

Contents

1	Weyl Algebras	1
1.1	Definition of a Weyl Algebra	1
1.2	The Algebraic Tensor Product	2
1.3	Wick's Theorem	5
1.4	Basis of a Weyl Algebra	8
1.5	Gaussian Functionals	10
1.6	Multisets	12
1.7	Finite Sets of Creation and Annihilation Operators	16
2	Continuous Sets of Creation and Annihilation Operators	25
2.1	Creation and Annihilation Operators on Fock Space	25
2.2	The Sum-Integral Lemma for Measures	26
2.3	Creation and Annihilation Operators on Locally Compact Spaces	31
2.4	Introduction of Point Measures	34
3	One-Parameter Groups	39
3.1	Resolvent and Generator	39
3.2	The Spectral Schwartz Distribution	46
4	Four Explicitly Calculable One-Excitation Processes	55
4.1	Krein's Formula	55
4.2	A Two-Level Atom Coupled to a Heat Bath of Oscillators	57
4.2.1	Discussion of the Model	57
4.2.2	Singular Coupling Limit	61
4.2.3	Time Evolution	66
4.2.4	Replacing Frequencies by Formal Times	68
4.2.5	The Eigenvalue Problem	70
4.3	A Two-Level Atom Interacting with Polarized Radiation	79
4.3.1	Physical Considerations	79
4.3.2	Singular Coupling	83
4.3.3	The Hamiltonian and the Eigenvalue Problem	86
4.4	The Heisenberg Equation of the Amplified Oscillator	88

4.4.1	Physical Considerations	88
4.4.2	The Singular Coupling Limit, Its Hamiltonian and Eigenvalue Problem	90
4.5	The Pure Number Process	94
5	White Noise Calculus	97
5.1	Multiplication of Diffusions	97
5.2	Multiplication of Point Measures	99
5.3	White Noise Operators	101
5.4	Wick's Theorem	107
5.5	Representation of Unity	109
5.6	Duality	111
6	Circled Integrals	113
6.1	Definition	113
6.2	A Circled Integral Equation	114
6.3	Functions of Class \mathcal{C}^1	117
7	White Noise Integration	123
7.1	Integration of Normal Ordered Monomials	123
7.2	Meyer's Formula	127
7.3	Quantum Stochastic Processes of Class \mathcal{C}^1 : Definition and Fundamental Properties	129
7.4	Ito's Theorem	131
8	The Hudson-Parthasarathy Differential Equation	139
8.1	Formulation of the Equation	139
8.2	Existence and Uniqueness of the Solution	140
8.3	Examples	141
8.3.1	A Two-Level Atom in a Heatbath of Oscillators	141
8.3.2	A Two-Level Atom Interacting with Polarized Radiation	143
8.3.3	The Heisenberg Equation of the Amplified Oscillator	144
8.3.4	A Pure Number Process	144
8.4	A Priori Estimate and Continuity at the Origin	145
8.5	Consecutive Intervals in Time	150
8.6	Unitarity	152
8.7	Estimation of the Γ_k -Norm	154
8.8	The Hamiltonian	161
8.8.1	Definition of the One-Parameter Group $W(t)$	161
8.8.2	Definition of \hat{a} , \hat{a}^+ and $\hat{\delta}$	163
8.8.3	Characterization of the Hamiltonian	169
9	The Amplified Oscillator	179
9.1	The Quantum Stochastic Differential Equation	179
9.2	Closed Solution	180
9.3	The Unitary Evolution	192
9.4	Heisenberg Equation	196
9.5	The Hamiltonian	203

9.6	Amplification	207
9.7	The Classical Yule-Markov Process	208
10	Approximation by Coloured Noise	213
10.1	Definition of the Singular Coupling Limit	213
10.2	Approximation of the Hudson-Parthasarathy Equation	215
	References	225
	Index	227

A Measure Theoretical Approach to Quantum
Stochastic Processes

von Waldenfels, W.

2014, XVII, 228 p., Softcover

ISBN: 978-3-642-45081-5