

Contents

1. Philosophy and Overview of the Book	1
1.1 Underlying Philosophy	1
1.2 Peculiarities of Style	1
1.3 What This Book Is <i>Really</i> About	2
1.4 Gaussian Processes and the Generic Chaining	3
1.5 Random Fourier Series and Trigonometric Sums, I	5
1.6 Matching Theorems, I	6
1.7 Bernoulli Processes	7
1.8 Trees and the Art of Lower Bounds	7
1.9 Random Fourier Series and Trigonometric Sums, II	8
1.10 Processes Related to Gaussian Processes	8
1.11 Theory and Practice of Empirical Processes	9
1.12 Partition Scheme for Families of Distances	9
1.13 Infinitely Divisible Processes	10
1.14 The Fundamental Conjectures	10
1.15 Convergence of Orthogonal Series; Majorizing Measures	10
1.16 Matching Theorems II: Shor's Matching Theorem	11
1.17 The Ultimate Matching Conjecture in Dimension ≥ 3	11
1.18 Applications to Banach Space Theory	11
1.19 Appendix B: Continuity	12
Reference	12
2. Gaussian Processes and the Generic Chaining	13
2.1 Overview	13
2.2 The Generic Chaining	13
2.3 Functionals	32
2.4 Gaussian Processes and the Mysteries of Hilbert Space	40
2.5 A First Look at Ellipsoids	51
2.6 Proof of the Fundamental Partitioning Theorem	55
2.7 A General Partitioning Scheme	61
2.8 Notes and Comments	70
References	73

3. Random Fourier Series and Trigonometric Sums, I	75
3.1 Translation Invariant Distances	75
3.2 The Marcus-Pisier Theorem	77
3.3 A Theorem of Fernique	86
3.4 Notes and Comments	88
References	89
4. Matching Theorems, I	91
4.1 The Ellipsoid Theorem	91
4.2 Matchings	100
4.3 The Ajtai, Komlós, Tusnády Matching Theorem	106
4.4 The Leighton-Shor Grid Matching Theorem	115
4.5 Notes and Comments	125
References	127
5. Bernoulli Processes	129
5.1 Boundedness of Bernoulli Processes	129
5.2 Chaining for Bernoulli Processes	131
5.3 Fundamental Tools for Bernoulli Processes	141
5.4 Control in ℓ^∞ Norm	146
5.5 Latała's Principle	149
5.6 Chopping Maps and Functionals	152
5.7 The Decomposition Lemma	161
5.8 Notes and Comments	170
References	170
6. Trees and the Art of Lower Bounds	173
6.1 Introduction	173
6.2 Trees	173
6.3 A Toy Lower Bound	180
6.4 Lower Bound for Theorem 4.3.2	183
6.5 Lower Bound for Theorem 4.4.1	192
Reference	197
7. Random Fourier Series and Trigonometric Sums, II	199
7.1 Introduction	199
7.2 Families of Distances	200
7.3 Statement of Main Results	203
7.4 Proofs, Lower Bounds	206
7.5 Proofs, Upper Bounds	212
7.6 Proofs, Convergence	221
7.7 Explicit Computations	227
7.8 Notes and Comments	231
References	232

8. Processes Related to Gaussian Processes	233
8.1 p -Stable Processes	233
8.2 Order 2 Gaussian Chaos	243
8.3 Tails of Multiple Order Gaussian Chaos	255
8.4 Notes and Comments	269
References	269
9. Theory and Practice of Empirical Processes	271
9.1 Discrepancy Bounds	271
9.2 How to Approach Practical Problems	282
9.3 The Class of Squares of a Given Class	283
9.4 When Not to Use Chaining	303
9.5 Notes and Comments	310
References	310
10. Partition Scheme for Families of Distances	313
10.1 The Partition Scheme	313
10.2 The Structure of Certain Canonical Processes	318
References	330
11. Infinitely Divisible Processes	331
11.1 A Well-Kept Secret	331
11.2 Overview of Results	332
11.3 Rosinski's Representation	344
11.4 The Harmonic Case	348
11.5 Proof of the Decomposition Theorem	354
11.6 Proof of the Main Lower Bound	358
References	369
12. The Fundamental Conjectures	371
12.1 Introduction	371
12.2 Selector Processes	371
12.3 The Generalized Bernoulli Conjecture	372
12.4 Positive Selector Processes	384
12.5 Explicitly Small Events	386
12.6 Classes of Sets	393
References	398
13. Convergence of Orthogonal Series; Majorizing Measures	399
13.1 Introduction	399
13.2 Chaining, I	408
13.3 Proof of Bednorz's Theorem	412
13.4 Permutations	420
13.5 Chaining, II	429
13.6 Chaining, III	443

13.7	Notes and Comments	444
	References	445
14.	Matching Theorems, II: Shor's Matching Theorem	447
14.1	Introduction	447
14.2	The Discrepancy Theorem	448
14.3	Decomposition of Functions of \mathcal{H}	453
14.4	Discrete Fourier Transform	461
14.5	Main Estimates	464
14.6	Proof of Proposition 14.2.4	472
14.7	Notes and Comments	474
	References	474
15.	The Ultimate Matching Theorem in Dimension ≥ 3	475
15.1	Introduction	475
15.2	The Crucial Discrepancy Bound	480
15.3	Cleaning up φ	484
15.4	Geometry	488
15.5	Probability, I	494
15.6	Haar Basis Expansion	500
15.7	Probability, II	506
	References	513
16.	Applications to Banach Space Theory	515
16.1	Cotype of Operators from $C(K)$	515
16.2	Computing the Rademacher Cotype-2 Constant	527
16.3	Classifying the Elements of B_1	533
16.4	1-Unconditional Bases and Gaussian Measures	536
16.5	Restriction of Operators	546
16.6	The $\Lambda(p)$ -Problem	554
16.7	Proportional Subsets of Bounded Orthogonal Systems	561
16.8	Embedding Subspaces of L^p into ℓ_N^p	572
16.9	Gordon's Embedding Theorem	588
16.10	Notes and Comments	591
	References	592
A.	Appendix: What This Book Is <i>Really</i> About	595
A.1	Introduction	595
A.2	The Kolmogorov Conditions	595
A.3	More Chaining in \mathbb{R}^m	597
A.4	The Garsia-Rodemich-Rumsey Lemma	598
A.5	Chaining in a Metric Space	599
A.6	Two Classical Inequalities	601

B. Appendix: Continuity	607
B.1 Introduction.....	607
B.2 Continuity Under Metric Entropy Conditions	607
B.3 Continuity of Gaussian Processes	613
References	617
Index	625

<http://www.springer.com/978-3-642-54074-5>

Upper and Lower Bounds for Stochastic Processes

Modern Methods and Classical Problems

Talagrand, M.

2014, XV, 626 p., Hardcover

ISBN: 978-3-642-54074-5