951$ | 1.003951 | 1              | 1              |
| Steady state slope of external finance premium     | $\nu'(\gamma_n^{-1}) = 0.0337$  | 0.0337   | 0              | 0              |
| Taylor rule parameters                             |                                 |          |                |                |
| Policy inertia                                     | $\rho_i = 0$                    | 0        | 0              | 0              |
| Sensitivity to inflation                           | $\phi_\pi = 1.5$                | 1.5      | 1.5            | 1.5            |
| Sensitivity to output                              | $\phi_y = 0.5$                  | 0.5      | 0.5            | 0.5            |

Table 1 (Continued)

| Parameters                               | BGG model                  | FA model        | DNK model       | RBC model       |
|------------------------------------------|----------------------------|-----------------|-----------------|-----------------|
| Shock process parameters                 |                            |                 |                 |                 |
| Persistence of the TFP shock             | $\rho_a = 0.8789$          | 0.8789          | 0.8789          | 0.8789          |
| Volatility of the TFP shock              | $\sigma_a = \exp(-0.3967)$ | $\exp(-0.3967)$ | $\exp(-0.3967)$ | $\exp(-0.3967)$ |
| Persistence of the monetary policy shock | $\rho_m = 0.8753$          | 0.8753          | 0.8753          | 0.8753          |
| Volatility of the monetary policy shock  | $\sigma_m = \exp(-0.4660)$ | $\exp(-0.4660)$ | $\exp(-0.4660)$ | $\exp(-0.4660)$ |

This table defines the benchmark parameterization of the BGG model as well as the FA, DNK and RBC model variants used in my simulations  
a This superscript indicates a parameter value change that is used to eliminate entrepreneurs from the model and to simplify its set-up whenever financial frictions are excluded

The Calvo price stickiness parameter,  $\alpha$ , is assumed to be 0.75. This parameter value implies that the average price duration is 4 quarters. The (inverse of the) leverage or gearing ratio of the entrepreneurs,  $\gamma_n \equiv \frac{N}{K}$ , is set at 0.5 and the entrepreneurs' quarterly survival rate in each quarter,  $\zeta$ , is chosen to be 0.9728. All the parameter choices so far are taken directly from Bernanke et al. (1999), but for the remaining structural parameters I use additional sources to select their values. The elasticity of substitution across varieties,  $\theta > 1$ , is set to 10. This parameter characterizes the (gross) price mark-up  $\mu \equiv \frac{\theta}{\theta-1} > 1$  and its value is consistent with a plausible net mark-up of 11 % (documented in the U.S. data, for instance, by Basu 1996). Notice here that the structural parameters  $\alpha$ ,  $\gamma_n$ ,  $\zeta$  and  $\theta$  do not affect the aggregate dynamics of the frictionless RBC economy and that only a subset of them are needed for the parameterization of the FA and DNK variants.

I choose a tiny share of 0.01 for the steady state entrepreneurial consumption,  $\gamma_{ce}$ , in the financial accelerator model and set this share to 0 in the absence of financial frictions. This modification of the Bernanke et al. (1999) set-up ensures that consumption is essentially determined by the households' Euler equation and that entrepreneurs' consumption is negligible for the dynamics of the model, as discussed before. I set the monetary policy inertia,  $\rho_i$ , to 0, the response of the monetary policy rule to fluctuations in inflation,  $\phi_\pi$ , to 1.5 and the response to fluctuations in output,  $\phi_y$ , to 0.5 to be consistent with the policy recommendation of Taylor (1993). Although the proposal for  $(\rho_i, \phi_\pi, \phi_y)$  in Taylor (1993) was based on a reaction function fitted with year-over-year inflation and detrended output, I impose the same parameter values in (20) in all cases—even when the policy rule reacts to annualized quarter-over-quarter inflation and/or output gap measures (which is closer to how this policy rule is often specified in the literature for quantitative and theoretical work).

The steady state external finance premium,  $v(\gamma_n^{-1}) \equiv \frac{R^k}{R}$ , is set to 1.003951 in the financial accelerator model, which corresponds to the average quarterly ratio between the Baa corporate yield and the 20-year Treasury yield during the Great Moderation period from 1984:I until 2007:IV (see Appendices A and B for further discussion on how it is calculated). This ratio is consistent with a spread,  $R^k - R \equiv (\frac{R^k}{R} - 1)R$ , of approximately 160 basis points at an annualized rate given that  $R = \beta^{-1} = \frac{1}{0.99}$ .<sup>13</sup> This is a bit smaller than the 200 basis points of the historical average spread between the prime lending rate and the six-month Treasury bill rate that Bernanke et al. (1999) used to parameterize their model, but I believe it offers a cleaner measure of the risks modeled. Absent the financial friction, the steady state external finance premium  $v(\gamma_n^{-1})$  is simply set to 1 and—accordingly—the spread  $R^k - R$  becomes equal to 0.

Meier and Müller (2006) estimated a similar financial accelerator model and reported plausible values for the composite coefficient  $\vartheta \equiv (\frac{v'(\gamma_n^{-1})\gamma_n^{-1}}{v(\gamma_n^{-1})})$  around 0.0672, which is close to the value implied by the parameterization of Bernanke

<sup>13</sup>Note that I multiply  $(\frac{R^k}{R} - 1)R$  by 400 in order to express the spread on an annual basis and in percentages.

et al. (1999). I adopt the value suggested by the estimates of Meier and Müller (2006) which implies that a 1 % increase in the leverage ratio,  $(\frac{\Delta \gamma_n^{-1}}{\gamma_n^{-1}})$ , is on average associated with a 6.72 basis points increase in the interest rate ratio,  $(\frac{\Delta \frac{R^k}{R}}{\frac{R^k}{R}})$ . Therefore, given my parameterization of the entrepreneurs' leverage ratio,  $\gamma_n^{-1} = 2$ , and the external finance premium,  $v(\gamma_n^{-1}) = 1.003951$ , the slope coefficient  $v'(\gamma_n^{-1}) \equiv \frac{\partial v(\gamma_n^{-1})}{\partial \gamma_n^{-1}} = \vartheta \frac{v(\gamma_n^{-1})}{\gamma_n^{-1}}$  is set equal to 0.0337 in order to match the value of 0.0672 of the composite coefficient  $\vartheta$ . In the frictionless RBC case and the DNK case, I set the steady state slope of the external finance premium,  $v'(\gamma_n^{-1})$ , to 0 in order to bring  $\vartheta$  to 0 as well shutting down the financial frictions of the model.

It is worthwhile to consider here the implications of the parameterization on the long-run allocation of expenditures. The steady investment share in the model of Bernanke et al. (1999),  $\gamma_x \equiv \delta(\frac{\psi}{\mu(v(\gamma_n^{-1})\beta^{-1} - (1-\delta))})$ , is a composite coefficient of structural parameters that is distorted by monopolistic competition and by the long-run external finance premium. In the frictionless RBC steady state, the investment share is simply given by  $\delta(\frac{\psi}{\beta^{-1} - (1-\delta)})$  which takes the value of 0.25 under the parameterization I adopt here. The monopolistic competition mark-up,  $\mu \equiv \frac{\theta}{\theta-1}$ , is a function of the elasticity of substitution across retail varieties,  $\theta$ , which does not appear anywhere else in the log-linearized equilibrium conditions. Imposing a plausible mark-up of approximately 11 % alone reduces the steady state investment share to 0.22 for the DNK case.

The investment share in steady state is also affected by the size of the external finance premium in steady state,  $v(\gamma_n^{-1}) \equiv \frac{R^k}{R}$ . This distortion does not only affect the steady state investment share, because it also enters in the elasticity of the external finance premium to changes in the leverage of the borrowers,  $\vartheta \equiv (\frac{v'(\gamma_n^{-1})\gamma_n^{-1}}{v(\gamma_n^{-1})})$ , as well as in the weight of capital gains in the returns to capital,  $\epsilon \equiv (\frac{1-\delta}{v(\gamma_n^{-1})\beta^{-1}})$ . In any case, the combined effect of the monopolistic competition mark-up and the external finance premium reduces the investment share in the BGG model to just 0.20, which implies a very significant shift away from investment over the long-run.<sup>14</sup>

Finally, I assume a share of entrepreneurial consumption,  $\gamma_{ce}$ , of 0.01 for the financial accelerator model—as a modeling simplification relative to the Bernanke et al. (1999) set-up—and of 0 in the absence of financial frictions. As a result, the households' consumption share,  $\gamma_c$ , becomes equal to 0.79 in the model variants with financial frictions and 0.80 otherwise. I do not incorporate government consumption in the model, as noted earlier, so the consumption and investment shares are related to their counterparts in the data based on real private output (excluding government compensation).

<sup>14</sup>The quarterly share of real investment over private real output is broadly consistent with the U.S. data. The average quarterly share in the U.S. for the period between 1971:III and 2007:IV is 0.1757. The average for the Great Moderation between 1984:I and 2007:IV is similar at 0.1719. For more details on the dataset, see Appendices A and B.

### 3.2 *Shock Processes and Macro Observables*

I parameterize the financial accelerator model to be consistent with the existing literature and comparable with Bernanke et al. (1999). The parameters that characterize the monetary policy regime and the shock processes depart somewhat from those of Bernanke et al. (1999) to conform with the long-run features of the data observed during the Great Moderation. To be more precise, the policy specification is set as in Taylor (1993) to describe the prevailing monetary policy regime. The features of the TFP and the monetary shock processes are estimated from actual data on the Solow residual and the deviations between the Federal Funds rate and the Taylor (1993)'s prescribed policy rates during the Great Moderation period (as detailed in Appendices A and B).

The observable data that the model tries to explain is detrended—or demeaned, as the case might be. The estimates of the trend or the level of these macro variables are based on data for the Great Moderation period, which I project forwards but also backwards to get longer time series to work with. Before anything else, of course, I need to clarify what I consider to be the time span of the Great Moderation. While different authors date the start at different times, most authors agree that the major decline in macro volatility began in 1984. McConnell and Pérez-Quirós (2000) estimate a break date of 1984:I using quarterly real output growth data between 1953:II and 1999:II. As it has become common practice to follow the dating of McConnell and Pérez-Quirós (2000), I also adopt 1984:I as the starting quarter of the Great Moderation for the purposes of this paper.

Given that the policy framework in the model is geared towards describing the post-Bretton Woods era, I do not attempt to assess in this paper the path of the macro series prior to 1971:III.<sup>15</sup> Moreover, I also abstract from discussing in great depth the structural changes that took place during the 1970s. In turn, I focus solely on the period since the onset of the Great Moderation. This avoids the structural breaks found in the data prior to the 1980s as well as the two consecutive recessions of the early 1980s, so it makes more straightforward the mapping of the data into the model. The Great Moderation period since 1984:I is largely characterized by stable trends, except in the aftermath of the 2007 recession. In fact, that is the only break that I consider here. I investigate the 2007 recession as a break with the Great Moderation allowing explicitly for the possibility of a level—but not a growth—shift on the long-run path along which the U.S. economy evolved. For my analysis, I compute detrended (or demeaned) variables that incorporate and ignore that level shift.

I settle with 2007:IV as the end of the Great Moderation period in order to ensure that my estimation results of the underlying trends of the data and, more generally,

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<sup>15</sup>The dollar became a fiat currency after the U.S. unilaterally terminated convertibility of the U.S. dollar to gold on August 15, 1971, ending the Bretton Woods international monetary system that had prevailed since the end of World War II. Floating exchange rates and increasing capital account openness characterize the post-Bretton Woods period in the U.S., in a major break with the prevailing monetary policy regime under Bretton Woods.

the parameterization of the model would not be driven by a few observations during and after the 2007 recession. I consider the possibility of a level shift occurring after the 2007 recession with the implication of increasing the size of the interest rate spreads, while lowering the share of hours worked and the levels of real private output (excluding government compensation), real private investment and real private consumption. The break itself is dated where the fall in real private output (excluding government compensation) is the highest in percentage terms—in the dataset described in Appendices A and B, by this metric, the level shift occurs in 2009:II.

Table 2 summarizes the empirical estimates of the Solow residual and monetary shock processes, as well as the detrending (or demeaning) of the observable macro variables of interest—the data includes real private output (excluding government compensation), real private investment, real private consumption, and the share of hours worked all expressed in per capita terms, as well as year-over-year consumption (of nondurables and services) inflation and the spread between the Baa corporate and the 20-year Treasury yield. Each specification is set in state space form and estimated by Maximum Likelihood with data for the Great Moderation period between 1984:I and 2007:IV.

Subsequently, I fix the coefficients of the specification at their estimated values for [1984:I, 2007:IV] and add a recession dummy that takes the value of 1 from 2009:II onwards to take account of the possibility of a level shift. I expand the estimation sample to go up to 2012:I and estimate the coefficient on the recession dummy to determine both the size and the significance (if any) of the break. This estimation strategy implemented in two stages preserves the estimates obtained with data prior to the 2007 recession as such, while allowing me to incorporate the data available up to 2012:I in order to test the hypothesis that a level shift may have occurred in the aftermath of the 2007 recession. Table 2 summarizes the empirical evidence for such structural shift whenever it is statistically significant in the data and reports the size of the break based on the currently available data.

**Macro Observables** Figure 1 illustrates the behavior of the share of hours worked, consumer (nondurables and services) price inflation in year-over-year rates and the quarterly interest rate spread around a constant level. I use historical estimates of the mean during the Great Moderation period for the share of hours worked and the interest rate spread, while the inflation level is set to the implicit monetary policy target of 2 percent sought over the past three decades and assumed by Taylor (1993). I maintain the description of the monetary policy framework invariant in my current analysis even after the 2007 recession, so the inflation target of 2 percent is unchanged before and after 2009:II. In turn, I allow for the possibility of a level shift in both hours worked as well as the quarterly interest rate spread that is shown in Table 2 to be statistically significant. The interest rate spread went up by around 10 basis points on average after 2009:II, while the share of hours worked in logs declined on average around 5.78 percent.

Figure 2 illustrates the path of the time series for real private output in logs, real private investment in logs and real private consumption in logs along a linear trend. In linearly detrending these macro time series, however, I impose a priori only a



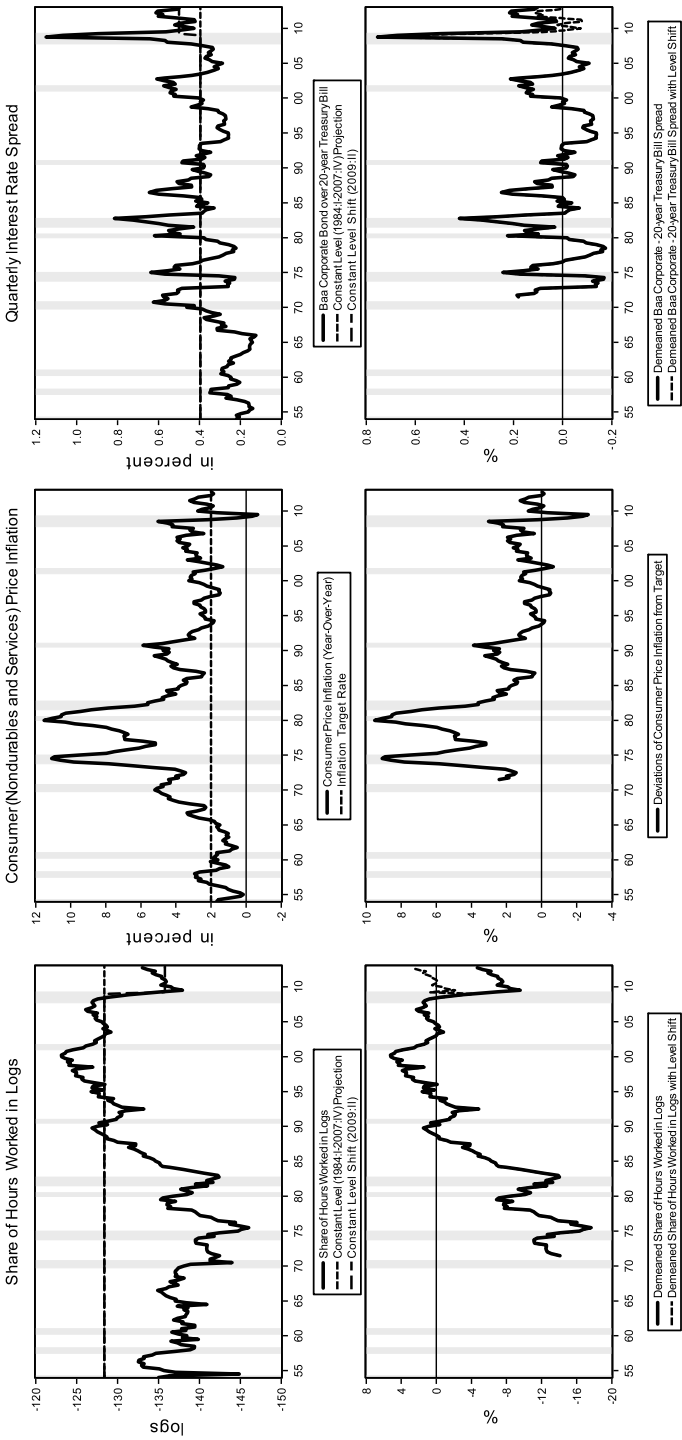
Table 2 Summary of maximum likelihood estimates

| Dependent variable                                                                                     | Regression estimates                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | Sample         |
|--------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------|
| Endogenous macro aggregates                                                                            |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |                |
| $\bar{h}_t \equiv \ln(h_t) 100$                                                                        | $\bar{h}_t = -128.3841 + u_t^h, \ln \sigma(u_t^h) = 1.104195$<br>(0.344779) (0.076884)                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | 1984:I–2007:IV |
|                                                                                                        | $\bar{h}_t = -128.3841 - 7.4254401_{(t \geq 2009:II)} + u_t^h, \ln \sigma(u_t^h) = 1.104195$<br>(2.827576)                                                                                                                                                                                                                                                                                                                                                                                                                                              |                |
| $\bar{\pi}_t \equiv (\frac{P_t - P_{t-4}}{P_{t-4}}) 100$                                               | $\bar{\pi}_t = 2 + u_t^\pi, \ln \sigma(u_t^\pi) = 0.394633$<br>(0.081194)                                                                                                                                                                                                                                                                                                                                                                                                                                                                               | 1984:I–2012:I  |
| $spread_t \equiv (\rho_t^{Baa Corp} - \rho_t^{20\text{ yr } T\text{-Bill}}) 100$                       | $spread_t = 0.395125 + u_t^{spread}, \ln \sigma(u_t^{spread}) = -2.377567$<br>(0.010858) (0.088871)                                                                                                                                                                                                                                                                                                                                                                                                                                                     | 1984:I–2007:IV |
|                                                                                                        | $spread_t = 0.395125 + 0.1045791_{(t \geq 2009:II)} + u_t^{spread}, \ln \sigma(u_t^{spread}) = -2.377567$<br>(0.043423)                                                                                                                                                                                                                                                                                                                                                                                                                                 |                |
| $\rho_t^j \equiv 0.25 \ln(1 + \frac{R_t^j}{100}),$<br>$j \in \{\text{Baa Corporate, 20 yr Treasury}\}$ | $\begin{pmatrix} 100 \frac{\bar{c}_t}{\bar{y}_t} \\ 100 \frac{\bar{x}_t}{\bar{y}_t} \end{pmatrix} = \begin{pmatrix} 63.91049 \\ 17.16505 \end{pmatrix} + \begin{pmatrix} u_t^{c\_share} \\ u_t^{x\_share} \end{pmatrix},$                                                                                                                                                                                                                                                                                                                               | 1984:I–2007:IV |
| $\bar{y}_t \equiv \ln(y_t) 100$                                                                        | $\begin{pmatrix} \bar{y}_t \\ \bar{c}_t \\ \bar{x}_t \end{pmatrix} = 100 \ln(10934.29) + 100 \begin{pmatrix} 0 \\ \ln(\frac{1}{100} (63.91049)) \\ \ln(\frac{1}{100} (17.16505)) \end{pmatrix} + 0.431212_t + \begin{pmatrix} u_t^y \\ u_t^c \\ u_t^x \end{pmatrix},$<br>(110.5673) (0.138189) (0.019412)<br>$\ln \sigma(u_t^{c\_share}) = 0.612087, \ln \sigma(u_t^{x\_share}) = 0.168934,$<br>(0.277974) (0.257966)<br>$\ln \sigma(u_t^y) = 1.047145, \ln \sigma(u_t^c) = 2.124132, \ln \sigma(u_t^x) = 0.269371$<br>(0.516894) (0.328568) (0.135221) | 1984:I–2012:I  |

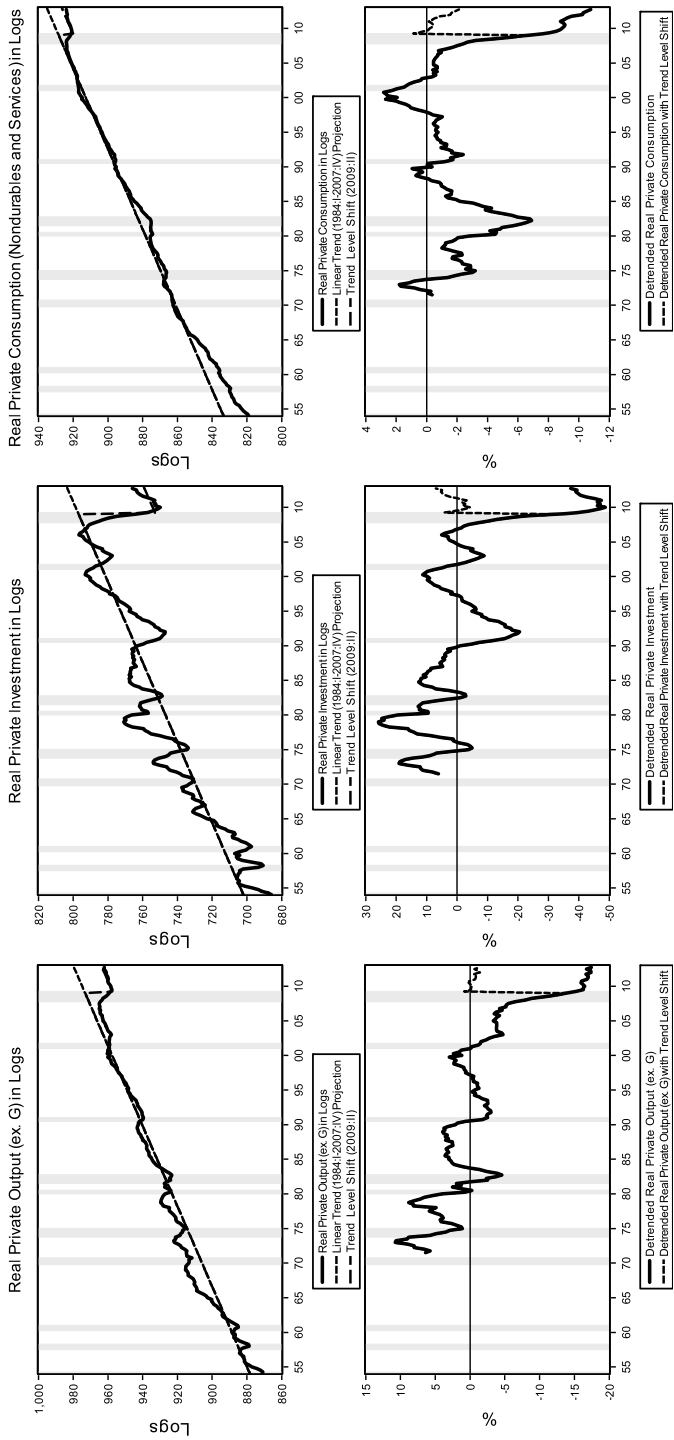
Table 2 (Continued)

| Dependent variable                               | Regression estimates                                                                                                                                                                                                                                                                                                                                                                                                                           | Sample         |
|--------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------|
| $\bar{c}_t \equiv \ln(c_t)/100$                  | $\begin{pmatrix} 100 \frac{\bar{c}_t}{y_t} \\ 100 \frac{\bar{x}_t}{y_t} \end{pmatrix} = \begin{pmatrix} 63.91049 + 4.9547861_{(t \geq 2009:II)} \\ (2.370044) \\ 17.16505 - 4.219118_{(t \geq 2009:II)} \\ (0.668740) \end{pmatrix} + \begin{pmatrix} u_t^{c,share} \\ u_t^{x,share} \end{pmatrix},$ $\begin{pmatrix} \bar{y}_t \\ \bar{c}_t \\ \bar{x}_t \end{pmatrix} = 100 \ln(10934.29) - 16.12661_{(t \geq 2009:II)} + 0.431212t + \dots$ | 1984:I–2007:IV |
| $\bar{x}_t \equiv \ln(x_t)/100$                  | $100 \begin{pmatrix} 0 \\ \ln(\frac{1}{100} (63.91049 + 4.9547861_{(t \geq 2009:II)})) \\ \ln(\frac{1}{100} (17.16505 - 4.219118_{(t \geq 2009:II)})) \end{pmatrix} + \begin{pmatrix} u_t^y \\ u_t^c \\ u_t^x \end{pmatrix},$ $\begin{pmatrix} \ln \sigma(u_t^{c,share}) = 0.612087, \ln \sigma(u_t^{x,share}) = 0.168934, \\ \ln \sigma(u_t^y) = 1.047145, \ln \sigma(u_t^c) = 2.124132, \ln \sigma(u_t^x) = 0.269371 \end{pmatrix}$          | 1984:I–2012:I  |
| Exogenous shock processes                        |                                                                                                                                                                                                                                                                                                                                                                                                                                                | 1984:I–2007:IV |
| $\bar{s}_t \equiv (\ln(\frac{S}{S_0}))/100$      | $\bar{s}_t = 55.73225 + 0.18507t + \hat{a}_t$ $(1.589296) \quad (0.026276)$                                                                                                                                                                                                                                                                                                                                                                    |                |
| $i_t^{AR} \equiv$ federal funds rate (effective) | $\hat{a}_t = 0.878870\hat{a}_{t-1} + \varepsilon_t^a, \ln \sigma(\varepsilon_t^a) = -0.396746$ $(0.055162) \quad (0.059299)$ $i_t^{AR} = 2 + \bar{\pi}_t + 0.5(\bar{\pi}_t - 2) + 0.5\hat{y}_t + \hat{m}_t$ $\hat{m}_t = 0.875284\hat{m}_{t-1} + \varepsilon_t^m, \ln \sigma(\varepsilon_t^m) = -0.465995$ $(0.040514) \quad (0.062270)$                                                                                                       | 1984:I–2007:IV |

This table reports the Maximum Likelihood estimates with standard errors between parentheses. The E-views 8 codes to generate the estimates are available from the author upon request



**Fig. 1** Deviations from constant level on hours worked, inflation and interest rate spreads. This graph plots the share of hours worked, the consumer (nondurables and services) price inflation in year-over-year terms, and the Baa corporate spread over the 20-year Treasury Bill in deviations. I include a constant level estimated over the Great Moderation period (1984:I–2007:IV) projected backwards and forwards allowing for a level shift on 2009:II (except for inflation), and the corresponding variables in deviations since 1971:III. For more details, see Appendix B ‘U.S. Dataset’ and the estimates reported in Table 2. The shaded areas represent NBER recessions



**Fig. 2** Deviations from trend on output, investment and consumption. This graph plots the detrended real private output (ex. G), real private investment and real private consumption. I include the log-linear trend over the Great Moderation period (1984:I–2007:IV) projected backwards and forwards allowing for a level shift on 2009:II, and the corresponding detrended variables since 1971:III. For more details, see Appendix B ‘U.S. Dataset’ and the estimates reported in Table 2. The shaded areas represent NBER recessions

minimal set of theoretical constraints with the aim to limit the violence done to the data. In particular, I only require the following two model-consistent features to be satisfied by the specification: First, the financial accelerator model that I use is consistent with a balanced growth path in which consumption and investment grow at the same rate as output, so I assume a time trend that is linear and has a common slope on all three variables (as described in Table 2 and in Appendices A and B).

Second, any structural change that affects the steady state of the financial accelerator model can result in a level shift in output that is consistent with an economy growing along the new path at the same constant rate as before but from a different level. Structural changes could also produce shifts in the steady state consumption and investment shares, though. I, therefore, impose the constraint that intercept of the linear time trend be consistent with the historical shares for consumption and investment observed during the Great Moderation period. I allow in the specification for the possibility that the level shift after the 2007 recession resulted in a decline in the level of output (the intercept) as well as resulted in a change in the long-run shares of consumption and investment.

The evidence reported in Table 2 seems to be consistent with a statistically-significant level shift in the trend specification for real private output, consumption and investment after 2009:II. As seen in Figure 2, the break is basically matched by the decline in real private output and appears to be largely permanent. A simultaneous downward shift of the long-run investment share of around 4 percentage points absorbed by a similar increase in the consumption share accounts for the large decline observed in investment and the more moderate downward shift on consumption.<sup>16</sup>

Adjusting for the estimated level shift and the perceived decline in the investment share, real private output and consumption remain below their new long-run path while real private investment bounced-back above trend by 2012:I. The evidence does not necessarily suggest that there is a break in trend growth in the data in the aftermath of the 2007 recession. In turn, ignoring the possibility of a level shift altogether would produce deviations from the Great Moderation trend that are unprecedented—for the post-Bretton Woods period since 1971:III.

**Solow Residual** Since the model abstracts from population growth, the Solow residuals are computed from data on the stock of capital, the share of hours worked, and private output expressed in per capita terms. For exact details on the calculation of the U.S. Solow residual, see the data description in Appendices A and B. I extract the relevant features of the stationary shock process  $\hat{a}_t$  used in the model taking into

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<sup>16</sup>The basic intuition of the permanent income hypothesis implies that consumption, as a fraction of the permanent income of households, moves in sink with permanent income changes. In the context of the model I use here that same logic implies that consumption shifts should follow from permanent changes in output. However, that is not the full story, as consumption declines depend on the long-run consumption and investment shares which also appear to have shifted around with the break.

account that what is observed is the Solow residual,  $S_t$ , and this measure includes a trend component that arises from labor-augmenting growth. Hence, I specify a deterministic linear time trend for  $S_t$  with autoregressive residuals to recover  $\hat{a}_t$  and to estimate the persistence and volatility of this stationary process. I cast the model for the Solow residual into state space form and estimate it by Maximum Likelihood for the period of the Great Moderation between 1984:I and 2007:IV.

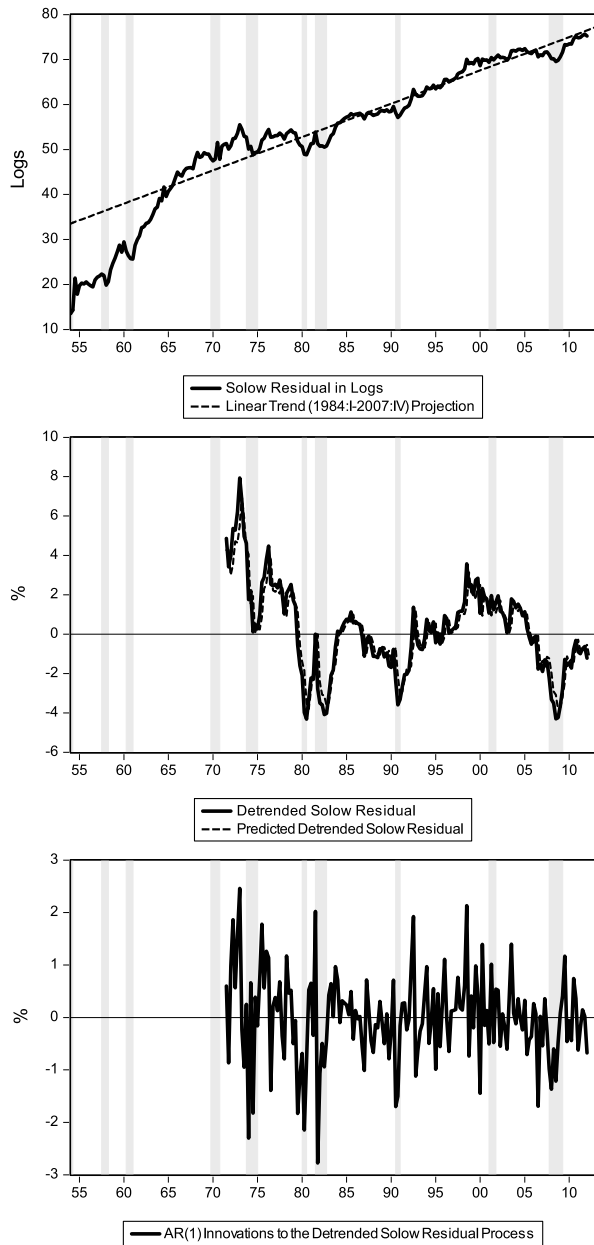
The estimates of the stationary part of the Solow residual in logs are fitted to an  $AR(1)$  process which characterizes the TFP shock dynamics of the model for  $\hat{a}_t$  described in Eq. (9). These features of the TFP shock process are common knowledge and economic agents factor that information in forming their own expectations. The persistence of TFP given by  $\rho_a$  is, therefore, set at the estimated value of 0.878870. Similarly, the volatility of the shock  $\sigma_a$  is equal to  $\exp(-0.396746) = 0.6725$ . Figure 3 illustrates the linear time trend and the stationary components of the actual series for the Solow residual in logs,  $S_t$ .

I consider the possibility of a level shift in the path of  $S_t$  after 2009:II as well. However, the p-value on the coefficient of the recession dummy that indicates the possibility of this level shift comes at 0.8577, so I cannot reject the null hypothesis that there is no such shift in the trend of the observed Solow residuals. The lack of evidence of a level shift in  $S_t$  suggests that the fall documented in real private output (excluding consumption) as well as on other macro variables does not follow from a level shift in productivity, but must be the result of structural changes that affect the steady state of the financial accelerator model. One distinctive possibility that would be consistent with the model and the data showing higher interest rate spreads and lower investment shares since the 2007 recession is this: external funding costs may have become significantly higher in the aftermath of the recession, making investment costlier, and therefore reducing the long-run capital-to-labor ratio and the level of economic activity.

The evidence gathered in the data also shows that trend growth has been noticeably higher for real private output than for the Solow residual. However, this fact can also be accounted by theory—this could be the case because output growth itself does not have to grow at the same rate as the Solow residual by the contribution of other growth factors (e.g., a trend decline in the relative price of capital goods) as discussed in Appendices A and B.

**Monetary Policy Shock** I define the monetary policy rule in the spirit of Taylor (1993), where the monetary policy instrument is the (effective) Federal Funds rate in percent per annum. As in Taylor (1993), the central bank reacts to the percentage inflation rate over the previous four quarters and to the percent deviation of real GDP from a log-linear trend (where the trend of real private output is estimated with data for the Great Moderation period only). I also maintain the parametric assumptions of Taylor (1993) implying that the response to fluctuations in inflation,  $\phi_\pi$ , is 1.5, the response to fluctuations in detrended output,  $\phi_y$ , is 0.5, and the interest rate smoothing parameter,  $\rho_i$ , is set to 0. All the sources on U.S. monetary policy rates are described in Appendices A and B.

**Fig. 3** Solow residuals: trend and stationary components. This graph plots the U.S. Solow residual, detrended and fitted to follow an  $AR(1)$  process. I include the log-linear trend over the Great Moderation period (1984:I–2007:IV) projected backwards and forwards, the corresponding detrended variables and the innovations of the  $AR(1)$  since 1971:III. For more details, see Appendix B ‘U.S. Dataset’ and the estimates reported in Table 2. The *shaded areas* represent NBER recessions



The Taylor (1993) implied annualized rates (in percentages),  $i_{t+1}^{AR}$ , are calculated with the following mathematical formula,

$$\begin{aligned}
 i_t^{AR} &= 2 + \bar{\pi}_t + 0.5(\bar{\pi}_t - 2) + 0.5\hat{y}_t + \hat{m}_t \\
 &= 4 + 1.5(\bar{\pi}_t - 2) + 0.5\hat{y}_t + \hat{m}_t,
 \end{aligned} \tag{28}$$

where  $\bar{\pi}_t$  is the rate of inflation over the previous four quarters in percentages, and  $\hat{y}_t$  is the detrended real private output in logs expressed in percentages. Taylor (1993) sets the implicit inflation rate at 2 percent and also adds a long-run (annualized) real interest rate  $r^{AR}$  of 2 percent in the specification of the rule in (28). Hence, if the inflation rate is on target (i.e., if  $\bar{\pi}_t = 2$ ) and real output is on trend (i.e., if  $\hat{y}_t = 0$ ), the Taylor rate would be equal to  $i_t^{AR} \equiv 2 + 2 = 4$ —two percentage points from the inflation target and two percentage points from the real rate.

I derive the monetary policy deviations  $\hat{m}_t$  using the formula in (28) and the same parameterization as Taylor (1993) to calculate the Taylor rates, but at least three caveats are in order: First, the conventional assumption underlying the class of models with nominal rigidities that I investigate here is that the long-run inflation rate and the inflation target are 0. Given that, the real and nominal interest rates must be equal along the balanced growth path—assuming that the unconditional mean of the deviations between the (effective) Fed Funds rate and the Taylor rates is 0 as well. This implies that the steady state nominal interest rate  $i^{AR}$  and the steady state real rate  $r^{AR}$  are equal to 4 percent annualized by consistency with a parameterization of the time discount factor,  $\beta$ , at 0.99. In other words, while the rule is unchanged, the interpretation of the long-run inflation and interest rates is conceptually different from that postulated by Taylor (1993) in his empirical work.

Second, while Taylor (1993) assumes the inflation target to be 2 percent, I observe that the actual inflation average over the Great Moderation period is 3.14 %. To treat the data on inflation and extract the cyclical component, I assume nonetheless that the inflation rate moves around the target of 2 percent set by Taylor (1993)—instead of demeaning the data.

Third, consistency between the model definitions and the data is maintained throughout the paper. For instance, I define real private output to be real GDP excluding government compensation of employees to be consistent with the model definition of output. I calculate my own measure of the log-linear trend of real private output (excluding government compensation) in order to fit the estimated trend for the Great Moderation period.<sup>17</sup> I also calculate the relevant inflation rate in terms of the consumption (nondurables and services) price deflator for the same reason. Hence, I depart from the preferred measures of real GDP and the GDP deflator used in Taylor (1993) solely to facilitate the mapping between the data and the model.

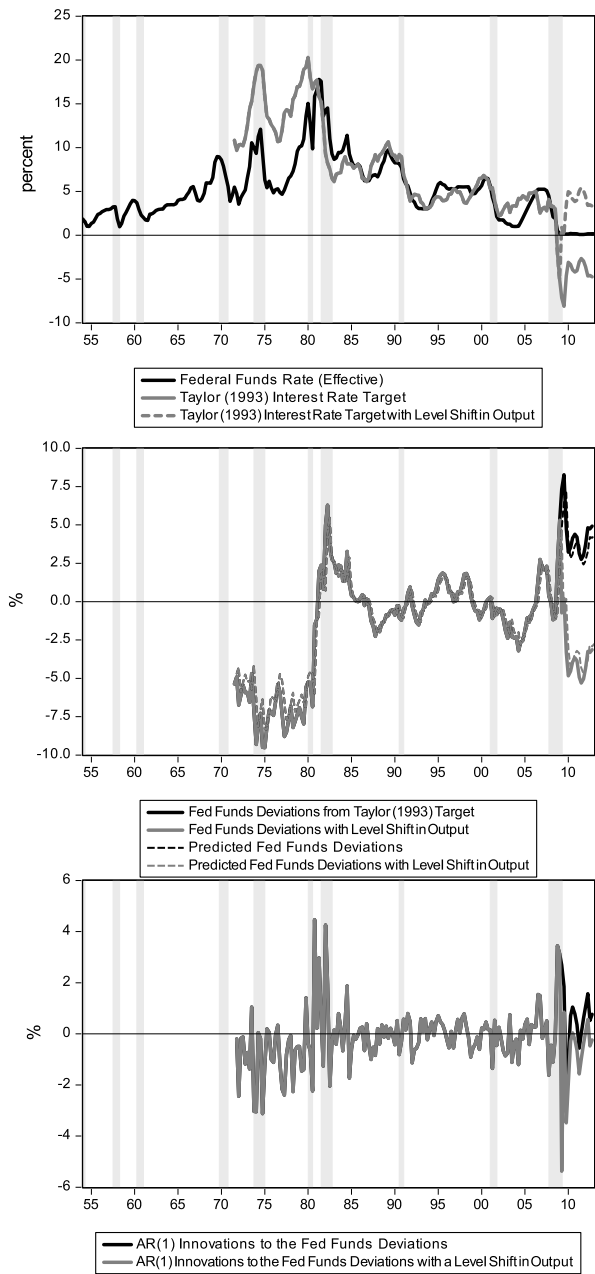
Monetary policy shocks are defined by the residual  $\hat{m}_t$ , implied by the deviations between the (effective) Federal Funds rate and the policy rule in (28). The performance of the rule is illustrated in Figure 4. As can be seen, even though I have used different data sources than those preferred by Taylor (1993), the long-held view that the rule provides a good description of most of chairman Greenspan's tenure at the helm of the Federal Reserve between 1987 and 2002 remains unchanged.

A number of further qualifications need to be made regarding the conduct of monetary policy during the Great Moderation period and about the interpretation of the monetary shocks derived in this way. First, I am surely missing some transitional

<sup>17</sup>Taylor (1993) estimates trend real GDP with a shorter sample from 1984:I till 1992:III.



**Fig. 4** U.S. monetary policy under the Taylor (1993) rule. This graph plots the Federal Funds rate (effective, annualized), as the policy instrument of reference. It also includes the Taylor rule rates based on the Taylor (1993) specification of the monetary policy rule reacting to changes in the year-over-year inflation rate and detrended output. The Taylor rule residuals are treated as exogenous and defined as the difference between the Federal Funds rate effective and the Taylor rule rates. The Taylor rule residuals are fitted to an  $AR(1)$  process over the period of the Great Moderation (1984:I–2007:IV) and then projected backwards and forwards. I also include the corresponding predicted Taylor residuals and the innovations of the  $AR(1)$  since 1971:III. For more details, see Appendix B ‘U.S. Dataset’. The shaded areas represent NBER recessions



dynamics in the first half of the 1980s. The 1970s and part of the early 1980s was a convulse period of time that saw significant structural and trend changes, none of which is fully captured by the model as it stands. Implicitly it is being assumed

that the new trends for the entire period were already known at the onset of the Great Moderation. The transitional dynamics could, perhaps, account for some of the discrepancies between the Taylor rule and the (effective) Fed Funds rate in the early-to-mid-1980s. I do not explore the issue further in the paper and, therefore, treat the resulting deviations as purely exogenous monetary shocks—in any event, their impact does not appear too large for the relevant period after 1984:I.

Second, there is a sizeable and systematic downward deviation from the rule after 2002. This coincided in time roughly with the aftermath of the Asian Crisis of 1997, the LTCM bailout in 1998, the 9/11 events, and the subsequent recession of 2001. It has resulted in a prolonged period where the Federal Funds rate has been kept too low relative to the prescriptions of the Taylor (1993) rule. This fact has been noted and extensively discussed before, but the model laid down here allows me to investigate its implications for the U.S. business cycle in a general equilibrium setting. One possible interpretation is that these systematic deviations of the policy rule could be indicative of a change in monetary policy regime that occurred in the late 1990s, leading to an environment with systematically lower interest rates. Many factors can contribute to such a regime change, for instance, a change in the weight policymakers assign to fighting inflation and promoting sustainable growth, a change in the long-run inflation target, or a change in the long-run real rates.

Distinguishing whether the deviations from the rule are exogenous after 2002 or reflect some sort of policy shift (or regime change) is probably one of the key challenges to determine the contribution to the U.S. business cycle that monetary policy has had over this period. I leave the exploration of alternative explanations for future research, and I treat the observed deviations under the Taylor rule in (28) as realizations coming from the same exogenous process for the monetary policy shocks as prior to 2002. I also assume that economic agents did not perceive those deviations as implying a regime shift for monetary policy.

Finally, there is the crucial issue of how to handle monetary policy at the zero-lower bound, especially since 2007. Based on my dataset for the U.S. economy and my characterization of the Taylor rule, the prescribed rate should have become negative in the fourth quarter of 2008 hitting a low point of  $-8.11\%$  in the third quarter of 2009 and would have remained in negative territory for the rest of my sample. In turn, if the Taylor rule had been followed recognizing the possibility of a level shift in output taking place around 2009:II, the prescribed path would have looked rather different as detrended output would look very different (see Figure 2). The prescribed Taylor rate would have still dipped below the zero-line in the fourth quarter of 2008 reaching a low point of  $-5.13\%$  in the first quarter of 2009 but would have returned to positive territory after that.

The financial accelerator model is unconstrained in the setting of the policy rate and, therefore, entails that no agent incorporates in its decision-making the practical fact that nominal rates are bounded below by zero.<sup>18</sup> This is an issue that cannot be disregarded even in the case the central bank would have recognized the possibility

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<sup>18</sup>Unless some unorthodox measures are put in place by central banks that I am not considering here either.

of a level shift in output from very early on. In any event, I will leave the exploration of the zero-lower bound for further research. Instead, the deviations between the unconstrained Taylor rate and the constrained (effective) Federal Funds rate are merely treated as realizations of the same exogenous monetary policy shock process.

With all those caveats in mind about what constitutes a monetary policy shock, I fit the series of Taylor rule deviations to an  $AR(1)$  process. The persistence of the monetary shock process given by  $\rho_m$  is, therefore, set at the estimated value of 0.875284. Similarly, the volatility of the monetary shock  $\sigma_m$  is equal to  $\exp(-0.465995) = 0.6275$ . This estimated process characterizes the dynamics of the monetary policy shock described in (22). I maintain the conventional assumption that all agents know about these shock dynamics and factor them into their decision-making process in forming their expectations.

## 4 Simulation and Quantitative Findings

In this paper I investigate the strengths and weaknesses of the financial accelerator mechanism of Bernanke et al. (1999) to account for the business cycle fluctuations observed in the U.S. data during the Great Moderation period and the 2007 recession. I focus my attention primarily on real private output per capita, real private consumption per capita, real private investment per capita, the share of hours worked per capita, and (year-over-year) inflation, since the path of these variables often provides a useful gauge of the model's overall performance and the effectiveness of monetary policy. I also track the quarterly interest rate spreads as a key indicator of the financial mechanism that the model is trying to describe.

Then, I ask the following questions from the Bernanke et al. (1999) framework: (a) to what extent does the financial accelerator model replicate the path followed by the macro variables of interest during the Great Moderation?; and (b) to what extent can a first-order approximation of the financial accelerator model such as the one proposed in the paper account for the unusual path that the U.S. economy has taken since the 2007 recession? In other words, is the financial accelerator model of Bernanke et al. (1999) a good benchmark to interpret the Great Moderation and the 2007 recession?

Given some initial conditions, the linearized equilibrium equations and the stochastic shock processes described in Section 2 constitute a fully specified linear rational expectations model. To answer my own questions about the model, I first derive the policy functions implied by the linear rational expectations model laid out in Section 2.<sup>19</sup> I use those policy functions to map the realizations of the detrended U.S. Solow residual in logs (the TFP shock) and the U.S. monetary policy deviations presented in Section 3.2—and also discussed in Appendices A and B—into measures of the cyclical behavior of (per capita) real private output, consumption

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<sup>19</sup>All the policy functions used in my simulations are derived using the software package Dynare. The parameterization satisfies the Blanchard-Kahn conditions, so a solution exists and is unique.

and investment, the share of hours worked per capita, year-over-year inflation and quarterly interest rate spreads (the external finance premium). I then compare the model simulations against the U.S. data also presented in Section 3.2.

To initialize each simulation, I assume that the economy is growing at (or near) its balanced growth path at the starting quarter. Prior to the Great Moderation period, 1983:IV stands out as a quarter where actual real private output per capita is approximately equal to its potential (as implied by the log-linear trend estimated for the period 1984:I–2007:IV). Hence, I take that quarter to be the initial period in all the simulations and set the exogenous state variables to match the values for the detrended Solow residual in logs and the monetary policy deviation for that quarter. In turn, all endogenous state variables of the model are set to zero in 1983:IV. For every subsequent quarter, the state variables are simulated using the realizations of the detrended Solow residual in logs and the monetary policy deviations obtained from the U.S. data. I only report the simulated endogenous series for the relevant period since 1984:I onwards.

I run a number of policy experiments and counterfactual simulations intended to gauge the strength of the financial accelerator mechanism, the contribution of TFP versus monetary shocks over the business cycle, and the sensitivity of the predictions to some key modelling assumptions. In order to test the robustness of the results, I specifically explore changes to the benchmark model that have been suggested already in the literature. More concretely, I investigate the role of the inflation rate measure (year-over-year versus quarter-over-quarter rates) and output fluctuations (detrended output versus the output gap) to which monetary policy reacts, the degree of nominal rigidities, and the sensitivity of the external finance premium to monetary shocks.

#### ***4.1 Model Comparison and Assessment***

To establish a clear point of reference, I evaluate the framework laid-out here during the Great Moderation—as this is the time period that my parameterization is meant to characterize. I simulate the financial accelerator model under the Bernanke et al. (1999) set-up specification—the benchmark BGG model—together with three nested variants—that include the financial accelerator model without nominal rigidities (FA), the standard New Keynesian model without financial frictions (DNK), and the standard Real Business Cycle (RBC) model without nominal rigidities or financial frictions. I compare all of those simulations against the observed data along three conventional dimensions: in their ability to match the standard business cycle moments documented in the data (Table 3), on the evidence of comovement between the simulations and the data (Tables 4A and 4B), and in their contribution to account for the movements of the data (illustrated by Figure 5 including the post-2007 recession period).

All simulations evaluated here are derived under the assumption that the central bank’s monetary policy during the Great Moderation can be well-approximated

by the Taylor (1993) rule introduced in its general form in Eq. (20). Subsequently, I will assess simple departures from this particular conception about the conduct of monetary policy that illustrate the importance of policy over the business cycle in the presence of different frictions. I consider the role of monetary shocks as drivers of business cycles (Figure 6), the economic significance of price stickiness—which introduces monetary non-neutrality into the model—and its interaction with the financial friction (Figure 7), and experiment with different measurements of inflation and output fluctuations in the policy reaction function (Figure 8).

Tables 3, 4A and 4B are constructed with actual and simulated data from 1984:1 until 2007:IV which excludes entirely the 2007 recession and its aftermath—it also excludes the period of nominal policy rates at the zero-lower band that followed. As indicated earlier, the simulations take as given the Taylor (1993) specification introduced in Eq. (20) and the parameterization in Table 1. The rule seems to track fairly well the path of the (effective) Federal Funds rate during most of the Great Moderation period since the mid-1980s until around 2002 and, therefore, can be thought of as a referent for the prevailing monetary policy regime for the period of interest in this study. Tables 3, 4A and 4B, therefore, provide insight on the key question of how well does the BGG model or any of its nested variants do in explaining the Great Moderation era under a conventional characterization of the policy regime.

In Table 3, the reported moments are all unconditional—for the simulated data these unconditional moments are all derived under the exact same realization of the TFP shock and the monetary policy shock derived earlier. I review standard business cycle moments (such as standard deviations, autocorrelations and correlations) for the main macro variables of interest—output, investment, consumption, hours worked, inflation and the external finance premium. The standard deviations summarize the volatility inherent in the actual and simulated data. This metric reveals important differences across models and shortcomings in accounting for the volatility observed during the Great Moderation. The BGG model or the FA variant without nominal rigidities tend to provide a better match for the standard deviations found in the data during the Great Moderation period, but it is notable that the patterns of volatility are altered by the combination of frictions—nominal rigidities and financial frictions—in rather complex ways.

The frictionless model, that represents the standard RBC way of interpreting business cycles, deviates from the data primarily because it severely undershoots the volatility of hours worked (0.426 versus 3.033 in the data) and investment (3.466 versus 8.540 in the data). This means that output is somewhat smoother than the data too, while the volatility of consumption is pretty much right on target. The RBC model is subject to both TFP and monetary policy shocks. However, the monetary shocks only have an impact on the nominal variables and all real variable are driven by the realization of the TFP shock because monetary policy is neutral in this case. Naturally, the specification of the systematic part of the Taylor rule itself only has consequences for the nominal variables as well. The resulting volatility of inflation overshoots that found in the data (2.041 versus 0.955 in the data). Absent any financial frictions, the RBC model is silent about the external finance premium.

Relative to the frictionless equilibrium—of the RBC model—the New Keynesian model (DNK) with monopolistic competition and sticky prices introduces monetary

**Table 3** Business cycle moments: simulated vs. empirical

|                                                                  | Data   | BGG model | FA model | DNK model | RBC model |
|------------------------------------------------------------------|--------|-----------|----------|-----------|-----------|
| Std. deviations                                                  |        |           |          |           |           |
| $\sigma(\hat{y}_t)$                                              | 2.856  | 1.565     | 1.955    | 1.521     | 1.806     |
| $\sigma(\hat{x}_t)$                                              | 8.540  | 12.373    | 4.630    | 3.008     | 3.466     |
| $\sigma(\hat{c}_t)$                                              | 1.367  | 2.078     | 1.204    | 1.110     | 1.284     |
| $\sigma(\hat{h}_t)$                                              | 3.033  | 3.052     | 0.602    | 1.012     | 0.426     |
| $\sigma(\hat{p}_t - \hat{p}_{t-4})$                              | 0.955  | 1.447     | 2.038    | 1.529     | 2.041     |
| $\sigma(\hat{s}\hat{p}_t)$                                       | 0.093  | 0.549     | 0.069    | –         | –         |
| Autocorrelation                                                  |        |           |          |           |           |
| $\rho(\hat{y}_t, \hat{y}_{t-1})$                                 | 0.951  | 0.840     | 0.885    | 0.895     | 0.880     |
| $\rho(\hat{x}_t, \hat{x}_{t-1})$                                 | 0.973  | 0.904     | 0.863    | 0.865     | 0.861     |
| $\rho(\hat{c}_t, \hat{c}_{t-1})$                                 | 0.910  | 0.940     | 0.913    | 0.917     | 0.900     |
| $\rho(\hat{h}_t, \hat{h}_{t-1})$                                 | 0.914  | 0.831     | 0.851    | 0.452     | 0.848     |
| $\rho(\hat{p}_t - \hat{p}_{t-4}, \hat{p}_{t-1} - \hat{p}_{t-5})$ | 0.888  | 0.965     | 0.898    | 0.954     | 0.896     |
| $\rho(\hat{s}\hat{p}_t, \hat{s}\hat{p}_{t-1})$                   | 0.890  | 0.959     | 0.852    | –         | –         |
| Correlations                                                     |        |           |          |           |           |
| $\sigma(\hat{y}_t, \hat{x}_t)$                                   | 0.628  | 0.804     | 0.986    | 0.988     | 0.987     |
| $\sigma(\hat{y}_t, \hat{c}_t)$                                   | 0.192  | –0.325    | 0.983    | 0.992     | 0.989     |
| $\sigma(\hat{y}_t, \hat{h}_t)$                                   | –0.181 | 0.721     | 0.962    | 0.291     | 0.944     |
| $\sigma(\hat{y}_t, \hat{p}_t - \hat{p}_{t-4})$                   | 0.173  | 0.333     | –0.704   | –0.523    | –0.699    |
| $\sigma(\hat{y}_t, \hat{s}\hat{p}_t)$                            | 0.210  | –0.727    | –0.986   | –         | –         |

These moments are based on the Taylor (1993) specification of the monetary policy rule reacting to changes in the year-over-year inflation rate and detrended output. The moments are calculated for detrended real private output (ex. Government compensation) per capita, detrended real private investment per capita, real private consumption per capita, demeaned hours worked per capita, cyclical inflation—computed as the deviation from a 2 percent target—and the demeaned quarterly interest rate spread between the Baa corporate bond yield and the 20-year Treasury bill rate. The full sample covers the period between 1984:I and 2007:IV

This table reports the theoretical moments for each series given my parameterization. All statistics on simulations are computed after each series is H-P filtered (smoothing parameter = 1600). I use Matlab R2012a (7.14.0.739) and Dynare v4.3.2 for the stochastic simulation

non-neutrality. Hence, that means monetary shocks now contribute to the fluctuations of all real variables—output, investment, consumption and hours worked. Based on data for the Great Moderation period, the DNK model generates somewhat lower volatility on all variables (nominal and real) except on hours worked where the volatility jumps from 0.426 in the RBC case to 1.012 in the DNK model. The distortion that nominal rigidities introduce in the dynamics of the economy causes mainly a static response in hours worked, while investment becomes somewhat smoother as capital accumulation is favored more to distribute the impact of the TFP and monetary shocks intertemporally. Hence, the DNK model does not re-

**Table 4A** Time series correlations: simulated and empirical data

|                                                     | Data | BGG model | FA model | DNK model | RBC model |
|-----------------------------------------------------|------|-----------|----------|-----------|-----------|
| Real private output (ex. G) per capita, $\hat{y}_t$ |      |           |          |           |           |
| Data                                                | 1    | −0.216    | 0.117    | 0.096     | 0.141     |
| BGG model                                           |      | 1         | −0.164   | 0.151     | −0.157    |
| FA model                                            |      |           | 1        | 0.895     | 0.998     |
| DNK model                                           |      |           |          | 1         | 0.888     |
| RBC model                                           |      |           |          |           | 1         |
| Real private investment per capita, $\hat{x}_t$     |      |           |          |           |           |
| Data                                                | 1    | −0.510    | 0.477    | 0.369     | 0.460     |
| BGG model                                           |      | 1         | −0.546   | −0.134    | −0.558    |
| FA model                                            |      |           | 1        | 0.835     | 0.998     |
| DNK model                                           |      |           |          | 1         | 0.827     |
| RBC model                                           |      |           |          |           | 1         |
| Real private consumption per capita, $\hat{c}_t$    |      |           |          |           |           |
| Data                                                | 1    | 0.322     | 0.500    | 0.545     | 0.512     |
| BGG model                                           |      | 1         | 0.573    | 0.459     | 0.641     |
| FA model                                            |      |           | 1        | 0.936     | 0.995     |
| DNK model                                           |      |           |          | 1         | 0.930     |
| RBC model                                           |      |           |          |           | 1         |

These correlations are based on the Taylor (1993) specification of the monetary policy rule reacting to changes in the year-over-year inflation rate and detrended output. The correlations correspond to detrended real private output (ex. Government compensation) per capita, real private investment per capita and real private consumption per capita for the entire period between 1984:I and 2007:IV. This table reports the theoretical moments for each series given my parameterization. All statistics on simulations are computed after each series is H-P filtered (smoothing parameter = 1600). I use Matlab R2012a (7.14.0.739) and Dynare v4.3.2 for the stochastic simulation

solve the two big “misses” of the RBC model—it only seems to improve relative to the low volatility of hours worked of the RBC model, but at the expense of worsening the predicted volatility of investment. Absent any financial frictions, the DNK model is also silent about the external finance premium.

The model with financial frictions alone (FA) preserves the monetary non-neutrality of the standard RBC model, so all real variables respond solely to the realization of the TFP shock and only nominal variables are affected by the realization of the monetary shock. As in the RBC case, different specifications of the systematic part of the Taylor rule that respond to inflation and output fluctuations will only affect the path of the nominal variables. However, relative to the frictionless equilibrium—of the RBC model—the FA model introduces an interest rate spread (the external finance premium) between the borrowing costs and the real risk-free rate that distorts the investment decisions. The result is that the FA model matches fairly well the demeaned quarterly interest rate spread between the Baa corporate

**Table 4B** Time series correlations: simulated and empirical data

|                                                                               | Data | BGG model | FA model | DNK model | RBC model |
|-------------------------------------------------------------------------------|------|-----------|----------|-----------|-----------|
| Share of hours worked per capita, $\hat{h}_t$                                 |      |           |          |           |           |
| Data                                                                          | 1    | −0.189    | 0.235    | 0.015     | 0.227     |
| BGG model                                                                     |      | 1         | −0.611   | 0.751     | −0.632    |
| FA model                                                                      |      |           | 1        | −0.194    | 0.992     |
| DNK model                                                                     |      |           |          | 1         | −0.219    |
| RBC model                                                                     |      |           |          |           | 1         |
| Inflation (year-over-year rate), $\hat{p}_t - \hat{p}_{t-4}$                  |      |           |          |           |           |
| Data                                                                          | 1    | 0.666     | 0.703    | 0.687     | 0.721     |
| BGG model                                                                     |      | 1         | 0.690    | 0.710     | 0.735     |
| FA model                                                                      |      |           | 1        | 0.980     | 0.997     |
| DNK model                                                                     |      |           |          | 1         | 0.980     |
| RBC model                                                                     |      |           |          |           | 1         |
| Quarterly interest rate spread (external finance premium), $\hat{s}\hat{p}_t$ |      |           |          |           |           |
| Data                                                                          | 1    | 0.093     | −0.058   | −         | −         |
| BGG model                                                                     |      | 1         | −0.712   | −         | −         |
| FA model                                                                      |      |           | 1        | −         | −         |
| DNK model                                                                     |      |           |          | 1         | −         |
| RBC model                                                                     |      |           |          |           | 1         |

These correlations are based on the Taylor (1993) specification of the monetary policy rule reacting to changes in the year-over-year inflation rate and detrended output. The correlations correspond to demeaned hours worked per capita, year-over-year inflation and the quarterly interest rate spread (or external finance premium) for the entire period between 1984:I and 2007:IV

This table reports the theoretical moments for each series given my parameterization. All statistics on simulations are computed after each series is H-P filtered (smoothing parameter = 1600). I use Matlab R2012a (7.14.0.739) and Dynare v4.3.2 for the stochastic simulation

bond yield and the 20-year Treasury bill rate (0.069 versus 0.093 in the data) and increases the investment volatility from 3.466 in the RBC model to 4.630 in the FA case. The increased investment volatility relative to the RBC case brings with it an increase in the volatility of hours worked and output, but—in spite of that—the volatility of investment, hours, and output still remains too low when compared against the actual data.

The BGG model of Bernanke et al. (1999) combines the two broad types of frictions highlighted by the FA model (financial frictions) and the DNK model (monopolistic competition and price stickiness). However, the interaction of both frictions often is more than just the sum of their separate effects. Nominal rigidities break-down with the monetary neutrality of the frictionless RBC model, while financial frictions imply that borrowing costs—or the opportunity cost of investment—is no longer equal to the real risk-free rate as in the frictionless RBC case. Moreover, with nominal rigidities in the BGG model, monetary non-neutrality implies that the



spread—the external finance premium—responds to the realization of the monetary shock and not just to the realization of the TFP shock. In other words, the external finance premium and ultimately the path of investment itself will be amplified by the realization of the monetary shock process that describes the exogenous deviations from the policy rate target set according to Taylor (1993)’s rule.<sup>20</sup> The amplification of the external finance premium will, in turn, crucially depend on the importance of the demand distortion caused by the degree of price stickiness, as I discuss in a later subsection.

The volatility magnification attained by the BGG model with a combination of monetary and TFP shocks under the benchmark parameterization cannot be more dramatic as it rises the volatility of the external finance premium from 0.069 in the FA case to 0.549 in BGG—well above the 0.093 seen in the data—and the volatility of investment from 4.630 in the FA case to 12.373 in BGG—also above the 8.540 observed in the data. Consumption and hours worked become somewhat more volatile in the BGG model than in the FA case, while output and inflation are less so. In other words, while variants of the model with financial frictions such as the BGG and FA ones tend to generate volatility patterns closer to those observed in the data for the Great Moderation period, I interpret from the results reported in Table 3 that breaking away from monetary neutrality by incorporating nominal rigidities alters the financial accelerator mechanism in a fundamental way by allowing monetary shocks to have real effects and to influence the external finance premium by distorting the demand allocation.<sup>21</sup> The resulting amplification of the spread (the external finance premium) generates a large increase in the volatility of investment as well.

The first-order autocorrelations also reported in Table 3 offer one simple measure of the persistence of fluctuations in the actual and simulated data. The evidence I have collected reveals that most variants of the model generate very persistent dynamics that are not far from those actually observed in the data. I find interesting that the persistence of hours worked under the DNK model is 0.452 while it is 0.831 or above in all other cases. However, aside from this noticeable difference, there is no strong support in Table 3 for the view that the addition of nominal rigidities and/or financial frictions changes the propagation of shocks—which are themselves quite persistent, as noted before—endogenously generated by the frictionless RBC model in any significant or systematic way.

The correlations between output and all other macro variables of interest—investment, consumption, hours worked, inflation and the external finance premium—provide a sense of the cyclicity of the actual and simulated data. Unlike the

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<sup>20</sup>I explore this issue further in Figure 6 and in the next subsection where I consider explicitly the impact that monetary shocks have on the dynamics of the BGG model.

<sup>21</sup>In the BGG model under the Taylor rule specified in (20), the decomposition of the variance of the spread is 35.48 from TFP innovations and 64.52 from monetary policy innovations. Needless to say, both the contribution of monetary shocks to the external finance premium as well as the impact that monetary shocks have on other variables will depend on the systematic part of the rule too. I explore that issue a little closer in the remainder of the paper.

situation described with the first-order autocorrelations, the results I report in Table 3 suggest that the combination of financial frictions and nominal rigidities results in major differences relative to the cyclical patterns of the frictionless RBC model or even relative to the variants that include solely nominal rigidities (DNK) or only financial frictions (FA). The RBC model under monetary neutrality generates a strong positive comovement between output and all other real variables—investment at 0.987 versus 0.628 in the data, consumption at 0.989 versus 0.192, and hours worked at 0.944 versus  $-0.181$ . These strong correlations are at odds with the data, but the RBC model also delivers a countercyclical inflation of  $-0.699$  that is far away from the mild procyclicality observed in the data (with a positive correlation between output and inflation of 0.173). The negative correlation between output and inflation is to be expected whenever inflation is primarily driven by real or supply-side shocks (TFP shocks in the framework that I am investigating here), but is otherwise inconsistent with the empirical evidence and therefore indicative that the contribution of TFP and monetary shocks is not well-captured in the frictionless RBC setting.

Breaking from the monetary neutrality of the RBC model with the addition of nominal rigidities only seems to weaken both the procyclicality of the hours worked— $-0.291$  in the DNK model versus 0.944 in the RBC model and  $-0.173$  in the actual data—and the counter-cyclical behavior of inflation. Adding financial frictions as in the FA model preserves the monetary neutrality of the RBC specification, and does very little to change the cyclicity of investment, consumption, hours worked and inflation. However, the FA model introduces an interest rate spread (the external finance premium) between the borrowing costs and the real risk-free rate that is solely driven by TFP shocks, as discussed before. In this case, a very strong negative correlation emerges between output and the external finance premium ( $-0.986$  in the FA model versus 0.210 in the data). In other words, while the model implies that “low” spreads ought to be expected in “good” times and vice versa, the data suggests that spreads have been mildly pro-cyclical. This counterfactual evidence is precisely one of the major challenges that the financial accelerator mechanism embedded in the framework of Bernanke et al. (1999) confronts—the strong countercyclicity of the external finance premium is at odds with the empirical evidence.

Combining both frictions—the nominal rigidities of the DNK model and the financial friction of the FA model—into the BGG framework, however, significantly alters the cyclicity patterns that arise from the RBC model and from any of the variants that incorporate just one of the frictions at a time. The external finance premium is now influenced by monetary shocks since the nominal rigidities incorporated by the BGG model break the monetary neutrality of the RBC and FA models. However, the external finance premium remains strongly countercyclical although less so than in the FA case—at  $-0.727$  in the BGG case versus  $-0.986$  in the FA case and 0.210 in the data. Hours worked and output also remain far from what I observe in the data, but improve relative to what the FA model can deliver by weakening their strong procyclicality bias to some extent.

The two major shifts are to be found on inflation and consumption. In regards to inflation, the BGG reverts the countercyclicity found in the RBC, DNK and FA

cases—at 0.333 versus 0.173 in the data. This suggests that the contribution of monetary shocks to the dynamics of inflation overwhelms the strength of the TFP shocks unlike what happened in other variants of the model. In regards to consumption, the BGG model also reverts the procyclicality of all other model variants with a correlation of  $-0.325$  versus  $0.983$  in the FA case and  $0.192$  in the data. This suggests an interesting reading of the model predictions: whenever monetary shocks have real effects as in the BGG model, the financial accelerator mechanism gets accentuated and this, in turn, has a substitution effect.

In “good” times, the external finance premium tends to be “low” and there is a strong incentive to postpone consumption for later in order to invest more now—taking advantage of the fact that it is relatively less expensive to borrow for investment purposes. Similarly, in “bad” times, the external finance premium tends to be “high” and there is a strong incentive to consume now and invest later. However, while the pattern is well understood, it is nonetheless counterfactual for the Great Moderation period. The implication of all these results is that the BGG model is an incomplete framework with which to account for the business cycle features in the U.S. since the mid-1980s. More research still is needed to understand the role of other frictions and shocks, and to quantify their contribution to the cyclical fluctuations during this period.

I explore in Table 3 the performance of each of the models discussed in this paper by investigating their strengths and weaknesses in attempting to match key features of the data (that is, in trying to match key business cycle moments). Tables 4A and 4B illustrate now the evidence of comovement between the simulations across all model specifications and the data for each one of the macro variables of interest. The goal of this exercise is to provide a simple metric to assess the ability of the simulated data to track the path of the actual data or the path of data simulated by other models—in other words, the exercise provides some insight into the similarities across model simulations and with respect to the actual data.

The conclusions inferred from Tables 4A and 4B reinforce the perception that the BGG model set-up of Bernanke et al. (1999) dramatically alters the dynamics of the frictionless RBC model (even those of simpler models with either nominal rigidities or financial frictions alone), but remains inadequately prepared to account for the Great Moderation period. The first notable observation here is that the correlation between the simulated data from the RBC model, the DNK model with nominal rigidities and the FA model with financial frictions is very high for all variables except for hours worked. This is consistent with the evidence reported in Table 3 that shows the business cycle moments from the DNK model differ from those of the frictionless model (RBC) or those of the model augmented with financial frictions (FA) largely in regards to hours worked.

Absent other frictions in the model, the impact of the distortion caused by the presence of nominal rigidities (monopolistic competition and price stickiness) is absorbed primarily by the intratemporal margin provided by hours worked. For the Great Moderation period, the correlation of simulated hours worked between the DNK and RBC model stands at  $-0.219$  while the correlation between the DNK and FA models is  $-0.194$ . However, the correlation between the simulated data from the DNK model and actual data is almost negligible at  $0.015$ , while the correlations

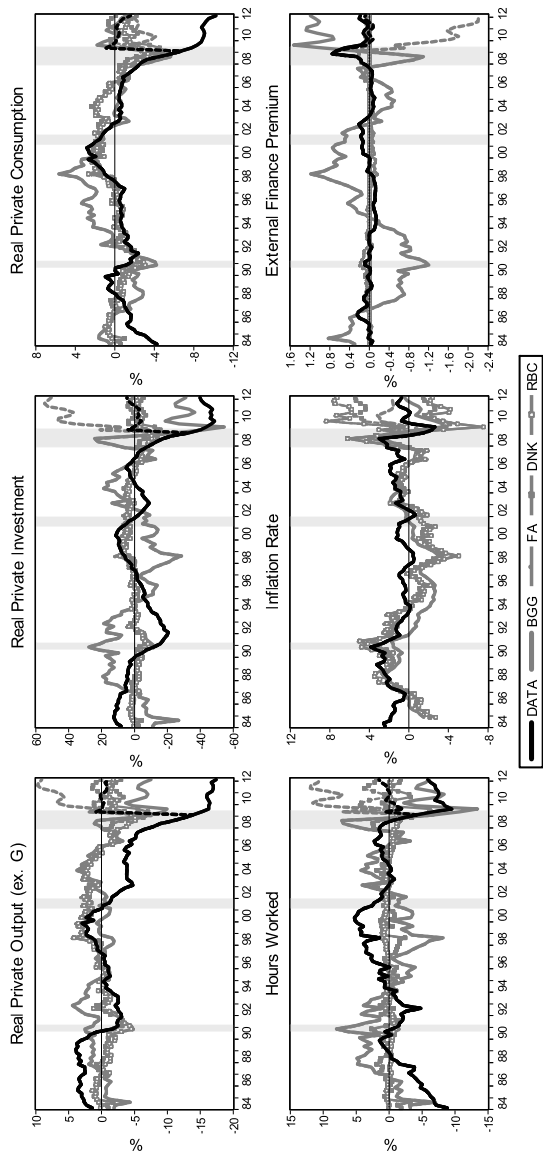
of the RBC and FA simulations with the actual data still have some information content at 0.227 and 0.235 respectively. This, in turn, translates into a weaker correlation between simulated and actual data on output for the DNK model than for the alternative RBC and FA variants.

A second notable observation is that the RBC, DNK and FA model simulations track better the data on inflation than on consumption and investment. These three models do worst matching the path of the actual data on hours worked and output, with the DNK model providing less information than the other two models for these variables. At its best, the correlation between actual and simulated output is merely 0.141 for the RBC model. Moreover, the FA model with financial frictions shows also a very poor result in tracking the external finance premium—as the correlation with the actual data is merely  $-0.058$ . However, the BGG model does generally worst than any of the other three models considered here and in most cases it generates a radically different path that moves in the opposite direction as the observed data. This happens, for instance, with consumption (where the correlation with the data is  $-0.510$ ), with output (where the correlation stands at  $-0.216$ ) and with hours worked (at  $-0.189$ ). The correlation between the external finance premium in the BGG model and the data becomes positive, but it is still pretty low at 0.093 and strongly negatively correlated with the external finance premium simulated by the FA model at  $-0.712$ .

In other words, the accentuation of the financial distortion mechanism by the combined effect of TFP and monetary shocks when nominal rigidities are added to the BGG model to break away from monetary neutrality produces a large deviation from the dynamics to be expected from the frictionless RBC model or from the FA and DNK variants. This departure amplifies the volatility of the model (especially on investment and the external finance premium), and it introduces a strong motive to substitute away from consumption and into investment. This pattern, however, does not fit well with the evidence during the Great Moderation period as the data in Tables 4A and 4B clearly indicate.

Figure 5 plots the actual data for each one of the macro variables of interest—output, investment, consumption, hours worked, inflation and the external finance premium—together with the simulations from each one of the model variants I have considered in the paper. Aside from illustrating some of the distinctive features that I already noted based on the evidence reported in Tables 3, 4A and 4B for the Great Moderation period between 1984:I and 2007:IV, it also includes observations and simulations for the period of the 2007 recession and its aftermath up to 2012:I. The actual data is plotted based on the trends from the Great Moderation period ending in 2007:IV (black solid line) and also adjusted to account for the possibility of a level shift in 2009:II discussed earlier (black dashed line). Similarly, the simulated series are reported using a realization of the monetary shock inferred under the assumption that output continued evolving along the same trend that prevailed prior to the 2007 recession (black solid line) and using a realization of the monetary shock that is derived from the assumption that output experienced a level shift after 2009:II (black dashed line).

It is worth pointing out that the pattern of investment and hours worked generated by the BGG model (gray line) during the entire period is clearly dominated by the



**Fig. 5** Comparison across model variants: simulations vs. data. This graph plots in black the detrended per capita real private output (ex. G) in logs, the detrended per capita real private investment in logs, the detrended per capita real private consumption in logs, the demeaned share of hours worked per capita in logs, the cyclical deviations of consumer (nondurables and services) price inflation in year-over-year terms relative to a 2 percent inflation target, and the demeaned Baa corporate quarterly spread over the 20-year Treasury Bill. I estimate the trends and means in the data for the Great Moderation period (1984:I–2007:IV), allowing for a level shift on 2009:II (except for the inflation variable). The black solid line represents the variables in deviations with respect to their Great Moderation trends, while the black dashed line indicates the variables in deviations accounting for a level shift in 2009:II. The shaded areas represent NBER recessions. This graph plots in solid gray the simulation of the BGG model with flexible prices and perfect competition but with financial frictions, in solid gray with square filled markers the Dynamic New Keynesian (DNK) model without financial frictions, and in solid gray with circle markers the standard Real Business Cycle (RBC) model. The dashed line represents the simulation of the corresponding model whenever the monetary shocks are derived under a Taylor rule which responds to a measure of output that has incorporated the adjustment for a level shift in economic activity occurring in 2009:II. For more details on the derivation of the realization of the monetary shock, see Appendix B ‘U.S. Dataset’. I use Matlab R2012a (7.14.0.739) and Dynare v4.3.2 for the stochastic simulation

long and deep swings in the external finance premium. These swings are strongly counter-cyclical—unlike the actual data—and much more sizeable than anything I observe in the quarterly interest rate spread between the Baa corporate bond yield and the 20-year Treasury bill rate. There is an evident comovement between these three simulated series (external finance premium, investment and hours worked) showing that whenever the external finance premium is “high”, both investment and hours worked are “low”. This, however, produces counterfactual predictions for all three variables that further limit the ability of the BGG model to account for the observations in the data, as noted earlier.

As expected, the RBC, DNK and FA variants of the model show a similar pattern among them when displayed together in Figure 5. Their simulations appear also to be quite different from the BGG simulation. In summary, the key margin that the financial accelerator model of Bernanke et al. (1999) introduces is given by the external finance premium which influences the investment path in the model. Without nominal rigidities as in the FA variant, the magnitude of the endogenous spread generated is somewhat lower than that observed in the data but the spread itself is strongly counter-cyclical while the data shows a mildly pro-cyclical pattern. The distortion that this margin adds to investment is rather small, so the discrepancies in the path of the real variables with respect to the frictionless RBC model are of second order importance and with respect to the DNK model they are only significant for hours worked.

With nominal rigidities as in the BGG model, monetary shocks have real effects and price stickiness distorts the demand allocation. Under the benchmark parameterization, this results in endogenous fluctuations of the external finance premium that are almost 10 times as volatile as in the FA case and still strongly counter-cyclical. The large movements of the external finance premium in this case end up dominating the evolution of investment (and, by extension, that of hours worked) while favoring a substitution away from consumption and towards investment. These features are all intimately connected, but generate simulated data that is largely at odds with the observed data—so generally the BGG model path is a worst one for the data than the path implied by the alternative models (RBC, FA or even DNK). Hence, a successful model of the business cycle that builds upon the financial mechanism of Bernanke et al. (1999) first and foremost needs to address the large amplification of the external finance premium under nominal rigidities and find an explanation for the mild procyclicality of the spread observed in the data.

One argument that has been forcefully discussed both among policymakers and scholars is that financial frictions—which are missing in the frictionless RBC framework or in the standard New Keynesian (DNK) model—may have been a crucial factor in the 2007 recession and its aftermath. One of the most popular models that accounts for the role of financial frictions explicitly in a general equilibrium setting is the Bernanke et al. (1999) model that I investigate here. In looking at the data after 2007 through this particular lens I explicitly take into account the possibility that a level shift may have occurred in the aftermath of the recession. I distinguish between actual or simulated series that are adjusted (dashed lines) and those that are not adjusted for such a level shift (solid line) in Figure 5 (as well as in Figures 6, 7 and 8 subsequently).

In any event, the BGG model turns out to display very large movements of the endogenous external finance premium in the post-2007 period that produce fluctuations in hours worked and investment that are hard to reconcile with the data. The model does well in tracking the consumption series adjusted for a level shift, but that is something that can also be attained with any of the alternative specifications. Hence, the shortcomings of the BGG model that make it insufficient to account for the business cycle fluctuations during the Great Moderation period also limit the insight that the framework can provide for the observed macro data in the U.S. since 2007.

These findings give some perspective and set the stage for a further exploration of the role of monetary policy and monetary policy shocks in the financial accelerator model of Bernanke et al. (1999).

## 4.2 *Claim 1: The Role of Monetary Shocks*

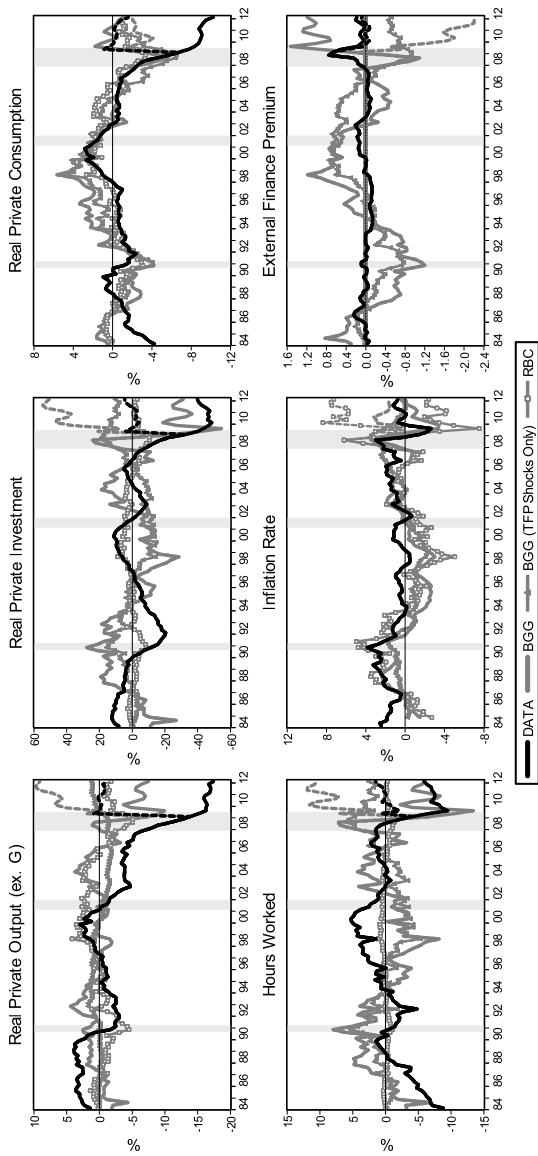
The benchmark financial accelerator model assumes price stickiness implying an average duration of each price spell of 4 quarters (i.e.,  $\alpha = 0.75$ ) and a parameterization of the sensitivity of the external finance premium implying that ceteris paribus a one percent increase in the leverage of borrowers raises the cost of external finance by almost 7 basis points per quarter (i.e.,  $\vartheta \equiv (\frac{v'(\gamma_n^{-1})\gamma_n^{-1}}{v(\gamma_n^{-1})}) = 0.0672$ ). The model is also endowed with a Taylor (1993)-type monetary policy rule as described in Eq. (20). All these features are viewed as consistent over the Great Moderation period with the empirical evidence available on the monetary policy regime and the features of the nominal rigidities and financial frictions.

In the experiment plotted in Figure 6, I look at the simulation of the benchmark financial accelerator model—the BGG model developed by Bernanke et al. (1999)—and compare it against the RBC model and a variant of the BGG model driven exclusively by TFP shocks in order to gauge the role played by monetary shocks in this environment. All three models are compared against the data to evaluate the strength of the quantitative findings.

In the frictionless RBC case, monetary neutrality ensures that all real variables are entirely driven by the realization of the TFP shock (gray line with square outlined markers). The BGG model incorporates nominal rigidities that introduce monetary non-neutrality, as I discussed earlier. The variant of the BGG model driven solely by TFP shocks (solid gray line with triangle markers) shows how the propagation of TFP differs relative to the RBC case whenever nominal rigidities and financial frictions are incorporated. The BGG model (solid gray line), in turn, introduces monetary shocks which have real effects as well. The BGG model, therefore, illustrates how the addition of monetary shocks further alters the dynamics of the macro variables of interest.

The monetary policy rule in all cases is described by Eq. (20) under the standard parameterization reported in Table 1. However, this requires some qualifications in the case of the BGG model driven exclusively by TFP shocks. In this particular





**Fig. 6** Claim 1: the role of monetary shocks.

This *graph plots in black* the detrended per capita real private output (ex. G) in logs, the detrended per capita real private investment in logs, the detrended per capita real private consumption in logs, the demeaned share of hours worked per capita in logs, the cyclical deviations of consumer (nondurables and services) price inflation in year-over-year terms relative to a 2 percent inflation target, and the demeaned Baa corporate quarterly spread over the 20-year Treasury Bill. I estimate the trends and means in the data for the Great Moderation period (1984:I–2007:IV), allowing for a level shift on 2009:II (except for the inflation variable). The *black solid line* represents the variables in deviations with respect to their Great Moderation trends, while the *black dashed line* indicates the variables in deviations accounting for a level shift in 2009:II. The *shaded areas* represent NBER recessions. This *graph plots in solid gray* the simulation of the BGG model with nominal rigidities (price stickiness and monopolistic competition) and financial frictions, in *solid gray with triangle markers* the same BGG model but driven solely by the realization of the TFP shock rather than by a combination of TFP and monetary shocks, and in *solid gray with square outlined markers* the standard Real Business Cycle (RBC) model. The *dashed line* represents the simulation of the corresponding model whenever the monetary shocks are derived under a Taylor rule which responds to a measure of output that has incorporated the adjustment for a level shift in economic activity occurring in 2009:II. For more details on the derivation of the realization of the monetary shock, see Appendix B ‘U.S. Dataset’. I use Matlab R2012a (7.14.0.739) and Dynare v4.3.2 for the stochastic simulation



variant, the specification of monetary policy involves the joint assumption that the central bank never deviates from the given Taylor rule and that all agents know (and believe) that the central bank is not going to deviate from that policy rule. Therefore, the comparison of the BGG model with and without monetary policy shocks provides further insight on the business cycle contributions from a combination of monetary policy deviations and TFP shocks beyond what can be accounted for with TFP shocks alone. To be precise, this does not isolate the effect of the monetary policy shocks but it shows how different would the endogenous dynamics of the model be including monetary shocks compared to the case where business cycles are entirely driven by TFP shocks.

It can be argued on the basis of the findings that I report here that the financial accelerator mechanism together with nominal rigidities has a strong amplification effect over the business cycle that is accentuated when I combine monetary and TFP shocks. Interestingly, I find that during the entire Great Moderation period and even in the 2007 recession, the external finance premium simulated by the model solely driven by TFP shocks tends to be more volatile and counter-cyclical than the actual spreads observed in the data—although it also noticeably differs from the endogenous spread under the BGG specification that combines both monetary and TFP shocks. It is interesting to see how smooth the simulated output series is relative to the standard BGG case with both monetary and TFP shocks, and nonetheless how different the path is relative to the standard frictionless RBC case.

By and large, the external finance premium in the variant of the BGG model without monetary shocks follows the same path as in the standard BGG model that includes monetary shocks as well. The two notable exceptions correspond to periods where monetary policy appears to have largely deviated from the Taylor rule prescription: the period of low interest rates between 2002 and 2006 that is often regarded as the build-up period for the 2007 recession and the period of interest rates at the zero-lower band since 2009:I.

In the early part of the 1980s, monetary policy deviations are larger in size—with the Federal Funds rate above the Taylor-implied rate—which neither variant of the model considered here is well-suited to account for. The deviations of monetary policy that I detect in the data after 1987 (as seen in Figure 4) are rather modest in size during most of chairman Greenspan's tenure at the Fed until 2002. In spite of that, the differences between the BGG model and the BGG model without monetary shocks are of first-order importance during those years (between 1987 and 2002).

The period of low interest rates between 2002 and 2006 is well-known for the size of the policy deviations—with the Federal Funds rate well below the Taylor-implied rate. It is precisely during this time that there is strong evidence of divergence between the BGG model and the BGG variant driven by TFP shocks alone—where the difference translates into a “high” spread (positive in deviations from its mean) if I look at the BGG model with TFP shocks alone and a “low” spread (negative in deviations from its mean) in the standard BGG case.

The other period of low interest rates corresponds with the time of policy rates set at the zero-lower band (since 2009). I explicitly allow for the possibility of a level shift in output in the calculation of the monetary policy deviations since 2009:II,

but this is only of relevance for the simulation of the BGG model when it includes monetary shocks as the BGG model with TFP shocks alone would be unaffected for obvious reasons. A significant discrepancy in the derived external finance premium and, by extension, on the dynamics of the economy emerges during this period nonetheless.

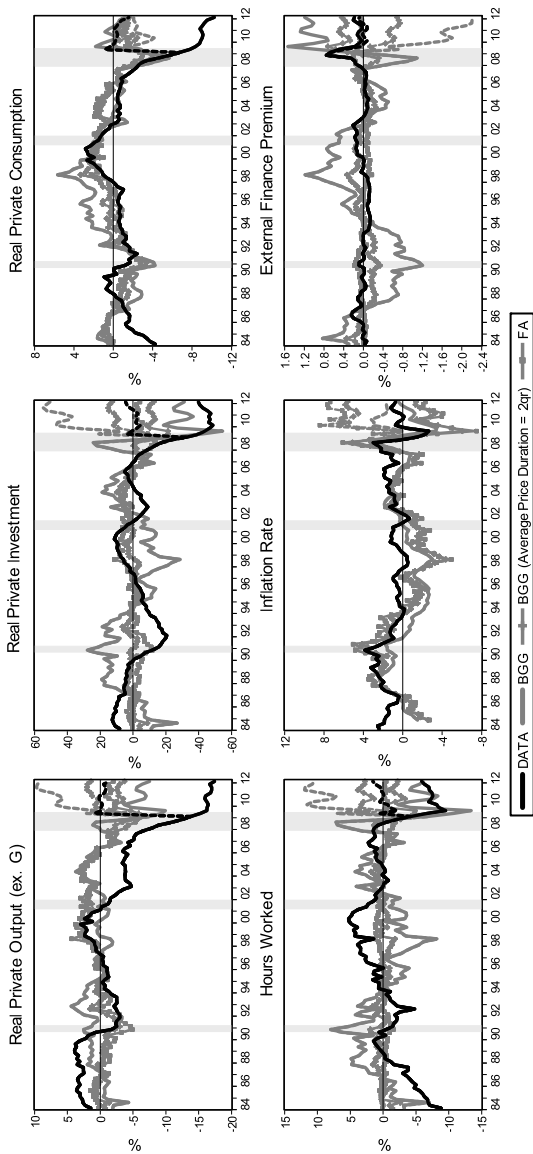
Neither of the instances of large monetary policy deviations reviewed here, though, suggests that combining monetary and TFP shocks helps improve the ability of the BGG model to capture the patterns observed in the data. These results highlight some of the inherent weaknesses of the financial accelerator model of Bernanke et al. (1999), but also indicate that neither variant of the model is capable of successfully explaining the turn of events during the 2007 recession.

### 4.3 Claim 2: The Role of Nominal Rigidities

As monetary non-neutrality in the standard New Keynesian (DNK) model rests on the assumption of nominal rigidities, the question arises as to how important is this feature for the strength of the financial accelerator mechanism in the BGG set-up. The simulations reported in this sub-section are all based on the same Taylor (1993) specification of monetary policy described in Eq. (20). Here, I compare the BGG model (gray line)—where the average duration of a pricing spell is of 4 quarters—against a BGG variant with lower nominal rigidities (gray line with cross markers)—where the average duration of a pricing spell is set at 2 quarters—and against the FA model specification (gray line with square filled markers) that abstracts entirely from nominal rigidities (that is, from price stickiness and monopolistic competition). All model simulations are still driven by the same combination of TFP and monetary policy shocks.

As can be seen in Figure 7, nominal rigidities play a crucial role in the dynamics of the BGG model. Reducing the degree of Calvo price stickiness from  $\alpha = 0.75$  (four quarters average duration) to  $\alpha = 0.5$  (two quarters average duration) alone dramatically reduces the magnitude of the fluctuations in the external finance premium to levels that are comparable with those observed in the quarterly interest rate spread between the Baa corporate bond yield and the 20-year Treasury bill rate. It is interesting to note, however, that the simulations of the BGG model with the benchmark parameterization of price stickiness and with low price stickiness are nonetheless highly correlated. In turn, the FA model that abstracts entirely from price stickiness and monopolistic competition but maintains the financial friction generates a significantly different external finance premium which is weakly (and negatively) correlated with the data, but strongly and negatively correlated with the BGG simulations under standard or low price stickiness. This can also be seen for the benchmark parameterization of the BGG model in the results of Table 4B.

The difference between the two versions of the BGG model and the FA model plotted in Figure 7 is that the FA model preserves monetary neutrality (and therefore spreads are entirely driven by TFP shocks) while the BGG model variants do



**Fig. 7** Claim 2: the role of nominal rigidities.

This *graph plots in black* the detrended per capita real private output (ex. G) in logs, the detrended per capita real private investment in logs, the detrended per capita real private consumption in logs, the demeaned share of hours worked per capita in logs, the cyclical deviations of consumer (nondurables and services) price inflation in year-over-year terms relative to a 2 percent inflation target, and the demeaned Baa corporate quarterly spread over the 20-year Treasury Bill. I estimate the trends and means in the data for the Great Moderation period (1984:I–2007:IV), allowing for a level shift on 2009:II (except for the inflation variable). The *black solid line* represents the variables in deviations with respect to their Great Moderation trends, while the *black dashed line* indicates the variables in deviations accounting for a level shift in 2009:II. The *shaded areas* represent NBER recessions. This *graph plots in solid red* the simulation of the BGG model with nominal rigidities (price stickiness implying an average price duration of 4 quarters and monopolistic competition) and financial frictions, in *solid gray with cross markers* the same model but assuming a lower degree of price stickiness that results in an average price duration of just 2 quarters, and in *solid gray with square filled markers* the FA model with flexible prices and perfect competition but with financial frictions. The *dashed line* represents the simulation of the corresponding model whenever the monetary shocks are derived under a Taylor rule which responds to a measure of output that has incorporated the adjustment for a level shift in economic activity occurring in 2009:II. For more details on the derivation of the realization of the monetary shock, see Appendix B ‘U.S. Dataset’. I use Matlab R2012a (7.14.0.739) and Dynare v4.3.2 for the stochastic simulation

not. The evidence, therefore, reveals that the magnitude and cyclical patterns of the external finance premium depend nonlinearly on whether nominal rigidities are included in the model. One may be tempted to argue that the strong negative correlation between the spread simulated by the FA model and the spread derived from the two versions of the BGG model is attributable to monetary shocks—as those have real effects and influence the spread in the BGG case with price stickiness, but not in the FA case. However, as it was discussed in the previous sub-section (in regards to Figure 6), the contribution of monetary shocks alone cannot explain everything. In fact, the distortion of the demand allocation resulting from nominal rigidities produces a similar path for the spread even when TFP shocks are the sole drivers of the cycle.

Hence, what really matters the most is the size of the demand distortion that price stickiness produces under the prevailing policy rule. The evidence illustrated through Figure 7 suggests that a range of plausible values for the Calvo price stickiness parameter  $\alpha$  that sets the average price duration between 2 and 4 quarters can nonetheless generate very significant differences in the simulated path of the macro variables of interest. The smaller the parameter  $\alpha$  (the shorter the average price duration), the smaller the fluctuations of the external finance premium and the smaller the impact those have on investment. As a result, the effects of the financial mechanism become rapidly similar to those of the FA model without price stickiness for output, investment, consumption and even hours worked. More broadly one could argue that a parameterization of the BGG model that is consistent with the fluctuations of the spreads actually observed in the data is likely to assign a much more modest role to fluctuations of the external finance premium in explaining the cyclical patterns of the macro variables of interest than that found under the benchmark BGG model.

One explanation often postulated for the 2007 recession is that borrowing costs may have substantially increased since the onset of the 2007 recession due to constraints on loan supply that resulted from the concurrent banking crisis (the bank lending channel may indeed have been impaired) or due to the increased burden of financial regulation. I explicitly account for that possibility in the data by allowing for a level shift to have occurred in 2009:II. The BGG model, even with low price stickiness, seems unable to match the path of the observed spread better than the FA model—but both models have shortcomings in explaining output, investment and hours worked.

One could argue on the basis of these observations that a change in the specification of the financial friction that introduces exogenous shocks to the external finance premium may help overcome the BGG model's apparent inability to account for the 2007 recession. In order to understand the 2007 recession, however, something more than augmenting the specification of the model with shocks is needed—a more fundamental question must be addressed first. Are the increases in borrowing spreads documented in the data (see Figure 1) better thought of as endogenous responses, or can they be modelled as random exogenous shocks to the spreads—or to the supply of loans from deposits—that are unpredictable for the economic agents? If one goes through the route of endogenizing the bank lending channel, then an extension of

the Bernanke et al. (1999) framework is clearly needed. That work is left for future research.

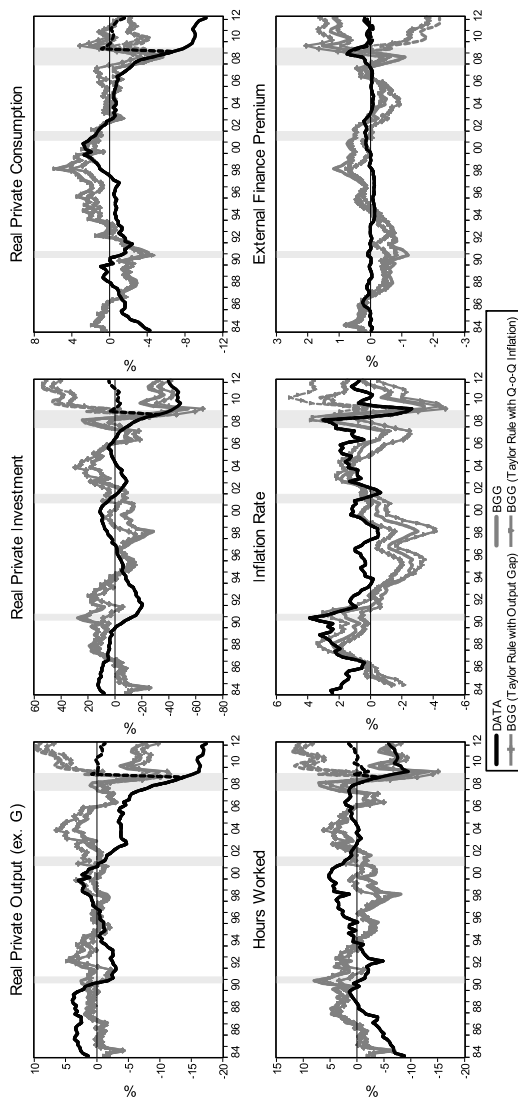
#### ***4.4 Claim 3: The Role of the Monetary Policy Rule***

Another thought experiment that one could consider is whether the measurement of inflation and output in the Taylor (1993) monetary policy rule has any bearing on the economic impact of following that policy rule. That is the purpose of the simulations illustrated in Figure 8. There are many ways in which this general question could be addressed, but I decided to restrict myself to just two very specific issues of measurement in the specification: whether measuring inflation in terms of year-over-year growth rates or annualized quarter-over-quarter rates matters; and whether to use deviations of output from trend or deviations of output from its frictionless potential (i.e., the output gap) matters. Most of the theoretical literature, after all, describes the reaction function of policymakers to inflation in terms of quarter-over-quarter rates and to output fluctuations in terms of the output gap—while part of the empirical literature on Taylor rules, including Taylor (1993) himself, looks at responses in terms of year-over-year inflation rates and detrended output.

In all simulations I have plotted so far in Figures 5–7, the implicit assumption is that the U.S. monetary policy targets the year-over-year growth rate and detrended output as explicitly stated in Eq. (20). I report in Figure 8 the simulation of the BGG model (gray line) under that specification of the Taylor rule for reference. But, then, how do I evaluate the BGG model under alternative specifications of the policy rule that may differ on the measurement of inflation or the measurement of output fluctuations?

One way to address the importance of using annualized quarter-over-quarter growth rates instead of year-over-year growth rates is by re-estimating the Taylor rule residuals under this alternative inflation rate measure in the specification, assuming that all economic agents know and believe that the response to inflation is set in terms of quarter-over-quarter rates. Then I could simulate the model again, but feeding these Taylor rule deviations derived under the alternative measure of inflation into the corresponding policy functions that solve the linearized rational expectations equilibrium of the model. The disadvantage of following this route is that it obscures the exact contribution of the measure of inflation to the dynamics since the simulation of the endogenous variables would jointly reflect the change in the inflation rate used to set the monetary policy target under the rule as well as a different shock process (and realization) for the exogenous monetary policy deviations. And then, how does one disentangle the contribution of one from the other?

A alternative approach could be followed to compare a policy rule specification that responds to the output gap instead of detrended output. Instead of following that route, I undertake here a much more modest thought-experiment. I will simply take as given the monetary policy shock process and assume it is exactly the same one I



**Fig. 8** Claim 3: the role of the monetary policy rule.

This *graph plots* in *black* the detrended per capita real private output (ex. G) in logs, the detrended per capita real private investment in logs, the demeaned share of hours worked, the demeaned price inflation in year-over-year terms relative to a 2 percent inflation target, and the demeaned Baa corporate quarterly spread over the 20-year Treasury Bill. I estimate the trends and means in the data for the Great Moderation period (1984:I–2007:IV), allowing for a level shift on 2009:II (except for the inflation variable). The *black solid line* represents the variables in deviations with respect to their Great Moderation trends, while the *black dashed line* indicates the variables in deviations accounting for a level shift in 2009:II. The *shaded areas* represent NBER recessions.

This *graph plots* in *solid gray* the simulation of the BGG model with nominal rigidities (price stickiness and monopolistic competition) and financial frictions under the Taylor (1993) rule that reacts to year-over-year changes in inflation and the detrended output, in *solid gray with cross markers* the same model under an alternative specification of the Taylor rule that reacts to year-over-year inflation and the output gap, and in *solid gray with inverted triangle markers* the same model under the alternative Taylor rule that reacts to quarter-over-quarter annualized inflation and detrended output. The *dashed line* represents the simulation of the corresponding model whenever the monetary shocks are derived under a Taylor rule which responds to a measure of output that has incorporated the adjustment for a level shift in economic activity occurring in 2009:II. For more details on the derivation of the realization of the monetary shock, see Appendix B ‘U.S. Dataset’. I use Matlab R2012a (7.14.0.739) and Dynare v4.3.2 for the stochastic simulation

have used thus far. Then I simulate the model under the assumption that monetary policy reacts to quarter-over-quarter annualized inflation rates and detrended output but with the same realization of the monetary shock (gray line with inverted triangle markers). Similarly, I take as given the monetary policy shock process and simulate the model under the assumption that monetary policy reacts to year-over-year inflation rates and the output gap with the same realization of the monetary shock (gray line with cross markers). This is a counterfactual exercise, but it gives me a sensible quantification of the impact that a change of this type in the monetary policy rule may have had.

Otherwise, the financial accelerator model that I simulate in all these variants corresponds exactly to the same parameterization that I describe in Table 1. One would conjecture that such a seemingly small change in the monetary policy rule specification cannot have major implications for the dynamics of the economy. The surprising thing is that just the opposite happens to be true, but not for all measurement changes considered. The plots in Figure 8 illustrate that the benchmark BGG model where monetary policy responds to year-over-year inflation, in fact, overlaps for the most part with the variant of the BGG model where monetary policy responds to quarter-over-quarter annualized inflation. Discrepancies between both simulations are only sizeable for the (year-over-year) inflation rate itself, but have only a marginal impact on all real variables.

In turn, the economic consequences of responding to the output gap instead of detrended output (as in the benchmark specification set in (20)) are clearly of first-order importance. I examine here the economy's response to two exogenous shocks—a monetary policy shock and a TFP shock—when the central bank follows the interest rate rule in (20) with output gap instead of detrended output on the right-hand side. Since output potential in a frictionless RBC environment under monetary neutrality is unaffected by the monetary policy shock, the response of output to monetary shocks matches that of the output gap. However, the same cannot be said for the productivity shocks. A positive TFP shock leads to a persistent decline in both inflation and the output gap, but increases output given the benchmark parameterization of the model (including the interest rate rule parameters).

Hence, whether the monetary policy rule responds to output or the output gap clearly matters when TFP shocks are one of the main drivers of business cycles because output and the output gap would tend to move in opposing directions. While the economic impact of both rule specifications is certainly important (as can be seen from Figure 8), the path of the macro variables of interest tends to be positively correlated under both specifications since the onset of the Great Moderation. As a result, it cannot be said that one specifications provides a clearly superior match for the observed data than the other one.

This counterfactual exercise is just one experiment on a broader set of questions about the role of monetary policy. While most of my previous experiments have been based on the interpretation of the discretionary component of monetary policy and how it is propagated in the presence of nominal rigidities, this counterfactual exercise comes to show that the systematic part of the policy rule can indeed have a major impact on the performance of the economic model. It also shows that issues



like the proper measurement of output deviations used in the policy rule can—in turn—be fundamental for the outcome of the model.

## 5 Concluding Remarks

I investigate a synthesis of the Bernanke et al. (1999) model—the BGG model—with leveraged borrowers (entrepreneurs), financial frictions, and nominal rigidities. I also consider three economically-relevant variants of this framework: the Real Business Cycle (RBC) model without nominal rigidities or financial frictions, the Dynamic New Keynesian (DNK) model with nominal rigidities but no financial frictions, and the Financial Accelerator (FA) model with financial frictions but without nominal rigidities. I parameterize the model to be consistent with Bernanke et al. (1999) and with the available data for the U.S. during the Great Moderation period.

In mapping the model to the data, I linearly detrend the nonstationary variables—output, investment and consumption—and demean or express in deviations the others—share of hours worked, inflation and quarterly interest rate spread—based on their Great Moderation trends but allowing for the possibility of a level shift in the aftermath of the 2007 recession. I also derive a realization of the detrended TFP shock and the monetary shock from U.S. data that I subsequently use to simulate U.S. business cycles with the different variants of the Bernanke et al. (1999) model over the Great Moderation period (from 1984:I until 2007:IV) and since the 2007 recession (2008:I–2012:I). I evaluate the performance of each model by comparing the business cycle features it generates and its fit against the observed data.

On the basis of these simulations, I argue that the characterization of the reaction function of monetary policy has non-trivial implications for the performance of the model and that the interpretation of all monetary policy deviations as exogenous shocks is anything but trivial. I also find that the degree of price stickiness is crucial for the accentuation of the economic impact of financial frictions, but that the interaction between these two frictions works nonlinearly for plausible values implying an average duration of prices between 2 and 4 quarters. The evidence reported suggests very stark differences in the dynamics implied by the BGG model at both ends of that range.

However, I find otherwise limited support in favor of the financial accelerator model as a superior framework to account for the U.S. business cycle during the Great Moderation period and—especially—during the 2007 recession and its aftermath. In fact, in some dimensions it becomes clear that a plain-vanilla RBC model (or the FA variant that includes the financial friction but no nominal rigidities) gets closer to accounting for the endogenous variables observed in the data than the model of Bernanke et al. (1999) does. In a nutshell, the problem with the BGG model is that it generates fluctuations of the external finance premium that are too large relative to the data under the standard parameterization of price stickiness. These fluctuations of the external finance premium largely drive investment and hours worked.



It is ultimately the amplitude of the external finance premium movements over the business cycle that explains the wedge between the BGG model and the other nested alternatives—the RBC, the DNK and the FA models. The BGG model, however, also implies that the external finance premium ought to be strongly counter-cyclical. The large and counter-cyclical fluctuations in the premium generate, in turn, a strong incentive to substitute resources away from consumption and into investment in periods when the spread is “low” and the cost of funding investment is “cheaper”. This tends to produce counter-cyclical consumption patterns on top of counter-cyclical spreads, both of which run contrary to the observed patterns in the U.S. data.

One can look at these broad results in two different ways. One can take the view that they cast the implications of the financial accelerator model of Bernanke et al. (1999) in a slightly less positive light and, therefore, that the BGG model is perhaps not yet ready for policy evaluation and analysis of the business cycles at the level we would like it to be. That is a reasonable reading of my results, but I would argue that it is still premature to claim on the basis of quantitative findings like the ones presented here that the financial accelerator mechanism is incompatible with the data or that it should be discarded altogether.

Another more sympathetic view would be that—indeed—there is a financial friction at play and it is important to account for it. The puzzle is, therefore, worse than it is conventionally thought because some source of randomness not accounted for or other features of the structural transmission mechanism that have not been explicitly modelled are still needed in order to bridge the gap between the model and the data. The problem could also be that monetary policy itself and monetary policy shocks in particular are not well-understood in this framework—e.g., if one estimates that the monetary policy regime may have shifted over time or doubts the extent to which policymakers have incorporated the possibility of a level shift in output in their calculations of the optimal target rate under Taylor (1993).

While the latter argument ends up creating more questions than it actually answers, the true fact is that more work needs to be done to better understand the role that financial frictions play on real economic activity and their interactions with monetary policy and with other frictions. My hope is that this paper will not be viewed as a closing chapter on the subject or on the model itself, but as an attempt to spur further interest towards a more quantitative evaluation of these questions and to encourage further development and integration of financial features in general equilibrium models.

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## Appendix A: The Log-Linearized Model

As a notational convention, all variables identified with lower-case letters and a caret on top are expressed in logs and in deviations relative to the their steady state values.

### A.1 The Financial Accelerator Model

#### Aggregate Demand Equations

$$\begin{aligned}
 \widehat{y}_t &\approx (1 - \gamma_x - \gamma_{ce})\widehat{c}_t + \gamma_x\widehat{x}_t + \gamma_{ce}\widehat{n}_{t+1}, \\
 \gamma_x &\equiv \delta \left( \frac{\psi}{\mu(v(\gamma_n^{-1})\beta^{-1} - (1 - \delta))} \right), \\
 \widehat{c}_t &\approx \mathbb{E}_t[\widehat{c}_{t+1}] - \sigma\widehat{r}_{t+1}, \\
 \mathbb{E}_t[\widehat{r}_{t+1}^k] &\approx \widehat{r}_{t+1} + \vartheta(\widehat{q}_t + \widehat{k}_{t+1} - \widehat{n}_{t+1}), \quad \vartheta \equiv \left( \frac{v'(\gamma_n^{-1})\gamma_n^{-1}}{v(\gamma_n^{-1})} \right), \\
 \widehat{r}_t^k &\approx (1 - \epsilon)(\widehat{m}\widehat{c}_t + \widehat{y}_t - \widehat{k}_t) + \epsilon\widehat{q}_t - \widehat{q}_{t-1}, \quad \epsilon \equiv \left( \frac{1 - \delta}{v(\gamma_n^{-1})\beta^{-1}} \right), \\
 \widehat{q}_t &\approx \chi(\widehat{x}_t - \widehat{k}_t).
 \end{aligned}$$

#### Aggregate Supply Equations

$$\begin{aligned}
 \widehat{y}_t &\approx \widehat{a}_t + \psi\widehat{k}_t + (1 - \psi - \varrho)\widehat{h}_t, \\
 \widehat{m}\widehat{c}_t &\approx \frac{1}{\varphi}\widehat{h}_t + \frac{1}{\sigma}\widehat{c}_t - (\widehat{y}_t - \widehat{h}_t), \quad \varphi \equiv \eta \left( \frac{1 - H}{H} \right), \\
 \widehat{\pi}_t &\approx \beta\mathbb{E}_t[\widehat{\pi}_{t+1}] + \left( \frac{(1 - \alpha\beta)(1 - \alpha)}{\alpha} \right)\widehat{m}\widehat{c}_t.
 \end{aligned}$$

#### Law of Motion for State Variables

$$\begin{aligned}
 \widehat{k}_{t+1} &\approx (1 - \delta)\widehat{k}_t + \delta\widehat{x}_t, \\
 \widehat{n}_{t+1} &\approx (\zeta\beta^{-1}\gamma_n^{-1})(\widehat{r}_t^k - \widehat{r}_t) + \widehat{r}_t + \widehat{n}_t + \dots \\
 &\quad (v(\gamma_n^{-1}) - 1)\gamma_n^{-1}(\widehat{r}_t^k + \widehat{q}_{t-1} + \widehat{k}_t) + \dots \\
 &\quad \frac{\varrho}{\psi}(v(\gamma_n^{-1})\beta^{-1} - (1 - \delta))\gamma_n^{-1}\widehat{y}_t + \widehat{m}\widehat{c}_t.
 \end{aligned}$$

## Monetary Policy Rule

$$\begin{aligned}
\widehat{r}_{t+1} &\equiv \widehat{i}_{t+1} - \mathbb{E}_t[\widehat{\pi}_{t+1}], \\
4\widehat{i}_{t+1} &\approx \rho_i 4\widehat{i}_t + (1 - \rho_i)[\phi_\pi \widetilde{\pi}_t + \phi_y \widetilde{y}_t] + \widehat{m}_t, \\
\widetilde{\pi}_t &\approx \begin{cases} (\widehat{p}_t - \widehat{p}_{t-4}) = \widehat{\pi}_t + \widehat{\pi}_{t-1} + \widehat{\pi}_{t-2} + \widehat{\pi}_{t-3}, \\ 4\widehat{\pi}_t, \end{cases} \\
\widetilde{y}_t &\approx \begin{cases} \widehat{y}_t, \\ \widehat{x}_t \equiv \widehat{y}_t - \widehat{y}_t^F. \end{cases}
\end{aligned}$$

## A.2 The Frictionless Model

### Aggregate Demand Equations

$$\begin{aligned}
\widehat{y}_t^F &\approx (1 - \gamma_x^F) \widehat{c}_t^F + \gamma_x^F \widehat{x}_t^F, \quad \gamma_x^F \equiv \delta \left( \frac{\psi}{\beta^{-1} - (1 - \delta)} \right), \\
\widehat{c}_t^F &\approx \mathbb{E}_t[\widehat{c}_{t+1}^F] - \sigma \widehat{r}_{t+1}^F, \\
\mathbb{E}_t[\widehat{r}_{t+1}^{kF}] &\approx \widehat{r}_{t+1}^F, \\
\widehat{r}_t^{kF} &\approx (1 - \epsilon^F)(\widehat{m}\widehat{c}_t^F + \widehat{y}_t^F - \widehat{k}_t^F) + \epsilon^F \widehat{q}_t^F - \widehat{q}_{t-1}^F, \quad \epsilon^F \equiv \left( \frac{1 - \delta}{\beta^{-1}} \right), \\
\widehat{q}_t^F &\approx \chi(\widehat{x}_t^F - \widehat{k}_t^F).
\end{aligned}$$

### Aggregate Supply Equations

$$\begin{aligned}
\widehat{y}_t^F &\approx \widehat{a}_t + \psi \widehat{k}_t^F + (1 - \psi) \widehat{h}_t^F, \\
\widehat{m}\widehat{c}_t^F &\approx \frac{1}{\varphi} \widehat{h}_t^F + \frac{1}{\sigma} \widehat{c}_t^F - (\widehat{y}_t^F - \widehat{h}_t^F), \quad \varphi \equiv \eta \left( \frac{1 - H}{H} \right), \\
\widehat{m}\widehat{c}_t^F &\approx 0.
\end{aligned}$$

### Law of Motion for State Variables

$$\widehat{k}_{t+1}^F \approx (1 - \delta) \widehat{k}_t^F + \delta \widehat{x}_t^F.$$

## Monetary Policy Rule

$$\begin{aligned}
\widehat{r}_{t+1}^F &\equiv \widehat{i}_{t+1}^F - \mathbb{E}_t[\widehat{\pi}_{t+1}^F], \\
4\widehat{i}_{t+1}^F &= \rho_i 4\widehat{i}_t^F + (1 - \rho_i)[\phi_\pi \widetilde{\pi}_t^F + \phi_y \widetilde{y}_t^F] + \widehat{m}_t, \\
\widetilde{\pi}_t^F &\approx \begin{cases} (\widehat{p}_t^F - \widehat{p}_{t-4}^F) = \widehat{\pi}_t^F + \widehat{\pi}_{t-1}^F + \widehat{\pi}_{t-2}^F + \widehat{\pi}_{t-3}^F, \\ 4\widehat{\pi}_t^F, \end{cases} \\
\widetilde{y}_t^F &\approx \begin{cases} \widehat{y}_t^F, \\ \widehat{x}_t^F \equiv \widehat{y}_t^F - \widehat{y}_t^F = 0. \end{cases}
\end{aligned}$$

### A.3 Shock Processes

$$\begin{aligned}\widehat{a}_t &= \rho_a \widehat{a}_{t-1} + \varepsilon_t^a, & \varepsilon_t^a &\sim N(0, \sigma_a^2), \\ \widehat{m}_t &= \rho_m \widehat{m}_{t-1} + \varepsilon_t^m, & \varepsilon_t^m &\sim N(0, \sigma_m^2).\end{aligned}$$

## Appendix B: U.S. Dataset

### B.1 Macro Aggregates

I adapt the work of Cociuba et al. (2009) to construct the quarterly series on hours worked and that of Gomme and Rupert (2007) to derive the quarterly measures of the U.S. stock of physical capital, U.S. investment, U.S. Total Factor Productivity (TFP) and other macro aggregates. The calculations of U.S. TFP are based on standard technological constraints which I summarize as follows,

Production Function:  $Y_t = A_t (K_t)^\alpha (\gamma^t H_t)^{1-\alpha}$ ,

Aggregate TFP:  $A_t = A_{t-1}^\rho e^{\varepsilon_t^a}$ ,  $|\rho| < 1$ ,  $A_0$  given,  $\varepsilon_t^a \sim N(0, \sigma_a^2)$ ,

Law of Motion for Capital in Structures:  $K_{st+1} = (1 - \delta_{st})K_{st} + X_{st}$ ,  $K_{s0}$  given,

Law of Motion for Capital in Equipment and Software:  $K_{et+1} = (1 - \delta_{et})K_{et} + X_{et}$ ,  $K_{e0}$  given,

Law of Motion for Capital in Housing:  $K_{ht+1} = (1 - \delta_{ht})K_{ht} + X_{ht}$ ,  $K_{h0}$  given,

Total Capital:  $K_{t+1} = K_{st+1} + K_{et+1} + K_{ht+1}$ ,

Total Investment:  $X_{t+1} = X_{st+1} + X_{et+1} + X_{ht+1}$ ,

where  $H_t$  denotes total hours worked,  $K_t$  is the total stock of physical capital,  $X_t$  is the total investment in physical capital,  $Y_t$  is real private output excluding government wages and salaries, and  $A_t$  is an aggregate TFP process that is thought to be stationary. I disaggregate capital and investment in three types:  $K_{st}$  is capital in market structures and  $X_{st}$  its corresponding real investment;  $K_{et}$  is capital in equipment and software and  $X_{et}$  real investment on equipment and software;  $K_{ht}$  is housing capital and  $X_{ht}$  is housing investment in real terms. I also consider labor-augmenting technological progress with a deterministic growth rate of  $\gamma$ .

The parameter  $\delta_{it}$  denotes the time-varying depreciation rate of capital by type  $i \in \{s, e, h\}$ . Real investment  $X_{it}$  is computed by deflating the nominal aggregate investment series by the consumption (nondurables and services) deflator, i.e.,

$$X_{it} \equiv (\gamma_q)^t Q_{it} I_{it}, \quad \text{for all } i \in \{s, e, h\},$$

where the relative price of investment in units of consumption grows at the deterministic rate of  $\gamma_q$  and its stationary component is denoted  $Q_{it}$  for each capital type  $i \in \{s, e, h\}$ . While current investment uses  $I_{it}$  units of current real private output per type of capital  $i$ , it yields  $X_{it}$  units of capital for production. In this sense, the

relative price of investment  $(\gamma_q)^t Q_{it}$  reflects the current state of the technology for producing the different types of capital. I recognize that capital depreciation  $\delta_{it}$  and the relative price of investment  $(\gamma_q)^t Q_{it}$  can differ across capital types, so I construct the stock of capital  $K_t$  by adding up its components  $(K_{st}, K_{et}, K_{ht})$ . In this situation, the disaggregated capital types are treated as perfect substitutes in obtaining the total stock of physical capital.

Since the model abstracts from population changes, then output, capital, investment and total hours worked should be expressed in per capita terms for consistency. I denote the population size as  $L_t$  and define the variables of interest in per capita terms as,

$$\begin{aligned} y_t &\equiv \frac{Y_t}{L_t}, & k_t &\equiv \frac{K_t}{L_t}, & x_t &\equiv \frac{X_t}{L_t}, & h_t &\equiv \frac{H_t}{L_t}, \\ x_{st} &\equiv \frac{X_{st}}{L_t}, & x_{et} &\equiv \frac{X_{et}}{L_t}, & x_{ht} &\equiv \frac{X_{ht}}{L_t}, \\ k_{st} &\equiv \frac{K_{st}}{L_t}, & k_{et} &\equiv \frac{K_{et}}{L_t}, & k_{ht} &\equiv \frac{K_{ht}}{L_t}. \end{aligned}$$

An implication of the Cobb-Douglas production function is that the specification admits a per-capita representation,

$$\begin{aligned} y_t &= A_t (k_t)^\alpha (\gamma^t h_t)^{1-\alpha}, \\ A_t &= A_{t-1}^\rho e^{\varepsilon_t^a} \end{aligned}$$

so the TFP measure  $A_t$  is unaffected by the population adjustment. Other endogenous variables of the model such as consumption  $c_t \equiv \frac{K_t}{L_t}$  are also expressed in per capita terms, while prices such as the interest rates or the consumer price index (CPI) do not admit a representation in per capita terms.

The strategy to recover the Solow residual (measured TFP) is to: (a) calculate total hours worked per capita,  $h_t$ ; (b) reconstruct the stock of total physical capital in per capita terms,  $k_t$ , with the perpetual inventory method using the aggregate investment series  $(X_{st}, X_{et}, X_{ht})$  given the vector of depreciation rates  $(\delta_s, \delta_e, \delta_h)$  and the population series  $L_t$ ; and (c) identify aggregate TFP from the production function as the part of real private output excluding government wages and salaries in per capita terms,  $y_t$ , that cannot be accounted for by the aggregate factors of production,  $h_t$  and  $k_t$ , given the capital income share  $\alpha$ . In the process to calculate the Solow residual, I also derive the relevant macro aggregates for real private output per capita, consumption per capita, investment per capita, hours worked per capita, and inflation as discussed here.

### B.1.1 Average Hours Worked and Working-Age Population

**Working-Age Population** Source: U.S. Department of Labor, Bureau of Labor Statistics (BLS). Employment and Earnings—Household Survey (Tables A-13 and A-22)

1. Download the following BLS series:

Employment and Earnings—Household Survey, Selected Labor Statistics by Sex and Detailed Age Group (NSA, Monthly, Thous): Civilian Noninstitutional Population: 16 Years and Over; Civilian Noninstitutional Population: 65 Years and Over.

2. Compute quarterly data of Population 16 and over and Population 65 and over by averaging over the monthly data.
3. Obtain quarterly civilian noninstitutional population ages 16 to 64 by subtracting one series from the other.
4. The series obtained is seasonally-adjusted using the Census X-12, multiplicative seasonal adjustment method.

**Average Hours Worked per Capita** Source: U.S. Department of Labor, Bureau of Labor Statistics (BLS). Current Population Survey (CPS)

1. Download the following BLS series:

(Unadj.) Average Hours, Total At Work, All Industries

(Unadj.) Number Employed, At Work

Both data series are at monthly frequency since June 1976. I complete the series with the historical data from July 1947 to May 1976 collected by Cociuba et al. (2009).

2. Convert the monthly series into data on a quarterly basis (by averaging the monthly numbers).
3. Seasonally-adjust the quarterly series using the Census X-12, multiplicative seasonal adjustment method.
4. Total hours worked per quarter are given by the product of employed persons at work on a quarterly basis times the average hours worked per week on a quarterly basis times  $\frac{52}{4}$ .
5. Quarterly average hours worked per capita can be computed by dividing the total civilian hours worked by the civilian noninstitutional working-age population (16–64 years old). The quarterly hours worked per capita is divided by  $\frac{5200}{4}$  ( $\frac{52}{4}$  weeks per quarter times 100 productive hours per week) to express the per capita hours worked as a ratio.

### B.1.2 Consumption (Nondurables and Services) Deflator and Inflation Rate

Source: U.S. Department of Commerce, Bureau of Economic Analysis (BEA), National Income and Wealth Division. National Income and Product Accounts, Domestic Product and Income (Table 1)

1. Download the following BEA series:

Domestic Product and Income: Table 1.1.5, Gross Domestic Product (Bil. \$, Annual): Personal Consumption Expenditures: Nondurable Goods; Personal Consumption Expenditures: Services.

Domestic Product and Income: Table 1.1.5, Gross Domestic Product (Bil. \$, Quarterly, SAAR): Personal Consumption Expenditures: Nondurable Goods; Personal Consumption Expenditures: Services.

Domestic Product and Income: Table 1.1.6, Real Gross Domestic Product (Bil. Chn. 2005. \$, Annual): Real Personal Consumption Expenditures: Nondurable Goods; Real Personal Consumption Expenditures: Services.

Domestic Product and Income: Table 1.1.6, Real Gross Domestic Product (Bil. Chn. 2005. \$, Quarterly, SAAR): Real Personal Consumption Expenditures: Nondurable Goods; Real Personal Consumption Expenditures: Services.

2. Construct the annual consumption (nondurables and services) deflator. Divide ('personal consumption expenditures: nondurable goods' plus 'personal consumption expenditures: services') by ('real personal consumption expenditures: nondurable goods' plus 'real personal consumption expenditures: services'), using annual data.
3. Construct the quarterly consumption (nondurables and services) deflator. Divide ('personal consumption expenditures: nondurable goods' plus 'personal consumption expenditures: services') by ('real personal consumption expenditures: nondurable goods' plus 'real personal consumption expenditures: services'), using quarterly data.
4. Construct the year-over-year inflation rate for the consumption (nondurables and services) deflator in percentages,

$$\bar{\pi}_t \equiv \left( \frac{P_t - P_{t-4}}{P_{t-4}} \right) 100.$$

### B.1.3 Private Output, Consumption and Investment

Source: U.S. Department of Commerce, Bureau of Economic Analysis (BEA), National Income and Wealth Division. National Income and Product Accounts, Domestic Product and Income (Table 1)

1. Download the following BEA series:

Domestic Product and Income: Table 1.1.5, Gross Domestic Product (Bil. \$, Quarterly, SAAR): Gross Domestic Product; Personal Consumption Expenditures: Private Nonresidential Investment: Structures; Private Nonresidential Investment: Equipment and Software; Private Residential Investment.

Domestic Product and Income: Table 1.1.6, Real Gross Domestic Product (Bil. Chn. 2005. \$, Quarterly, SAAR): Real Personal Consumption Expenditures: Nondurable Goods; Real Personal Consumption Expenditures: Services.

Domestic Product and Income: Table 1.12, National Income by Type of Income (Bil. \$, Quarterly, SAAR): Government Wages and Salaries.

2. Construct the quarterly real output series for the U.S. Subtract 'government wages and salaries' from the 'gross domestic product'. Divide the resulting series by the quarterly consumption (nondurables and services) deflator computed before.

3. Construct the quarterly real consumption series for the U.S. Add ‘real personal consumption expenditures: nondurable goods’ plus ‘real personal consumption expenditures: services’.
4. Construct the quarterly real investment series for structures, equipment and software, and housing. Use the quarterly (nominal) investment series for each type of capital (‘personal consumption expenditures: private nonresidential investment: structures’, ‘private nonresidential investment: equipment and software’, and ‘private residential investment’) and divide them by the quarterly consumption (nondurables and services) deflator computed before. The real investment sample starts in the first quarter of 1947. Calculate total real investment as the sum of the real investment for all three types of capital (structures, equipment and software, and housing).
5. Construct the series for real output, real consumption and real investment by capital type in per capita terms and at quarterly rates. Divide the quarterly real output, real consumption and real investment computed before by the civilian noninstitutional population between the ages of 16 and 64 (which was derived earlier). The civilian noninstitutional population is expressed in thousands and must be multiplied by 1000 to express it in number of individuals. The real output, consumption and investment series are expressed in billions of 2005 dollars and must be multiplied by  $10^9$  to express everything in units of 2005 dollars. Divide the resulting ratios by 4 to express the quarterly per capita real output, real consumption and real investment on a quarterly basis—rather than at an annualized rate.

#### **B.1.4 Total Factor Productivity**

**Capital’s Share of Income** Source: U.S. Department of Commerce, Bureau of Economic Analysis (BEA), National Income and Wealth Division. National Income and Product Accounts, Domestic Product and Income (Table 1) and Supplemental Tables (Table 7)

1. Download the following BEA series:

Domestic Product and Income: Table 1.12, National Income by Type of Income (Bil. \$, Annual): Compensation of Employees, Paid; Government Wages and Salaries; Rental Income of Persons, with Capital Consumption Adjustments; Corporate Profits with Inventory Valuation and Capital Consumption Adjustments; Net Interest and Miscellaneous Payments on Assets.

Domestic Product and Income: Table 1.3.5, Gross Value Added by Sector, (Bil. \$, Annual): Gross Value Added: General Government.

Domestic Product and Income: Table 1.7.5, Relation of Gross Domestic Product, Gross National Product, Net National Product, National Income, and Personal Income (Bil. \$, Annual): Gross National Product; Net National Product.

Domestic Product and Income: Table 1.9.5, Net Value Added by Sector (Bil. \$, Annual): General Government: Net Domestic Product.



Supplemental Tables: Table 7.4.5, Housing Sector Output, Gross and Net Value Added (Bil. \$, Annual): Gross Housing Value Added; Housing: Compensation of Employees; Housing: Rental Income of Persons with Capital Consumption Adjustments; Housing: Corporate Profits with Inventory Valuation and Capital Consumption Adjustments; Housing: Net Interest; Net Housing Value Added.

2. Exclude government labor income to be consistent with the concept of output defined in the model. NIPA includes an imputed capital income flow for owner occupied housing, but omits the corresponding labor income flows. This omission can introduce an upward bias in the derivation of the capital income share  $\alpha$ . Instead of attempting to correct or adjust the data to account for the omission of labor income flows for owner occupied housing, exclude housing imputed rents from the capital income series for the purpose of computing the capital's share of income. Calculate nominal labor income,  $Y^{LP}$ , as 'compensation of employees, paid' minus 'housing: compensation of employees' minus 'government wages and salaries.' Calculate nominal capital income including depreciation,  $Y^{KP_d}$ , as 'rental income of persons, with capital consumption adjustments' plus 'corporate profits with inventory valuation and capital consumption adjustments' plus 'net interest and miscellaneous payments on assets' minus 'housing: rental income of persons with capital consumption adjustments' minus 'housing: corporate profits with inventory valuation and capital consumption adjustments' minus 'housing: net interest' plus depreciation. Compute depreciation as ('gross national product' minus 'gross value added: general government' minus 'gross housing value added') minus ('net national product' minus 'general government: net domestic product' minus 'net housing value added').
3. Under the standard assumptions of an aggregate Cobb-Douglas production function and perfect competition, the capital share in the production function  $\alpha$  can be computed as the ratio of all capital income sources divided by private output. Compute the capital's share of income for each year as,

$$\alpha = \frac{Y^{KP_d}}{Y^{LP} + Y^{KP_d}}.$$

Calculate the average for the entire sample period after the Korean War (starting in 1954) in order to pin down the capital and labor shares in the production function (i.e., the constant parameter  $\alpha$ ).

**Depreciation Rate** Source: U.S. Department of Commerce, Bureau of Economic Analysis (BEA), National Income and Wealth Division. National Income and Product Accounts, Domestic Product and Income (Table 1) and Fixed Assets and Consumer Durable Goods (formerly called Fixed Reproducible Tangible Wealth in the U.S.), Capital Stock (Tables 4 and 5)

1. Download the following BEA series:

Domestic Product and Income: Table 1.1.5, Gross Domestic Product (Bil. \$, Quarterly, SAAR): Personal Consumption Expenditures: Private Nonresidential

Investment: Structures; Private Nonresidential Investment: Equipment and Software; Private Residential Investment.

Fixed Assets and Consumer Durable Goods, Capital Stock: Tables 4.1, 4.2 and 4.3, Net Stock of Private Fixed Nonresidential Assets by Legal Form and Industry, Year-end Estimates at Current Cost (Bil. \$, Annual): Net Stock: Private Fixed Nonresidential Structures; Net Stock: Private Fixed Nonresidential Equipment and Software.

Fixed Assets and Consumer Durable Goods, Capital Stock: 5.1, 5.2 and 5.3, Net Stock of Private Fixed Nonresidential Assets by Legal Form and Industry, Year-end Estimates at Current Cost (Bil. \$, Annual): Net Stock: Private Residential Fixed Assets.

2. Construct the annual (year-end) stock of real capital on structures, equipment and software, and housing. Use the annual (nominal) stocks of capital at current cost for each category ('net stock: private fixed nonresidential structures', 'net stock: private fixed nonresidential equipment and software', and 'net stock: private residential fixed assets') and divide them by the annual consumption (nondurables and services) deflator computed before.
3. Construct the quarterly real investment series for structures, equipment and software, and housing. Use the quarterly (nominal) investment series for each category ('personal consumption expenditures: private nonresidential investment: structures', 'private nonresidential investment: equipment and software', and 'private residential investment') and divide them by the quarterly consumption (nondurables and services) deflator computed before. The real investment sample starts in the first quarter of 1947. The quarterly real investment series are expressed at quarterly—rather than annualized—rates. In other words, the series of quarterly annualized investment deflated by the quarterly consumption (nondurable and services) deflator must be divided by 4.
4. Recover the fixed quarterly depreciation rates by year.  $K_t$  is the stock of capital and  $K_{it}$  is the stock for each capital type  $i \in \{s, e, h\}$  accumulated at the end of  $t - 1$  that becomes available for production during  $t$ . Consistently with this timing convention, assume that the annual year-end estimate of the stock of capital is equal to the capital available for production during the first quarter of the following year. Let  $\{t, t + 1, t + 2, t + 3\}$  be the quarters corresponding to a given year  $z$ , and  $\{t + 4, t + 5, t + 6, t + 7\}$  those corresponding to year  $z + 1$ . Then, for all  $i \in \{s, e, h\}$  the capital available in each quarter subject to the fixed depreciation rate  $\delta_{iz}$  can be expressed as,

$$t + 1 \text{ (Year } z, \text{ Second Quarter): } K_{it+1} = (1 - \delta_{iz})K_{it} + X_{it},$$

$$t + 2 \text{ (Year } z, \text{ Third Quarter): } K_{it+2} = (1 - \delta_{iz})K_{it+1} + X_{it+1},$$

$$t + 3 \text{ (Year } z, \text{ Fourth Quarter): } K_{it+3} = (1 - \delta_{iz})K_{it+2} + X_{it+2},$$

$$t + 4 \text{ (Year } z + 1, \text{ First Quarter): } K_{it+4} = (1 - \delta_{iz})K_{it+3} + X_{it+3},$$

which can be written recursively in the form of a quartic equation,

$$K_{it+4} = K_{it}(1 - \delta_{iz})^4 + X_{it}(1 - \delta_{iz})^3 + X_{it+1}(1 - \delta_{iz})^2 + X_{it+2}(1 - \delta_{iz}) + X_{it+3}.$$

The depreciation rates for each one of the three categories—structures, software and equipment and housing—are computed using a quartic solver in Matlab. The solver returns eight numbers (in the complex plane) that satisfy the formula,

$$a_4\lambda^4 + a_3\lambda^3 + a_2\lambda^2 + a_1\lambda + a_0 = 0.$$

Take the deflated year-end stock of capital for the previous year—which is available for production during the first quarter of the current year—to be  $a_4(\equiv K_{it})$ . Then,  $a_3(\equiv X_{it})$  represents the real investment in the first quarter of the year,  $a_2(\equiv X_{it+1})$  is the real investment in the second quarter,  $a_1(\equiv X_{it+2})$  is the real investment in the third quarter, and  $a_0 \equiv (X_{it+3} - K_{it+4})$  is the difference between the real investment in the fourth quarter and the deflated year-end stock of capital available for production during the first quarter of the following year. The depreciation rate for a given year  $z$  is computed as one minus the largest real root on the quartic polynomial (i.e.,  $\delta_{iz} = 1 - \lambda$ ). While the depreciation rates are invariant within a year, they are allowed to vary from one year to the next in the calculations of the quarterly stock of capital. For more details on finding the solution to the quartic polynomial, see e.g. Gomme and Rupert (2007).

### Stock of Capital

1. Construct the quarterly stock of real capital on structures, equipment and software, and housing. Start in the first quarter of 1947 with the available stock of capital for that quarter that corresponds to the deflated year-end stock of capital at current costs for each type of capital for the year 1946—which was calculated before. The first quarter of 1947 stock of capital net of depreciation (using the 1947 depreciation rates) plus the real investment in the first quarter of 1947 gives the capital available for production in the second quarter of 1947. Compute recursively the stock of capital available for production in a given quarter as the sum of the real investment in the previous quarter plus the stock of capital available in the previous quarter net of depreciation (at the corresponding depreciation rate of the year on which the previous quarter falls). The quarterly depreciation rates vary across years, but are constant within a given year and had been previously calculated. The quarterly real investment series by capital type have also been computed before.
2. Construct the quarterly stock of real capital on structures, equipment and software, and housing in per capita terms. Divide the quarterly real stock of capital for each type by the civilian noninstitutional population between the ages of 16 and 64 (which was derived before). The civilian noninstitutional population is expressed in thousands and must be multiplied by 1000 to express it in number of individuals. The capital stock and investment series are expressed in billions of 2005 dollars and must be multiplied by  $10^9$  to express them in units of 2005 dollars. Add the real stock of capital per capita disaggregated by type to obtain to total real stock of capital per capita available for production.

## Solow Residual

1. Construct the quarterly Solow residual using the calculated series for per capita output, hours worked and total capital. Without loss of generality, transform all series into indexes where the first quarter of 1948 takes the value of 1 (i.e., 1948:I = 1). The average capital income share determines the parameter  $\alpha$ . Then, calculate the Solow residual index ( $\frac{S_t}{S_0}$ ) as,

$$\frac{S_t}{S_0} \equiv \left( \frac{(\frac{y_t}{y_0})}{(\frac{k_t}{k_0})^\alpha (\frac{h_t}{h_0})^{1-\alpha}} \right),$$

where  $(\frac{y_t}{y_0})$  is the per capita real output index with the base year set at  $t = 0$ . Similarly, for the indexes of capital and hours worked in per capita terms, i.e. for  $(\frac{k_t}{k_0})$  and  $(\frac{h_t}{h_0})$  respectively. The level of the Solow residual does not convey any additional information beyond is contained in this index, so one can work directly with the index series.

2. Express the Solow residual in logs as,

$$\bar{s}_t \equiv \left( \ln \left( \frac{S_t}{S_0} \right) \right) 100,$$

and the levels of real per capita output (excluding government), real per capita consumption, real per capita total investment and total hours worked per capita in logs as,

$$\begin{aligned} \bar{y}_t &\equiv \ln(y_t) 100, & \bar{c}_t &\equiv \ln(c_t) 100, \\ \bar{x}_t &\equiv \ln(x_t) 100, & \bar{h}_t &\equiv \ln(h_t) 100. \end{aligned}$$

All these variables are multiplied by 100 to be able to quote them in percentages.

## B.2 Financial and Monetary Variables

### B.2.1 Interest Rate Spread

Source: Treasury Department, and Board of Governors of the Federal Reserve System, “Selected Interest Rates”, “Interest Rates Updated Before FRB Publication”, H.15 (415).

1. Download the following Treasury Department and Federal Reserve Board series: Selected Interest Rates—FRB H.15 (NSA, Quarterly Average of Daily Data, Yields in Percent Per Annum): 20-Year Treasury Note Yield at Constant Maturity; Baa Corporate Bonds, Moody’s Seasoned.

Treasury Long-Term Composite (over 10 years) is no longer available on the FRB H.15 release, but the series continuous to be regularly updated by the Treasury Department. The composite is an unweighted average of all issues outstanding of bonds neither due nor callable in less than 10 years.

2. Complete the 20-year Treasury series. The nominal 20-year Treasury was discontinued between January 1, 1987 through September 30, 1993. Data for this period is calculated as an average of the 10- and 30-year constant maturity yields. Data prior to April 1953 corresponds to the Treasury Long-Term composite (over 10 years) yield. This Long-Term composite index is an unweighted average of all issues outstanding of bonds neither due nor callable in less than 10 years.
3. Compute the yields for the 20-Year Treasury, and Moody's Baa corporate yields on a quarterly basis. These nominal yields,  $R_t^j$  for each type  $j = \{20\text{-year Treasury, Baa corporate}\}$ , are quoted per annum, in percent. A typical transformation is to compute the quarterly compounded rate  $\rho_{jt}$  as follows,

$$\rho_{jt} = 0.25 \ln \left( 1 + \frac{R_t^j}{100} \right).$$

4. Compute the spread between the Baa corporate yield and the 20-year Treasury yield. Moody's drops bonds if the remaining life falls below 20 years, if the bond is susceptible to redemption, or if the rating changes. Hence the spread with the 20-year Treasury indicates the risk of corporates at the margin (which are barely above investment grade), controlling for maturity. Compute the spread simply taking the difference between the quarterly compounded yields of two rates  $i, k$  as follows,

$$spread_t^{i,k} = 100 * (\rho_{it} - \rho_{kt}).$$

5. Compute the ratio  $\frac{R^k}{R}$  with the spread of the Baa corporate bond rate and 20-year Treasury rate as,

$$\frac{R^k}{R} = 1 + \frac{spread^{Baa\ Corporate, 20\text{-year Treasury}}}{100}.$$

Compute the historical averages in order to calibrate the model.

## B.2.2 Monetary Policy

Source: Board of Governors of the Federal Reserve System, "Selected Interest Rates", "Interest Rates Updated Before FRB Publication", H.15 (415).

1. Download the following Federal Reserve Board series:  
Selected Interest Rates—FRB H.15 (NSA, Quarterly Average of Daily Data, Yields in Percent Per Annum): Federal Funds (Effective).
2. Taylor (1993) proposes the rate of inflation over the previous four quarters as one of the key objectives for monetary policy. Construct the year-over-year inflation rate for the consumption (nondurables and services) deflator derived before in percentages,

$$\bar{\pi}_t \equiv \left( \frac{P_t - P_{t-4}}{P_{t-4}} \right) 100.$$

3. Taylor (1993) proposes detrended output as one of the key objectives for monetary policy. Express the quarterly real per capita output (excluding government compensation) in logs and multiplied by 100 as  $\bar{y}_t \equiv \ln(y_t)100$ . Then, estimate a linear time trend for the real per capita output index in logs as before (over the sample period: 1984:I–2007:IV),

$$\begin{aligned}\bar{y}_t &= \hat{\bar{y}}_t + u_t^y, \\ \hat{\bar{y}}_t &= \alpha_y + \beta_y t.\end{aligned}$$

Detrended output as the percentage deviation of per capita real output (excluding government compensation) relative to its trend can be calculated as

$$\hat{y}_t \equiv \bar{y}_t - \hat{\bar{y}}_t.$$

4. Compute the Taylor rule rate following the reaction function proposed in Taylor (1993),

$$\begin{aligned}i_t^{TR} &\equiv r + \bar{\pi}_t + \frac{1}{2}\hat{y}_t + \frac{1}{2}(\bar{\pi}_t - \pi) \\ &= (r + \pi) + \frac{1}{2}\hat{y}_t + \frac{3}{2}(\bar{\pi}_t - \pi),\end{aligned}$$

where  $i_t^{TR}$  is the target rate for monetary policy,  $\bar{\pi}_t$  defines the year-over-year inflation rate of consumption (nondurables and services) for quarter  $t$  (in percentages),  $(\pi_t - \pi)$  refers to the percentage deviation of inflation relative to its long-run target, and  $\hat{y}_t$  is the detrended output obtained as the percentage deviation of per capita real output (excluding government) relative to its log-linear trend. Implicit in this equation is the notion that the real (annualized) interest rate is  $r \equiv 2\%$  and the long-run inflation target is  $\pi \equiv 2\%$  (per annum), so the long-run nominal interest rate is equal to  $(r + \pi) = 4\%$ . This specification is otherwise isomorphic to an alternative policy that assumes a long-run inflation target of zero and a real interest rate of  $r - \frac{1}{2}\pi = 2\% - \frac{1}{2}2\% = 1\%$ .

5. Compute monetary policy shocks  $m_t$  relative to this policy rule as the difference between the federal funds rate (effective) and the Taylor-implied target rates in every period, i.e.  $m_t \equiv i_t - i_t^{TR}$ . Demean the monetary policy shock process, if the mean is statistically different from zero, in order to obtain  $\hat{m}_t$ .
6. Estimate an  $AR(1)$  process for the demeaned monetary policy shock series  $\bar{m}_t$  without a constant term,

$$\hat{m}_t = \rho_m \hat{m}_{t-1} + \varepsilon_t^m,$$

in order to characterize the monetary policy shock process in the model.

### B.3 Mapping the Non-stationary Data

The financial accelerator model of Bernanke et al. (1999) is stationary by construction, but most observed aggregate variables in the data—even in per capita terms—are not so. The non-stationary observed time series are mapped into the stationary series endogenously generated by the model by detrending the data first (to account for their upward trend). The way to solve the financial accelerator model which I use is to employ a log-linear approximation and express all variables in log deviations from their steady state. All stationary series in the data which are not subject to detrending, then, are demeaned to make them comparable with the endogenous series simulated by the model.

The implicit assumption about the long-run growth path is that the relative price of investment grows at the deterministic rate of  $\gamma_q$  for all  $i \in \{s, e, h\}$  and that labor-augmenting technological progress attains a rate of long-run growth of  $\gamma$ . These assumptions are consistent with a deterministic balanced growth path where all variables grow at a constant—but not necessarily common—rate over the long-run (see, e.g., Gomme and Rupert 2007). Hence, when detrending the data these differences in long-run growth rates across macro variables must be taken into account.

Similar to Gomme and Rupert (2007), the constant growth rate of real output per capita  $g$  should be given by  $g = (g_k)^\alpha (\gamma)^{1-\alpha}$  along a balanced growth path. For the growth rate of capital per capita  $g_k$  to be constant it must be the case that  $g_k = g\gamma_q$ . Hence, these balanced growth path relations imply that,

$$g = (\gamma_q)^{\frac{\alpha}{1-\alpha}} \gamma.$$

The long-run growth of the relative price of investment,  $\gamma_q$ , and the long-run rate of labor-augmenting technological progress,  $\gamma$ , both affect the balanced growth path of per capita real output  $y_t$ .

All real investment series— $x_{st}$ ,  $x_{et}$ ,  $x_{ht}$  and  $x_t$ —as well as real consumption per capita  $c_t$  must grow at the same common growth rate as real output, i.e. must grow at  $g$ . The stock of capital for each type ( $k_{st}$ ,  $k_{et}$ ,  $k_{ht}$ ) and total capital  $k_t$  grow in turn at a different fixed rate  $g_k = (\gamma_q)^{\frac{1}{1-\alpha}} \gamma$  (where  $g_k \neq g$  if  $\gamma_q \neq 1$ ). The share of hours worked per capita  $h_t$  is stationary, and bounded within the unit interval. The Solow residual  $S_t$  trends upwards at the rate of technological progress,

$$S_t \equiv \frac{y_t}{(k_t)^\alpha (h_t)^{1-\alpha}} = A_t (\gamma^{1-\alpha})^t,$$

even when working with variables in per capita terms (i.e.,  $y_t$ ,  $k_t$  and  $h_t$ ). However, the rate of long-run growth on real output per capita given by  $g = (\gamma_q)^{\frac{\alpha}{1-\alpha}} \gamma$  and the rate of growth of  $S_t$  given by  $\gamma^{1-\alpha}$  do not generally coincide.

I do not impose this long-run relationships directly on the data for detrending, but I infer from the logic of the argument that: hours worked do not require detrending, only need to be demeaned; that real output per capita,  $y_t$ , real investment per capita,  $x_t$ , and real consumption per capita,  $c_t$ , all should be detrended using a common

trend growth; and that the Solow residual measured as  $S_t$  is non-stationary as well but detrending it requires a different trend growth than that of output, investment and consumption.

I focus my investigation on the period of the Great Moderation since 1984:I. This period is characterized by stable trends in the data until the 2007 recession. I model the possible presence of nonlinearities in the data since 2007:IV by allowing for a break in the level, but not in the growth rate, of the trend component. For variables that do not trend upwards, I allowed for the possibility that the historical mean of the Great Moderation may have shifted as well.

## Detrending

1. Fit a linear time trend to the index series for the Solow residual,  $\bar{s}_t$ , in logs as follows,

$$\bar{s}_t \equiv \left( \ln \left( \frac{S_t}{S_0} \right) \right) 100,$$

$$\bar{s}_t = \alpha_s + \beta_s t + \hat{a}_t.$$

Estimate this linear trend for the sample period 1984:I–2007:IV, abstracting from the break in the data during the 1970s. While growth resumed during the Great Moderation period, it was at a slower pace than in the 1950s and 1960s. I do not account for that break or model explicitly the transition dynamics implied by it. Estimate an  $AR(1)$  process for the detrended Solow residual series without a constant term,

$$\hat{a}_t = \rho_a \hat{a}_{t-1} + \varepsilon_t^a.$$

This characterizes the aggregate TFP shock process in the model.

2. Fit a common linear time trend to the series for the real output (excluding government) per capita in logs,  $\bar{y}_t$ , for the real consumption per capita in logs,  $\bar{c}_t$ , and for the real investment per capita in logs,  $\bar{x}_t$ , as follows,

$$\bar{y}_t \equiv \ln(y_t) 100, \quad \bar{c}_t \equiv \ln(c_t) 100, \quad \bar{x}_t \equiv \ln(x_t) 100,$$

$$\begin{pmatrix} 100 \frac{c_t}{y_t} \\ 100 \frac{x_t}{y_t} \end{pmatrix} = \begin{pmatrix} \gamma_c \\ \gamma_x \end{pmatrix} + \begin{pmatrix} u_t^{c\_share} \\ u_t^{x\_share} \end{pmatrix},$$

$$\begin{pmatrix} \bar{y}_t \\ \bar{c}_t \\ \bar{x}_t \end{pmatrix} = \begin{pmatrix} 100 \ln(\alpha_y) \\ 100 \ln(\alpha_y \frac{\gamma_c}{100}) \\ 100 \ln(\alpha_y \frac{\gamma_x}{100}) \end{pmatrix} + \beta_y t + \begin{pmatrix} u_t^y \\ u_t^c \\ u_t^x \end{pmatrix},$$

adjusting the intercept to be consistent with the average consumption share,  $\gamma_c$ , and investment share,  $\gamma_x$ . Estimate this trend specification for the sample period 1984:I–2007:IV. Re-estimate the linear trend including all data available until now, but allowing for the possibility of a level shift in the intercept occurring



during the 2007 recession both resulting from a permanent change in the output level,  $\alpha_y$ , or a permanent shift in the consumption and investment shares,  $\gamma_c$  and  $\gamma_x$ .

### Demeaning

1. Transform the series on hours worked into an index where the first quarter of 1948 takes the value of 1 (i.e., 1948:I = 1). Demean the series on hours worked in logs and multiplied by 100 by estimating the following relationship,

$$\begin{aligned}\bar{h}_t &\equiv \ln(h_t)100, \\ \bar{h}_t &= \alpha_h + u_t^h,\end{aligned}$$

over the sample period 1984:I–2007:IV. Allow for the possibility of a level shift in the intercept occurring during the 2007 recession.

2. Compute the percentage deviation of inflation relative to the long-run inflation target inflation by subtracting  $\pi$  from the series  $\pi_t$ ,

$$\hat{\pi}_t \equiv (\pi_t - \pi) = \left( \frac{P_t - P_{t-4}}{P_{t-4}} \right) 100 - \pi,$$

where the standard practice is to set the long-run inflation target during the Great Moderation period (1984:I–2007:IV) at  $\pi \equiv 2\%$  (per annum).

**Sample Period** Notice that 1971:III signifies also the advent of a distinctly different monetary policy regime in the U.S. On February 1, 1970, Arthur F. Burns became chairman of the Fed replacing the long-serving William McChesney Martin. Then, on August 15, 1971, the U.S. unilaterally terminated convertibility of the U.S. dollar to gold. The dollar becoming a fiat currency in 1971:III ended the Bretton Woods international monetary system that had been in place since the end of World War II. The onset of the Great Moderation period of low macro volatility and low inflation is often traced back to the appointment of Paul Volcker in August 6, 1979, who decidedly brought the inflation of the 1970s under control. The start of the Great Moderation is generally dated to 1984:I, so it coincided for the most part with the tenure of Alan Greenspan as chairman of the Fed which began in August 11, 1987. And, as Taylor (1993) famously noted, the monetary policy during the period of the Great Moderation is fairly well-described by the simple policy rule that I have adopted in this paper.

1. Detrend/demean all series further back to 1971:III with the estimates of the linear trend/level obtained for the Great Moderation period (1984:I–2007:IV).
2. Set 1971:III as the initial period for the simulation of the model because actual output is closest to its Great Moderation trend at that point than at any other quarter prior to 1984:I and because it occurs after the trend break in productivity of the 1970s.

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