Preface

In our current understanding of subatomic particle physics, recently successfully tested with the discovery of the Higgs boson, processes involving elementary particles are described by quantum field theory. Scattering amplitudes are the central objects in perturbative quantum field theory as they link the theoretical description to experimental predictions. To a large degree the historical development of quantum field theory was driven by the need to compute scattering amplitudes. They constitute the integral building block for the construction of scattering cross sections determining the probabilities for scattering processes to occur at particle colliders. Non-abelian gauge field or Yang-Mills theories represent the backbone of high energy physics, as they provide the theoretical framework to describe the interactions of elementary particles in the standard model. The principles for computing scattering amplitudes in gauge theories have been settled since the mid 1970s: once the Lagrangian and its gauge fixing term has been constructed, one reads off the Feynman rules for the scalar, fermion and gauge fields. The scattering amplitude is then given by the sum of all contributing diagrams built from the Feynman rules with the subsequent integration over the internal loop momenta. Divergences in these integrals require regularization, and the ultraviolet divergences lead to a renormalization of the theory. So from a conceptual viewpoint one might consider this chapter of quantum field theory as complete.

However, it turns out that beyond the simplest examples the complexity of the Feynman diagrammatic computation quickly gets out of hand. Already at tree-level such a computation can become enormous if one is dealing with gauge interactions. For example, the number of Feynman diagrams contributing to a gluon tree-level amplitude $g + g \to n \cdot g$ grows factorially with n [1]. On the other hand, the final answer can be obtained in closed analytic form, and, when expressed in convenient variables, is remarkably simple. There are two complementary ways of understanding the simplicity of the final answer. A reason for the complexity of the Feynman diagram calculation is that individual Feynman diagrams are gauge variant and involve off-shell intermediate states in internal propagators. The amplitude, on the other hand, is gauge invariant and only knows about on-shell degrees of freedom. Hence, in going from Feynman diagrams to an amplitude the unphysical degrees of freedom cancel. On-shell approaches that focus on the analytic structure of the final result allow to circumvent these unnecessary complications. Another reason for the simplicity of the final answer has to do with symmetry properties and is more surprising: it turns out that besides the obvious symmetries of the Lagrangian, gluon amplitudes have additional, hidden symmetries, that constrain their form. Choosing appropriate variables makes the action of these symmetries transparent and thereby also simplifies the expressions. For these reasons, both analyticity and symmetry properties are two topics that will play an important role in these lecture notes.

In the past decade tremendous progress was made both in our understanding of the structure of scattering amplitudes and in the ability of computing the latter in gauge theories at high multiplicities (number of external legs) and beyond the one-loop order. These advances provided the field with a new set of tools that go beyond the textbook approaches. These methods build on using a color decomposition of the gauge theory amplitudes and on expressing them in a spinor helicity basis particularly suited for massless particles. Thinking about the analytic structure of tree-level amplitudes leads to novel on-shell recursion relations. They allow the analytic construction of tree-level amplitudes from atomistic three-point ones. At loop level unitarity-based techniques, combined with the knowledge of an integral basis for one-loop Feynman integrals, may be used to construct loop amplitudes from tree-level amplitudes. In summary, all amplitudes follow from the on-shell three-point vertices, and no reference to the complicated form of the Lagrangian, gauge fixing terms and ghosts is necessary.

Our main focus is on methods based on general principles such as analyticity and symmetries. The progress for generic gauge theories was often led by studies in the maximally supersymmetric gauge theory in four dimensions, $\mathcal{N}=4$ super Yang-Mills. It may be thought of as an idealized version of QCD and provides an ideal laboratory for the development and testing of new approaches. In fact, gluon tree-level amplitudes in both models are identical, but they can be easily solved for within the $\mathcal{N}=4$ super Yang-Mills framework, in part thanks to fascinating hidden symmetries. Discussing this theory in these lecture notes also allows us to have many examples where scattering amplitudes can be computed analytically, and we hope that this will help to illustrate the general properties.

These modern helicity amplitude or on-shell methods are so far not treated in depth in standard textbooks on quantum field theory. Yet they are of increasing importance in phenomenological applications, which face the demand of high precision predictions for high multiplicity scattering events at the Large Hadron Collider, as well as in foundational studies in quantum gauge field theory towards the structure, possible symmetries and construction of the perturbative S-matrix. In fact these developments have led to a cross-fertilization of high-energy phenomenology and formal theory in recent years. This has made the field of scattering amplitudes a very active and fascinating area of research attracting many new researchers and students. The goal of these lecture notes in physics is to serve these communities in bridging the gap from a textbook knowledge of quantum field theory to a working expertise for doing research in the domain of scattering amplitudes. Our aim was to keep these notes compact so that they are suitable for an advanced graduate lecture course. In order to make them more accessible for students, we have included a series of exercises, together with their solutions. Due to our wish to keep the volume of these notes amenable to a lecture course, a certain selection of topics, certainly biased by personal tastes, had to be made. At the end of each chapter we therefore give an account of related developments in the field not discussed in the lecture notes and give references to all relevant works.

The level of the text aims at advanced graduate students who have already followed an introductory course on quantum field theory, typically leading to QED scattering processes and a few simple one-loop computations therein. The basics of non-abelian quantum field theories are briefly reviewed in chapter one of this book. Here also the basic tools of the modern helicity amplitude approach are provided and some simple tree-level diagrams are computed in the conventional way, using color-ordered Feynman rules. Chapter two introduces the on-shell recursion and discusses universal factorization properties for color ordered amplitudes. After a discussion of Poincaré and conformal symmetry, the $\mathcal{N}=4$ super Yang-Mills theory is introduced, along with its super-amplitude formalism and a supersymmetric on-shell recursion relation allowing for an exact solution. In chapter three the loop-level structure of amplitudes is reviewed. Here the reduction of one-loop Feynman integrals to a basis of scalar integrals is discussed. The idea of (generalized) unitarity in constructing one-loop amplitudes from tree-level data by putting various internal legs on-shell is reviewed and a number of concrete examples are computed in detail. In the second part of chapter three we give an introduction to the evaluation of Feynman integrals at one and higher loop order. After reviewing their definition and useful parameter representations, such as the Feynman and Mellin representations, we give an introduction to the integration by parts techniques in conjunction with differential equations. The final chapter is devoted to advanced topics mostly within maximally supersymmetric gauge theory: recursion relations for loop integrands, the duality of scattering amplitudes to Wilson loops with light-like contours and correlation functions of local operators. Finally, the hidden dual conformal and Yangian symmetries of $\mathcal{N}=4$ super Yang-Mills amplitudes are discussed pointing towards a fascinating hidden integrability of this four-dimensional gauge theory.

Some elements of modern on-shell methods are also discussed in the recent quantum field theory textbooks of Srednicki [2] and Zee [3]. Introductory and in-depth reviews exist on the various topics presented in these lecture notes. These are [1, 4–6] on the topics of chapters one and two. The integral reduction and unitarity methods are pedagogically reviewed in [7, 8] from a phenomenological viewpoint. The comprehensive textbook of Smirnov [9, 10] discusses techniques for the computation of Feynman integrals in great detail. As for more formal aspects related to $\mathcal{N}=4$ super Yang-Mills, symmetries and dualities, there exists a special issue of J. Phys. A [11–24], as well as the very recent comprehensive review [25]. These references have overlaps with some of the material presented in these notes, but also discuss supergravity amplitudes, dualities of gauge and gravity amplitudes, amplitudes in twistor space, and the Grassmannian approach to superamplitudes. This is certainly not a complete list but reflects the texts from which we have learned a lot ourselves.

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