

Chapter 2

Stars as Globes of Gas

A Theory of the Stars

J. Homer Lane was the first person to investigate the details of the temperature distribution within a star. In 1870, he published a seminal paper in the *American Journal of Science and Arts*, entitled, ‘On the theoretical temperature of the Sun, under the hypothesis of a gaseous mass maintaining its volume by its internal heat, and depending on the laws of gases as known to terrestrial experiment’. Put simply, in this work Lane assumed that stellar matter behaved as an *ideal gas* and obeyed Boyle’s law, as terrestrial gases do. It is a different matter altogether whether this is a reasonable assumption or not, and we shall have occasion to return to this in a later chapter. But at the time, this was a significant piece of work, and this pioneering paper was followed by investigations by A. Ritter, in Germany, Lord Kelvin, in Britain, and others. The culmination of this line of investigation was the publication of the monumental book, *Gaskugeln*, by R. Emden, in 1907.

Hydrostatic Equilibrium

Before understanding Lane’s results, let us set up the equation for the mechanical stability of the star. Consider an imaginary concentric spherical surface of radius inside the star as shown in Fig. 2.1. Let us place on this surface a small cylinder, whose axis points along the outward radius at that point. The cross-section of this cylinder is of unit area of the base and length dr , and it contains stellar material. The density of this stellar material is $\rho(r)$, the value that obtains at that radius. The gravitational force on that cylinder would be due to the mass interior to the imaginary surface. Let us call this mass, $M(r)$.

As the area of cross-section of the cylinder is unity, and its length is dr , the mass of the infinitesimal cylinder is given by $\rho(r)dr$. The force of attraction between $M(r)$ and $\rho(r)dr$ is

$$\frac{GM(r)\rho(r)dr}{r^2}. \quad (2.1)$$

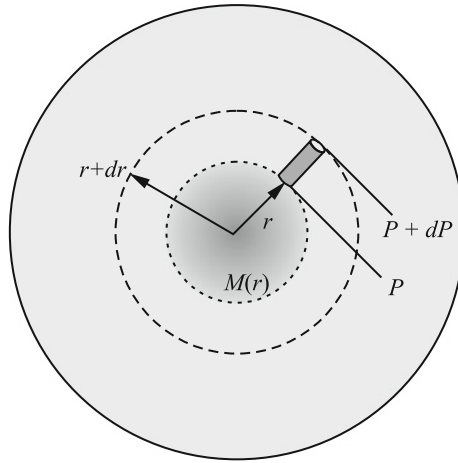


Fig. 2.1 Consider an infinitesimal cylinder at a distance r from the centre of unit cross-sectional area and of height dr . The gravitational force acting on it will arise from the mass $M(r)$ of material interior to the spherical shell on which it lies. This has to be balanced by the difference in pressure dP , which represents a force $-dP$ in the direction of increasing r (pointing outward from the centre). This is the condition for hydrostatic equilibrium of the *star*, and must be satisfied at every point in the *star*

As you know, in Newton's law the contribution to the force from the mass *exterior* to the surface cancels out. This is illuminating and quite simple to prove. If you know some calculus, I urge you try to prove this. The gravitational force on this infinitesimal cylinder has to be balanced by the pressure differential on it. This is just the *difference* in pressure at the two surfaces of the cylinder at a distance from the centre equal to the radius r and $r + dr$, respectively. Let us denote this by dP . This difference in pressure dP represents the force, $-dP$, acting on the cylinder in the direction of increasing r . Thus the equation for the equilibrium of the unit cylinder is

$$dP = -\frac{GM(r)\rho(r)dr}{r^2}$$

One can rearrange this as

$$\boxed{\frac{dP}{dr} = -\frac{GM(r)\rho(r)}{r^2}}. \quad (2.2)$$

The above equation is known as the *Equation of Hydrostatic Equilibrium*. For a star to be mechanically stable, this equation has to be satisfied at *every point in the star*. Otherwise, as a distinguished astronomer said, 'the punishment would be swift'. Any violation of this condition of *hydrostatic equilibrium* would result in motion within the star. For example, the material within our sample unit cylinder would either sink, or float up due to buoyancy.

Now, to go back to Lane, the pressure of the gas is to be calculated according to Boyle's law, namely,

$$P = nk_B T. \quad (2.3)$$

Here n is the number density of particles (number of constituent particles per unit volume).

The ultimate objective of any theory of the stars is to derive the radial profile of the density and temperature. This is not as simple as it sounds. While the right-hand side of Eq. (2.2) involves only the density, the left-hand side involves both the density and temperature; this is because gas pressure is determined by density, as well as temperature (see Eq. 2.3). Therefore, to derive how the stellar density varies as a function of r one has to know how the temperature varies with r . It should be clear from Eq. (2.3) that dP/dr involves both $d\rho/dr$ and dT/dr . As we shall see in the next chapter, one had to wait till the 1920s to be able to derive the radial profile of the density and temperature.

Why Does the Sun Shine?

Now that one has a mathematical framework to understand why the Sun is stable, the next question to answer is, 'what is the source of energy that makes the Sun shine?' Lane did not attempt to address this question. But his theory did make the following curious assertion. As the star radiates energy, its internal temperature must decrease (since the internal energy is decreasing). This will disturb the delicate balance between the gravitational force and the pressure force (According to thermodynamics, *the pressure of a gas is just two-thirds of its energy density*). Consequently, gravity will gain an upper hand. The star will therefore have no option but to contract. But this works to compress the gas even more, and the gas will get hotter and hotter. *So we have the paradox that as the star radiates energy, it will get hotter!* This is surely a violation of the laws of thermodynamics. As you know, any body that is capable of coming to thermal equilibrium with its surroundings must get *cooler* as it radiates heat to the surrounding. But the above mentioned paradoxical behaviour is what Lane's model predicts, and there is no escaping from this as long as the gas behaves as a perfect gas, and if the energy radiated is coming at the expense of the stored thermal energy. To put it differently, Lane's model star has *negative specific heat*. This discussion naturally leads us to ask the following questions.

The Pressure of an Ideal Gas

Consider an enclosure of volume V containing an ideal gas at a temperature T . Let N be the number of particles in the box. We want to calculate the pressure of the gas. First, what does one mean by an *ideal gas*? What Maxwell taught us is that the particles are in random motion, so that they collide with one another frequently.

Apart from these collisions, the particles can also *interact* with one another through interatomic forces; they may attract or repel one another according to some law. Thus the particles not only have kinetic energy but also potential energy. If the potential energy is negligible compared with the *kinetic energy*, one says that a gas is *ideal*.

$$\text{Kinetic energy} \gg \text{potential energy}$$

A normal gas, such as air, is very nearly ideal because the mean distance between the particles is large compared to the size of the molecules, and this condition is easily satisfied.

According to thermodynamics, the pressure of a gas is equal to two-thirds its energy density (i.e., its internal energy U per unit volume). We have already remarked that the average energy of the particles is $\frac{3}{2}k_B T$. Therefore, the total energy of the gas is $N \times \frac{3}{2}k_B T$. Thus the pressure of the gas is

$$P = \frac{2}{3} \frac{U}{V} = \frac{Nk_B T}{V} = nk_B T, \quad (2.4)$$

where n is the number density of particles. This is *Boyle's law*.

Source of Energy

Why does the Sun shine? And what keeps it shining?

One of the first persons to ponder about this was the English astronomer John Herschel (1792–1871). He was the son of William Herschel, whose name we have already encountered. In 1837, John Herschel exposed a bowl of water to sunshine for a fixed period of time and measured the rise in temperature. This enabled him to estimate the amount of energy radiated by the Sun per unit time. And it was staggering! At first he entertained the idea that the source of energy was some sort of combustion in which chemical energy is converted to heat. But he soon abandoned this idea when he realized that the Sun would not be able to sustain its energy

generation for a long time by consuming chemical fuels. He went on to a guess ‘*If a conjecture might be hazarded, we should look rather to the known possibility of an indefinite generation of heat by friction, or to its excitement by the electric discharge, than to any actual combustion of ponderable fuel, whether solid or gaseous, for the origin of solar radiation*’.

A few years later, in 1846, Julius Mayer (1814–1878), a German physicist, proposed that a continuous bombardment of the Sun by *meteorites* leftover from the time of formation of the solar system, could heat up the Sun. There were grave difficulties with this idea, although initially this idea met with the approval of Lord Kelvin, the high priest of physics at the time.

An alternative explanation was advanced by a Scottish engineer John Waterston. He proposed that gravitational contraction of the Sun at the rate of a hundred metres a year would provide an adequate supply of heat. This idea was picked up by the great German physicist Hermann von Helmholtz since this seemed natural to him. Prior to this, the German philosopher Immanuel Kant had proposed that the solar system formed due to the contraction of a giant cloud of gas (It is incredible that this is, in fact, the modern scenario for the formation of stars and their planets!). Helmholtz felt that this contraction must be continuing still. Lord Kelvin also became convinced of this and abandoned the meteorites hypothesis.

Let us make sure that we are clear about this idea. When a star contracts, matter moves towards the centre of the star; the *difference* in the gravitational potential energy between the old configuration and the new configuration is converted into heat. But there is curious twist to this. Remember what we said earlier. As long as the gas behaves as a *perfect gas* the star must get hotter as it radiates and contracts. So the heat generated as the star contracts must be sufficient not only to replace the heat lost as radiation but also to heat the star to a higher temperature. This is essential, for otherwise the star has no option but to collapse.

Helmholtz and Kelvin estimated that the Sun had been shining for about twenty million years, and will continue to shine for another twenty million years or so. Let us see how one may estimate this timescale. Recall our discussion of the *Virial Theorem* in the previous chapter. According to this theorem, when a star contracts, *only one-half of the gravitational potential energy released is available for radiation*. The other half is stored as thermal energy. The gravitational potential energy of the Sun is $\sim -2GM_{\odot}^2/R_{\odot}$. If we divide one-half of this by the rate at which the Sun has been radiating, then we can get an estimate for how long the Sun has been shining.

$$t \sim -\frac{1}{2} \frac{\text{gravitational potential energy}}{\text{luminosity}} \sim \frac{GM_{\odot}^2/R_{\odot}}{L_{\odot}} \quad (2.5)$$

This is just like estimating how long the money in your bank account will last; you have to divide the balance in your account by the rate at which you are spending the money. The present rate at which the Sun is losing energy is the *luminosity* of the Sun [$L_{\odot} = 4 \times 10^{33} \text{ erg s}^{-1}$]. Inserting the values for the mass and radius of the Sun, we come to the conclusion that if the Sun had been radiating at the present luminosity,

then it could have done so only for about 20 million years [Convince yourself of this by substituting the values]. This seemed a comfortably long time for Lord Kelvin. Even though the geologists were convinced (even at that time) that the Earth was older than 20 million years, Lord Kelvin was not bothered. He used his status to tell the geologists to confine themselves to this timescale!

But he should have been bothered. Most of the stars we see in the sky with the naked eye are far more luminous than the Sun; they radiate a hundred or a thousand times more than the Sun. If he had used the same argument, he would have come to the conclusion that these luminous stars were born only 100,000 years ago. He would then have had to wonder whether, ‘the antiquity of man is greater than that of the stars shining’, as Eddington put it.

The discovery of radioactivity was the last nail in the coffin for the contraction hypothesis. Using modern techniques, geologists were able to determine the age of the older rocks, and this turned out to be *more than a billion years*. This is how they did it. Uranium is seen to disintegrate into lead and helium at a known rate. Chemically, uranium and lead are very dissimilar; therefore they are unlikely to be deposited together. So if you find both uranium and lead in rocks, you can assume that the lead was formed due to the radioactive disintegration of uranium. Since one knows the rate at which this disintegration takes place, from the ratio of lead-to-uranium one can estimate how long ago the rock formed. Now, if the earth itself is several billion years old, the Sun must be even older than this. So we are back to square one as far as the source of energy in the Sun and the stars. But one can say this. If external source of energy (such as meteorite), as well as gravitational contraction, are ruled out then the *star must contain some hidden source of energy which enables it to shine for billions of years*.

Sir Arthur S. Eddington provided the breakthrough. Addressing the British Association in Cardiff on 24 August 1920, Eddington argued that only *subatomic energy* is available in unlimited quantity. To quote Subrahmanyan Chandrasekhar, ‘This address contains some of the most prescient statements in all of astronomical literature’. Eddington’s remarks are so bold and incredibly brilliant that I shall quote in full the relevant part of his address.

Only the inertia of tradition keeps the contraction hypothesis alive—or rather, not alive, but an unburied corpse. But if we decide to inter the corpse, let us frankly recognize the position in which we are left. A star is drawing on a vast reservoir of energy by means unknown to us. This reservoir can scarcely be other than the subatomic energy which, it is known, exists abundantly in all matter; we sometime dream that man will one day learn how to release it and use it for his service. The store is well-nigh inexhaustible, if only it could be tapped. There is sufficient in the Sun to maintain its output of heat for 15 billion years. . . .

Aston has further shown conclusively that the mass of the helium atom is even less than the sum of the masses of the four hydrogen atoms which enter into it—and in this, at any rate, the chemists agree with him. There is a loss of mass in the synthesis amounting to 1 part in 120, the atomic weight of hydrogen being 1.008 and that of helium just 4. I will not dwell on his beautiful proof of this, as you will no doubt be able to hear it from himself. Now mass cannot be annihilated, and the deficit can only represent the mass of the electrical energy set free in the transmutation. We can therefore at once calculate the quantity of energy liberated when helium is made out of hydrogen. If 5 per cent of a star’s mass consists initially of hydrogen atoms, which are gradually being combined to form more complex elements, the total heat

liberated will more than suffice for our demands, and we need look no further for the source of star's energy.

If, indeed, the subatomic energy in the stars is being freely used to maintain their great furnaces, it seems to bring a little nearer to fulfilment our dream of controlling this latent power for the well being of the human race—or for its suicide.

Reproduced from *Observatory*, **43**, 353–5, 1920.

To appreciate how extraordinarily prescient these predictions were we have to transform ourselves to the year 1920. At the time, our knowledge of the atom and its nucleus was very rudimentary. Electrons and protons were the only known elementary particles at that time. Ernest Rutherford had demonstrated that radioactive substances emitted three types of radiation, which he called *alpha*, *beta* and *gamma radiation*. Beta radiation turned out to be streams of electrons, while *gamma radiation* showed all the characteristics of electromagnetic radiation. By a series of careful experiments, Rutherford was able to demonstrate that the alpha radiation was nothing but the ions of helium atom; they were positively charged with a charge equal to twice the charge of the electron. After this came the brilliant experiments of Rutherford and Soddy. Soon they could draw definite conclusions regarding the atomic mass, or atomic weight, of the alpha particles. The helium atom or alpha particle had four units of atomic weight.

Rutherford then embarked on the now famous *scattering experiments*, in which he bombarded matter with alpha particles. Based on the results of these experiments, Rutherford was able to give a general picture of an atom—a *nuclear atom*. The atom consisted of a minute nucleus, containing practically all the mass of the atom, and a positive charge equal to the atomic number times the magnitude of the electronic charge. For electrical neutrality, the nucleus would have to be surrounded by a number of electrons equal to the atomic number. This deduction of the nuclear nature of the atom was one of the most important discoveries in the entire history of physics. Barely two years later, in 1913, Niels Bohr published his *theory of the hydrogen atom*.

By the year 1920, Rutherford and his brilliant students at the Cavendish Laboratory in Cambridge had succeeded in the artificial disintegration of the atoms. Around the same time, one of his students, Aston, invented the *mass spectrograph*. Using this Aston was able to measure the masses of the atoms. If we take the mass of the hydrogen atom (1.008 units) to be the unit with which to measure the masses of other nuclei, the mass of the helium atom would be exactly 4, and that of oxygen, 16.

This suggested to Eddington that the helium atom must have *formed* by the combination of four hydrogen atoms. But how could this happen? Although the mass of the helium atom made this hypothesis plausible, the question remained, 'is it possible'? How does one account for the fact that the atomic number (or charge) of the helium nucleus is only two? No one knew the answers to these questions, not even Eddington. He simply packed four protons and two electrons into the helium nucleus; this would fix the problem with the charge! Despite all this confusion, he attached supreme significance to Aston's discovery that *the mass of the helium nucleus is less than four times the mass of the hydrogen nucleus*. The energy equivalent of this

deficit mass, which we now call the binding energy, is presumably radiated away. According to Eddington, *this is the source of the energy that makes the stars shine*.

Neither the physicists nor the chemists, leave alone the astronomers, thought much of all this. To them, Eddington replied as follows.

To my mind the **existence** of helium is the best evidence we could desire of the possibility of the **formation** of helium. The four protons and two electrons constituting its nucleus must have been assembled at some time and place; and why not in the stars? When they were assembled the surplus energy must have been released, providing a prolific supply of heat. Prima facie this suggests the interior of a star as a likely locality, since undoubtedly a prolific source of heat is there in operation. I am aware that many critics consider the conditions in the stars not sufficiently extreme to bring about the transmutation—the stars are not hot enough. The critics lay themselves open to an obvious retort; we tell them to go and find a **hotter place**.

Well, once upon a time there was a hotter place! That was the very-early universe. But that is a different story, and we shall discuss that much later in our series of monographs. As to how the story of the energy production in the stars unfolded, we shall return to this in Chap. 5. In the meantime, let us go back to Lane's idea and continue with our discussion of the equilibrium and stability of the stars.

We have already discussed the condition for *hydrostatic or mechanical equilibrium*:

$$\frac{dP}{dr} = -\frac{GM(r)\rho}{r^2}. \quad (2.6)$$

To proceed further, we have to supplement this with an equation describing the *thermal equilibrium* of the star. Thermal equilibrium requires that the temperature distribution is capable of maintaining itself automatically even though there is a continuous transfer of heat from one part of the star to another. How does energy in the form of radiant heat flow from the interior to the surface from where it escapes into space? In principle, there are three modes of transfer of heat, viz. *conduction*, *convection*, and *radiation*. Lane considered convective transport in which heat is transported from one region to another by actual movement of material. This is what happens in our atmosphere. If convective transport is operative, then the star cannot possibly be in mechanical equilibrium.

If we discard this mechanism, we are left with *conduction* and *radiation*. It turns out that thermal conductivity of stellar matter is too small to be effective in transporting the radiant energy from the interior to the surface. That leaves us with radiation as the only mode of transfer of heat. The idea of *radiative equilibrium*, in which heat is transferred by radiation itself and the temperature distribution is controlled by the flow of radiation, was first invoked by R. A. Sampson in 1894. But a full theory had to wait till the subject of *radiative transfer* was developed by the great German physicist and astronomer, Karl Schwarzschild, in a fundamental paper published in 1906. We shall discuss this in Chap. 3.



Sir Arthur Stanley Eddington
(28 December 1882–22 November 1944)

Eddington was born in Kendal, England, the son of Quaker parents. While at school, he proved to be a brilliant scholar particularly in mathematics and English literature. His performance earned him a scholarship to Owens College, Manchester in 1898. Eddington was greatly influenced by his physics and mathematics teachers, *Arthur Schuster* and *Horace Lamb*. His progress was rapid, winning him several scholarships and he graduated with a B.Sc. in physics with First Class Honours in 1902.

Based on his performance at Owens College, he was awarded a scholarship at Trinity College in the University of Cambridge 1902. Two years later, Eddington became the first ever second-year student to be placed as **Senior Wrangler**—the highest academic distinction for undergraduates. After receiving his B.A. in 1905, he began research on *Thermionic Emission* in the Cavendish Laboratory. This did not go well for him, and he spent time teaching mathematics to first year engineering students, without much satisfaction. But fortunately, this unsatisfactory period was brief!

In 1907, he won the prestigious Fellowship of Trinity College, Cambridge. In December 1912, George Darwin, son of Charles Darwin, died suddenly and Eddington was promoted to his chair as the Plumian Professor of Astronomy and Experimental Philosophy in early 1913. Later that year Eddington was named Director of the Cambridge Observatory. He was elected a Fellow of the Royal Society shortly after.

During World War I, Eddington became embroiled in controversy within the British astronomical and scientific communities. Being a Quaker, Eddington was a pacifist. He struggled to keep wartime bitterness out of astronomy.

He repeatedly called for British scientists to preserve their pre-war friendships with German scientists. Eddington's pacifism caused severe difficulties during the war, especially when he was called up for military service in 1918. He claimed conscientious objector status, a position recognized by the law, but despised by the public. In 1918 the government sought to revoke this deferment, and only the timely intervention of the Astronomer Royal and other high-profile figures kept Eddington out of prison.

Relativity

Eddington was famous for his work in *General Theory of Relativity* which was published by *Einstein* in 1916. During World War I, Eddington was the **Secretary of the Royal Astronomical Society**, which meant he was the first to receive a series of letters and papers from Willem de Sitter regarding Einstein's General Theory of Relativity. Eddington was fortunate in being not only one of the few astronomers with the mathematical skills to understand this theory, but (owing to his international and pacifist views) one of the few at the time who was still interested in pursuing a theory developed by a German physicist! World War I severed many lines of scientific communication and new developments in German science were not well known in England. He quickly became the chief supporter and expositor of Theory of Relativity in Britain. Eddington wrote a number of articles which announced and explained Einstein's theory to the English-speaking world. He also became known for his popular expositions and interpretations of the theory.

He and Astronomer Royal Frank Dyson organized two expeditions to make observations on a solar eclipse in 1919 to conduct the first empirical test of Einstein's theory: *the measurement of the deflection of light by the Sun's gravitational field*. Dyson argued that Eddington's expertise was indispensable for this most important expedition. It was this argument, and the powerful connections that Dyson had, that allowed Eddington to escape prison during the war!

After the war, Eddington travelled to the island of Príncipe near Africa to observe the solar eclipse of 29 May 1919. During the eclipse, he took photographs of the stars in the region around the Sun. According to the theory of General Relativity, stars near the Sun would appear to have been slightly shifted because their light had been curved by Sun's gravitational field. This effect is noticeable only during an eclipse, since otherwise the Sun's brightness obscures the stars.

Eddington's observations, published the next year, confirmed Einstein's theory, and were hailed at the time as a conclusive proof of General Relativity. The news was reported all over the world as a major story. Afterwards,

Eddington embarked on a campaign to popularize relativity and the expedition as landmarks both in scientific development and international scientific relations.

Throughout this period Eddington lectured on relativity, and was particularly well known for his ability to explain the concepts in lay terms as well as scientific. He collected many of these into the publication, *Mathematical Theory of Relativity*, in 1923, which Albert Einstein suggested was ‘the finest presentation of the subject in any language’.

Fundamental Theory

During the 1920s, until his death, he increasingly concentrated on what he called the *Fundamental Theory*, which was intended to be a unification of quantum theory, relativity, and gravitation. At first, he progressed along ‘traditional’ lines, but turned increasingly to an almost numerological analysis of the dimensionless ratios of fundamental constants.

His basic approach was to combine several fundamental constants in order to produce a dimensionless number. In many cases, these would result in numbers close to 1040, its square, or its square root. He was convinced that *the mass of the proton and the charge of the electron were a natural and complete specifications for constructing a Universe* and that their values were not accidental. One of the discoverers of quantum mechanics, Paul Dirac, also pursued this line of investigation, which has come to be known as the Dirac large numbers hypothesis, and some scientists even today believe it has something to it.

Theory of the Stars

Eddington’s most important contributions were, however, concerning the nature of the stars. As we shall see in this volume, his insights were deep and his conjectures extraordinarily prescient. But he also made several serious errors of judgment. One of them concerned the fate of massive stars. He rejected the spectacular discovery made by young Chandrasekhar, resulting in a major controversy. We shall discuss this at great length in the next volume of this series. But this most unfortunate controversy did not affect Chandrasekhar’s opinion of Eddington. In a lecture delivered at Cambridge University to commemorate Eddington’s birth centenary, Subrahmanyan Chandrasekhar described Eddington as the ‘most distinguished astronomer of his time’.

Cycling

Eddington was very fond of cycling. He devised a measure of a cyclist's long-distance riding achievements. The Eddington Number in this context is defined as, n , the number of days a cyclist has cycled more than an equal number in distance, namely, n miles. For example an Eddington Number of 30 would imply that a cyclist has cycled more than 30 miles in a day on 30 occasions! Eddington must have been a very good cyclist, and very fond of it. To quote from one of the letters written to S. Chandrasekhar in the evening of his life:

My n is now 77. It made the last jump a few days ago when I took an 80-mile ride in the fen country. I have not been able to go on a cycling tour since 1940, because it is impossible to rely on obtaining accommodations for the night; so my records advance slowly.

Popular and Philosophical Writings

During the 1920s and 1930s, Eddington gave innumerable lectures, interviews, and radio broadcasts on astronomy, relativity, and later, quantum mechanics. Many of these were gathered into books, such as *Stars and Atoms*, *Nature of the Physical World*, and *New Pathways in Science*. His skillful use of literary allusions and humour helped make these famously difficult subjects quite accessible.

Eddington's books and lectures were immensely popular with the public, not only because of Eddington's clear and entertaining exposition, but also his willingness to discuss the philosophical and religious implications of the new physics. His popular writings made him, quite literally, a household name in Great Britain between the two world wars.

What are the Stars?

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