# Environmental Policy in a Dynamic Model with Heterogeneous Agents and Voting

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**Abstract** We consider a population of infinitely-lived households split into two: some agents have a high discount factor (the patients), and some others have a low one (the impatients). Polluting emissions due to economic activity harm environmental quality. The governmental policy consists in proposing households to vote for a tax to maintain environmental quality. By studying the voting equilibrium at steady states we show that the equilibrium maintenance level is the one of the median voter. We also show that (i) an increase in total factor productivity may produce effects described by the Environmental Kuznets Curve, (ii) an increase in the patience of impatient households may foster environmental quality if the median voter is impatient and maintenance positive, finally (iii) a decrease in inequality among the patient households leads to an increase in environmental quality if the median voter is patient and maintenance is positive. We show that, when the median income of the median voter is lower than the mean (which is empirically founded),

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our model with heterogeneous agents predicts a lower level of environmental quality than what the representative agent model would predict, and that increasing the public debt decreases the level of environmental quality.

## 1 Introduction

With the growing importance of global environmental issues, such as global warming, and the emphasis put on the general question of sustainable growth and development, environmental policies and their financing have become a major subject of concern in many developing or developed countries. As a response, economic theory, and especially in macro-economics, elaborated dynamic models based on the representative agent assumption to disentangle the nexus between economic growth and pollution, or more generally environmental quality (see among many others, Gradus and Smulders 1993; Stokey 1998, or Xepapadeas 2005). Though, it is striking to notice that the public debate about environmental policies and their financing very often focus on the distributive aspects of the policies, and more precisely on the distribution of their burden among heterogenous agents. To capture that dimension, economists must get rid of the representative agent and must start considering heterogeneous agents in their macrodynamic models. There exist several ways of introducing heterogeneity, e.g. in wealth (Kempf and Rossignol 2007), in individual labor productivity (Jouvet et al. 2008), or in age with overlapping generations (John and Pecchenino 1994; Jouvet et al. 2008).

In this paper we consider heterogeneity in agents' discount factors.[1] We assume that the population is exogenously divided into two groups, one with patient households and the other with impatient households. Each individual votes in favor, or against a public policy for environmental maintenance. Maintenance is a public policy financed by a tax on households, and pollution flows from firm's activity. We define a voting equilibrium and the related general equilibrium of the economy at the steady state.

Our main results can be summarized as follows. First, if the policy choice were one-dimensional (i.e. static with one homogeneous agent) then the median-voter theorem would straightforwardly apply. Unfortunately, it cannot be applied in our dynamic multidimensional. We show that, at the steady state, a voting equilibrium coincides with the solution the one that would result from the median voter theorem. We thus provide a logically consistent definition of the median voter theorem in a dynamic setting. This establishes the applicability of the median voter theorem on steady state equilibria. This is an important theoretical result because the current literature always *assumes* that the median voter theorem can be applied after the steady state is defined, though the steady state equilibrium should itself depend on the voting outcome (see e.g. Kempf and Rossignol 2007; Corbae et al. 2009).

[1] For a general survey of the literature on models of economic growth with consumers having different discount factors, see Becker (2006).

Our theoretical contribution is to prove that a dynamic voting equilibrium coincides with the application of the median voter theorem. Furthermore, to highlight the advantages of considering heterogenous agents, we compare our results with what the representative agent framework would provide. The results differ in many respects.

Beyond the theoretical aspects, we also contribute to the literature on political economy and environmental policy. With some comparative statics, we show many novel results. We first show that, if the median voter is impatient, she consumes all her revenue, and the maintenance level is zero. But if the median voter is patient, then maintenance is positive but not uniquely determined. Then we go further and stress that there exist two channels through which discount factors shape agents' choices on maintenance, a direct one and an indirect one. In our model, the higher the agents' discount factor, the larger is her desired level of maintenance (it is the direct channel). But in the same time, the richer the agent, the stronger her desired level for maintenance. It is well-established in the literature that only agents with a high discount factor have positive savings in the long run. Those with a low discount factor save nothing. Thus, the former become wealthy in the long run and are prone to ask for high levels of environmental maintenance. In the meantime the latter become poorer and ask for lower levels of maintenance (it is the indirect equilibrium channel). Combining these two channels provides us with new results about the relationship between economic development and environmental quality through the voting process, i.e. a new rationale for the so-called Environmental Kuznets Curve (see e.g. Dasgupta et al. 2002; Prieur 2009). As far as inequality among agents is concerned, we also show that when the median voter is patient, then reducing inequality has a positive effect on environmental quality.

Actually, this discussion also relates to the broad debate about how discounting impacts the choice of environmental policies.[2] Although discounting is generally considered as a normative issue, it also has a positive content, as stressed by Dasgupta: "discount rates on consumption changes combine *values* with *facts*. Dasgupta (2008, p. 144) or by Arrow et al. (1995) who distinguishes *prescriptive* and *descriptive* positions. In environmental economics, a high discount factor leads to modest and slow environmental maintenance levels, while a low discount rate leads to immediate and strong action. The common characteristics in all this literature is to rely on the assumption that there exists a *representative agent* in the economy whose preferences are considered as given by a benevolent social planner. This agent further acts as a benevolent social planner.[3] We depart from the representative agent hypothesis by considering an economy populated with heterogeneous agents. Then, we are able to provide a microeconomic rationale to determine the implicit global discount rate in this economy. This departs from the normative discussion on what the discount rate *should* be. In our analysis we take heterogeneous agents' preferences beforehand and we scrutinize how heterogeneity shapes the policy in the

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[2]Recently this debate has experienced a strong revival after the publication of the Stern Review (Stern 2006, 2008). Prominent economists have contributed to the debate, like Dasgupta (2008), Nordhaus (2008) or Weitzman (2007).

[3]Or the *social evaluator*, to take Dasgupta's words.

global economy. This is a novel contribution to the debate on discounting based on a positive approach.

Applying the median voter theorem to dynamic models requires a suitable analytical redesign of the political settings in this model. Models of such a kind are much harder to analyze than their static counterparts or than the usual intertemporal models without political ingredients. There is a growing interest in the recent literature on the analysis of the performance of majoritarian settings in dynamic frameworks, see e.g. Baron (1996), Krusell et al. (1997), Cooley and Soares (1999), Rangel (2003) and Bernheim and Slavov (2009). The stage of development of the theory is still in its infancy. In particular, there is no consensus about how to model dynamic majoritarian voting. Without going into detail in this introduction, it should be stressed that our approach to voting is different from the approaches used in the above-mentioned papers. We propose a novel definition of voting equilibrium, which is related to Kramer-Shepsle equilibrium concept (Kramer 1972; Shepsle 1979). This definition will allow us to provide new theoretical results about voting equilibrium in a dynamical setting.

These results also yield a discussion about alternative financing schemes of the environmental maintenance policy. We look at the different impacts on heterogenous households and especially on the median voter, of financing maintenance both with taxes and with issuance of public bonds. We show that, under common assumption about income distribution, an increase in the public debt leads to a lower environmental quality.

The paper is organized as follows. In Sect. 2 we present the model, define the competitive equilibria and describe steady-state equilibria for a given policy. In Sect. 3 we endogenize the voting procedure on environmental maintenance, define the intertemporal and steady state voting equilibria, and show the logical consistency between the median voter theorem and the voting equilibrium in dynamic general equilibrium. In Sect. 5 the comparison with the representative agent framework is proposed. In Sect. 4 we perform comparative statics exercises to analyze how environmental quality is impacted by an increase in total factor productivity, an increase in patience, and a decrease in inequality. The discussion about the impact of public debt on the environmental quality is carried out in Sect. 6. Section 7 is the conclusion.

## 2 The Model

Our objective is to define and to study the intertemporal competitive equilibria with voting on maintenance. We define voting equilibria in two steps. In this section, we do the first step as we determine the competitive equilibrium production and consumption paths for a given maintenance policy. The second step will be presented in the next section where, among these competitive equilibria, the ones for which a voting equilibrium exists will be selected. We use a discrete-time framework of infinitely-lived consumers who inelastically supply one unit of labor at each time period, with a representative polluting firm and a global public bad, a stock pollution.

## 2.1 Production and Pollution

Output is determined by means of a production function $aF(K_t, L_t) = Laf(k_t)$, where $a$ is total factor productivity, $K_t$ and $L_t$ are capital and labor at time $t$, $k_t = K_t/L$ is capital intensity, $f(k) = aF(k, 1)$ is the production function in intensive form. Capital is assumed to fully depreciate each period. Output can be used for consumption, investment or environmental maintenance. For the sake of simplicity we will forget about the total factor productivity TFP $a$ until Sect. 4 where it really becomes useful. The dynamics of capital is given by

$$K_{t+1} = F(K_t, L_t) - C_t - M_t,$$

where $C_t$ is aggregate consumption and $M_t$ is aggregate maintenance. The pollution flow, $P_t$, is proportional to output:

$$P_t = \lambda F(K_t, L) = \lambda Lf(k_t), \quad \lambda > 0. \tag{1}$$

Let $Q_t$ be an index of environmental quality defined as $\bar{Q} - S_t$, where $\bar{Q}$ is some pre-industrial (prior to global warming) quality level and $S_t$ is the cumulative pollution stock at time $t$. The dynamics of $Q_t$ is given by the following function:

$$Q_{t+1} = \Psi\left(Q_t - \kappa P_t + \frac{M_t}{\mu}\right), \tag{2}$$

where $\Psi : \mathrm{R}_+ \to \mathrm{R}_+$ is a concave increasing function, $\kappa > 0$ and $\mu > 0$ are two exogenously given coefficients (for dimensional issues). Because the "marginal environmental productivity" of maintenance ($\partial Q_{t+1}/\partial M_t = \Psi'(\cdot)/\mu$) depends negatively on $\mu$, one may interpret $1/\mu$ as the environmental efficiency of maintenance. Let $\bar{Q}$ be a unique positive solution to the following equation: $\Psi(Q) = Q$, i.e. the stationary value of environmental quality with no pollution and no maintenance. For example, the following specifications of $\Psi(X)$ can be used: $\Psi(X) = X^\nu \bar{Q}^{1-\nu}$, with $0 < \nu < 1$, or $\Psi(X) = \nu X + (1-\nu)\bar{Q}$, with $0 < \nu < 1$. Let $\Phi(\cdot) = \Psi^{-1}(\cdot)$. We can rewrite (2) as:

$$\mu\Phi(Q_{t+1}) = \mu(Q_t - P_t) + M_t.$$

It should be noticed that $\mu\Phi'(Q)$ is the marginal cost of quality improvement.

The representative firm maximizes its profit $\pi_t$ under the constraint of the technology $F(K_t, L_t)$ by choosing $K_t$ and $L_t$ and by taking real wage ($w_t$) and interest rates ($r_t$) as given. The firm's problem reads:

$$\max_{K_t, L_t} \pi_t = F(K_t, L_t) - (1 + r_t)K_t - w_t L_t. \tag{3}$$

The first-order conditions are $F'_K(K_t, L_t) = 1 + r_t$ and $F'_L(K_t, L_t) = w_t$, or in intensive terms: $f'(k_t) = 1 + r_t$ and $f(k_t) - f'(k_t)k_t = w_t$.

## 2.2 The Consumers

Population consists of $L$ consumers, with $L$ an integer and odd. Each consumer is endowed with one unit of labor force. The objective function of consumer $i$ is:

$$\sum_{t=0}^{\infty} \beta_i^t \big[u(c_t) + v(Q_t)\big],$$

where $c_t$ is consumption at time $t$ and $\beta_i$ is a discount factor. Let us assume that $u(c)$ and $v(Q)$ satisfy the following conditions:

$$\begin{aligned} &u'(c) > 0, \qquad u''(c) < 0, \qquad u'(0) = \infty, \\ &v'(Q) > 0, \qquad v''(Q) < 0, \qquad v'(0) = \infty. \end{aligned}$$

Each consumer $i$ is either patient ($\beta_i = \beta^h$) or impatient ($\beta_i = \beta^l$), with $0 < \beta^l < \beta^h < 1$. The set of patient consumers (with a discount factor equal to $\beta^h$) is $H_h$, and the set of impatient consumers (those with $\beta^l$) is $H_l$. Consumers pay a tax $m_t = M_t/L$ to finance the public provision of environmental maintenance. The budget constraint of a consumer at time $t$ is thus:

$$\begin{aligned} c_t + s_t + m_t &\le w_t + (1 + r_t)s_{t-1}, \\ c_t \ge 0,\ s_t &\ge 0, \end{aligned} \tag{4}$$

where $w_t$ is the wage rate, $r_t$ is the interest rate, and $s_t$ are her savings at time $t$.[4]

Consumers' utility depends on variables on which she has full control ($c_t$ and $s_t$) but also on maintenance $m_t$, which will be determined by voting (yet to be introduced). At this stage, the result of voting is taken as given by the agents. Hence, we need to solve the consumer's program (to choose the optimal values for $c_t$ and $s_t$, $\forall t$), considering $m_t$ as given.

Suppose that at time $\tau$ consumer $i$ is given her predetermined level of savings $\hat{s}^i_{\tau-1}$, the predetermined level of environmental quality $\hat{Q}_\tau$, the stream of pollution $(P_t)_{t=\tau}^{\infty}$ and some maintenance policy which is represented by a sequence $\mathbf{m} = (m_t)_{t=0}^{\infty}$ of non-negative numbers. The problem $\mathcal{P}_1(\tau)$ of this consumer reads as follows:

$$\max_{(c_t)_{t=\tau}^{+\infty}, (s_t)_{t=\tau}^{+\infty}} \sum_{t=\tau}^{\infty} \beta_i^t \big[u(c_t) + v(Q_t)\big]$$

subject to:

$$\begin{aligned} &\mu\Phi(Q_{t+1}) = \mu(Q_t - \kappa P_t) + Lm_t, \quad t = \tau, \tau+1, \\ &c_t + s_t + m_t \le w_t + (1 + r_t)s_{t-1}, \quad t = \tau, \tau+1, \\ &\quad s_{\tau-1} = \hat{s}^i_{\tau-1},\ Q_\tau = \hat{Q}_\tau, \\ &\quad c_t \ge 0, s_t \ge 0,\ t = \tau, \tau+1. \end{aligned}$$

[4]Consumers are forbidden to borrow against their future labor income. Hence, their savings must be non-negative.

It must be noticed that, since $\mathbf{m} = (m_t)_{t=0}^{\infty}$ is given, the sequence $(Q_t)_{t=\tau}^{+\infty}$ is in fact predetermined by $\hat{Q}_\tau$ and $\mathbf{m}$. Hence, the utility consumer $i$ derives from environmental quality, $\sum_{t=\tau}^{\infty} \beta_i^t v(Q_t)$, does not depend on her choice. In what follows, when we will define voting equilibrium, it will be key to keep in mind that, if $(s_{t-1}^{i*}, c_t^{i*}, Q_t^*)_{t=0}^{\infty}$ is a solution to problem $\mathcal{P}_1(0)$, then $(s_{t-1}^{i*}, c_t^{i*}, Q_t^*)_{t=\tau}^{\infty}$ is also a solution to problem $\mathcal{P}_1(\tau)$ at $\hat{s}_{t-1}^i = s_{t-1}^{i*}$ and $\hat{Q}_t = Q_t^*$.

## *2.3 Competitive Equilibrium Paths and Steady-State Equilibria*

Let at time 0 the environmental policy be represented by some given sequence $\mathbf{m} = (m_t)_{t=0}^{\infty}$ of non-negative numbers. Let an initial state $\{(\hat{s}_{-1}^i)_{i=1}^L, \hat{k}_0, \hat{Q}_0\}$ also be given. Here, $\hat{s}_{-1}^i \geq 0$ stand for the initial savings of consumers $i = 1, \ldots, L$, $\hat{k}_0 > 0$ is the initial per capita stock of capital ($\sum_{i=1}^L \hat{s}_{-1}^i = L\hat{k}_0$), and $\hat{Q}_0 > 0$ is the initial value of environmental quality.

**Definition 1** (Competitive equilibrium path) Given $\mathbf{m}$, the sequence $\mathcal{E}^{\mathbf{m}} = \{k_t^*, 1 + r_t^*, w_t^*, (s_{t-1}^{i*}, c_t^{i*})_{i=1}^L, P_t^*, Q_t^*\}_{t=0}^{\infty}$ is called a *competitive equilibrium path* starting from $\{(\hat{s}_{-1}^i)_{i=1}^L, \hat{k}_0, \hat{Q}_0\}$ if:

1. capital and labor markets clear at the following prices: $1 + r_t = 1 + r_t^* = f'(k_t^*)$, $w_t = w_t^* = f(k_t^*) - f'(k_t^*)k_t^*$, $t = 0, 1, \ldots$;
2. for each household $i = 1, \ldots, L$ the sequence $(s_{t-1}^{i*}, c_t^{i*}, Q_t^*)_{t=0}^{\infty}$ is a solution to problem $\mathcal{P}_1(0)$ at $1 + r_t = 1 + r_t^*$, $w_t = w_t^*$, $t = 0, 1, \ldots$;
3. $\sum_{i=1}^L s_{t-1}^{i*} = Lk_t^*$, $t = 0, 1, \ldots$;
4. $P_t^* = \lambda L f(k_t^*)$, $t = 0, 1, \ldots$;
5. $\mu\Phi(Q_{t+1}^*) = \mu(Q_t^* - \kappa P_t^*) + Lm_t$, $t = 0, 1, \ldots$.

Notice that, at each time $t$, maintenance $m_t$ is given and smaller than the wage rate $w_t$. We will not discuss the existence of equilibrium paths. Our main emphasis is on steady-state equilibria.

**Definition 2** (Competitive steady state equilibrium) Let an $m \geq 0$ be given and let $\mathbf{m} = (m_t)_{t=0}^{\infty}$, with $m_t = m$, $t = 0, 1, \ldots$. We call a tuple $E^m = \{k^*, 1 + r^*, w^*, (s^{i*}, c^{i*})_{i=1}^L, P^*, Q^*\}$ *a competitive steady-state equilibrium* if the sequence $\{k_t^*, 1 + r_t^*, w_t^*, (s_{t-1}^{i*}, c_t^{i*})_{i=1}^L, P_t^*, Q_t^*\}_{t=0}^{\infty}$ given for all $t = 0, 1, \ldots$ by

$$k_t^* = k^*, \qquad 1 + r_t^* = 1 + r^*, \qquad w_t^* = w^*, \tag{5}$$

$$\left(s_{t-1}^{i*}, c_t^{i*}\right)_{i=1}^L = \left(s^{i*}, c^{i*}\right)_{i=1}^L, \tag{6}$$

$$P_t^* = P^*, \qquad Q_t^* = Q^*, \tag{7}$$

is an equilibrium path starting from the initial state $\{(\hat{s}_{-1}^i)_{i=1}^L, \hat{k}_0, \hat{Q}_0\} = \{(s^{i*})_{i=1}^L, k^*, Q^*\}$.

Provided the above definition, the following proposition describes the structure of steady-state equilibria. It is an adaptation of the well-established results of Becker (1980, 2006) to our model.

**Proposition 1** (Structure of steady state equilibrium) *A tuple* $E^m = \{k^*, 1+r^*, w^*, (s^{i*}, c^{i*})_{i=1}^L, P^*, Q^*\}$ *satisfying* $m < w^*$ *is a steady-state equilibrium if and only if*

$$\beta^h = \frac{1}{1+r^*}, \quad 1+r^* = f'\big(k^*\big), \qquad w^* = f\big(k^*\big) - f'\big(k^*\big)k^*, \tag{8}$$

$$P^* = \lambda L f\big(k^*\big), \tag{9}$$

$$\mu\Phi\big(Q^*\big) = \mu\big(Q^* - \kappa P^*\big) + Lm, \tag{10}$$

$$s^{i*} = 0, \quad i \in H_l, \tag{11}$$

$$s^{i*} \geq 0, \quad i \in H_h, \tag{12}$$

$$\sum_{i=1}^{L} s^{i*} = \sum_{i \in H_h} s^{i*} = Lk^*, \tag{13}$$

$$c^* + s^* + m = w^* + \big(1+r^*\big)s^*. \tag{14}$$

*Proof* See Sect. 8.1. □

In this proposition, (8) shows that the steady-state capital intensity, interest rate, and the wage rate are determined by the discount factor of the patient consumer. Equations (11)–(12) tell us that impatient consumers have zero savings. It means that all the capital is owned by the patient consumers. As a consequence, in a steady-state equilibrium all impatient consumers have the same income and savings levels. In contrast, the distribution of savings among the patient consumers is indeterminate in a steady state. As shown by (13), only aggregate savings is determined.

## 3 Voting Equilibria

There is no reason for heterogenous agents to agree on the desired level of environmental maintenance. One way to solve this problem is to choose maintenance by majority voting. If policy choices were one-dimensional, one could refer to the median voter theorem. But this theorem does not apply here. In this section we propose a definition of voting equilibrium and we prove that the level of maintenance that comes out at the voting steady-state equilibrium is the one that would have been chosen by the median voter.

Let $\mathbf{m} = (m_t)_{t=0}^{\infty}$ be an environmental policy. The optimal value of problem $\mathcal{P}_1(\tau)$ for consumer $i$ is a function of $\hat{s}_{\tau-1}$, $\hat{Q}_\tau$ and $\mathbf{m}$. We will denote this optimal value by $V_{i,\tau}(\hat{s}_{\tau-1}, \hat{Q}_\tau, \mathbf{m})$.

**Definition 3** (Preferred change in environmental maintenance) Suppose that the environmental policy is represented by some sequence $\bar{\mathbf{m}} = (\bar{m}_t)_{t=0}^{\infty}$ of non-negative numbers and that at $\mathbf{m} = \bar{\mathbf{m}}$ the function $V_{i,\tau}(\hat{s}_{\tau-1}, \hat{Q}_\tau, \mathbf{m})$ is differentiable in $m_\tau$. We say that consumer $i$ is *in favor of increasing* $m_\tau$ if $\frac{\partial V_{i,\tau}(\hat{s}_{\tau-1}, \hat{Q}_\tau, \mathbf{m})}{\partial m_\tau} > 0$ and is *in favor of decreasing* $m_\tau$ if $\frac{\partial V_{i,\tau}(\hat{s}_{\tau-1}, \hat{Q}_\tau, \mathbf{m})}{\partial m_\tau} < 0$ and $\bar{m}_\tau > 0$.

Let us assume that, for an equilibrium path

$$\mathcal{E}^{\bar{\mathbf{m}}} = \left\{k_t^*, 1 + r_t^*, w_t^*, \left(s_{t-1}^{i*}, c_t^{i*}\right)_{i=1}^{L}, P_t^*, Q_t^*\right\}_{t=0}^{\infty}$$

the function $V_{i,\tau}(s_{\tau-1}^*, Q_\tau^*, \mathbf{m})$ is differentiable in $m_\tau$ at $\mathbf{m} = \bar{\mathbf{m}}$. We denote by $N_\tau^+(\mathcal{E}^{\bar{\mathbf{m}}})$ the number of consumers who are in favor of *increasing* $\bar{m}_\tau$, and by $N_\tau^-(\mathcal{E}^{\bar{\mathbf{m}}})$ the number of consumers who are in favor of *decreasing* $\bar{m}_\tau$. We are now equipped to define intertemporal voting equilibria.

**Definition 4** (Intertemporal voting equilibrium) Let $\mathbf{m}^* = (m_t^*)_{t=0}^{\infty}$ be a maintenance policy and $\mathcal{E}^{\mathbf{m}^*}$ be an equilibrium path constructed at this policy. We call the couple $(\mathbf{m}^*, \mathcal{E}^{\mathbf{m}^*})$ *an intertemporal voting equilibrium path* if at $\mathbf{m} = \mathbf{m}^*$ $\forall \tau = 0, 1, \ldots$ the function $V_{i,\tau}(s_{\tau-1}^*, Q_\tau^*, \mathbf{m})$ is differentiable in $m_\tau$, and

$$N_\tau^+\left(\mathcal{E}^{\mathbf{m}^*}\right) < \frac{L}{2}, \qquad N_\tau^-\left(\mathcal{E}^{\mathbf{m}^*}\right) < \frac{L}{2}, \quad \forall \tau = 0, 1, \ldots.$$

According to this definition, an intertemporal voting equilibrium is reached if, at each time period there exists neither a majority of agents who are in favor of increasing maintenance, nor a majority of agents who are in favor of decreasing maintenance. And because we take the number of agents as odd by assumption, then there exists an agent for whom the maintenance level is optimal in equilibrium.

This definition is in line with the usual way of defining intertemporal equilibrium, as articulated by Hicks (1936) and, more recently, by Grandmont (1983). In our model, any intertemporal voting equilibrium can be seen as a sequence of temporary voting equilibria in which agents perfectly anticipate the whole future, including voting results. Indeed, let $(\mathbf{m}^*, \mathcal{E}^{\mathbf{m}^*})$ be an intertemporal voting equilibrium. Suppose that at time $\tau$ the agents are asked to vote on $m_\tau$ and that they correctly anticipate $m_t^*$ for all $t = \tau + 1, \tau + 2, \ldots$. Then it is clear that all the conditions for the median voter theorem hold in this one-dimensional voting, and that the preferred value of $m_\tau$ for the median voter coincides with $m_\tau^*$. A key implication is that intertemporal voting equilibria are time consistent.

In the rest of the paper we shall focus on steady state voting equilibria. Consider a couple $(m^*, E^{m^*})$, where $m^* \geq 0$ and $E^{m^*} = \{k^*, 1 + r^*, w^*, (s^{i*}, c^{i*})_{i=1}^{L}, P^*, Q^*\}$ is a steady-state equilibrium constructed at the maintenance policy $\mathbf{m}^* = (m_0^*, m_1^*, \ldots)$, $m_t^* = m^*$, $t = 0, 1, \ldots$. Let $\mathcal{E}^{m^*}$ be an equilibrium path corresponding to $E^{m^*}$.

**Definition 5** (Steady state voting equilibrium) We call the couple $(m^*, E^{m^*})$ *a steady state voting equilibrium* if the couple $(m^*, \mathcal{E}^{m^*})$ is an intertemporal voting equilibrium path.

To answer the question of whether a couple $(m^*, E^{m^*})$ is a steady state voting equilibrium or not, it is sufficient to know which consumers are in favor of an increase in $m_0^* = m^*$ at time 0, and which ones are in favor of a decrease. We know that, for each consumer $i$, the sequence $(\tilde{s}_{t-1}^i, \tilde{c}_t^i, \tilde{Q}_t)_{t=0}^{\infty}$ given by

$$\tilde{s}_{t-1}^i = s^{i*}, \qquad \tilde{c}_t^i = c^{i*}, \qquad \tilde{Q}_t = Q^*, \tag{15}$$

is a solution to

$$\max_{(c_t)_{t=0}^{+\infty}, (Q_t)_{t=0}^{+\infty}} \sum_{t=0}^{\infty} \beta_i^t \big[u(c_t) + v(Q_t)\big], \tag{16}$$

$$\mu \Phi(Q_{t+1}) = \mu\big(Q_t - \kappa P^*\big) + L m_t^*, \quad t = 0, 1, \ldots, \tag{17}$$

$$c_t + s_t + m_t^* \le w^* + \big(1 + r^*\big) s_{t-1}, \quad t = 0, 1, \ldots, \tag{18}$$

$$s_{-1}^i = \hat{s}_{-1}^i, \;\; Q_0 = \hat{Q}_0, \tag{19}$$

$$c_t \ge 0, \;\; s_t \ge 0, \;\; Q_t \ge 0, \;\; t = 0, 1, \ldots \tag{20}$$

at $\hat{s}_{-1}^i = s^{i*}$, $\hat{Q}_0 = Q^*$.

**Lemma 1** (Differentiability of value function w.r.t. maintenance and sign of derivative) *Let for some $i$ the sequence $(\tilde{s}_{t-1}^i, \tilde{c}_t^i, \tilde{Q}_t)_{t=0}^{\infty}$ given by (15) be a solution to problem (16)–(20) at given $m_t^* = m^* \in [0, w^*)$, $t = 0, 1, \ldots$ and at $\hat{s}_{-1}^i = s^{i*}$, $\hat{Q}_0 = Q^*$. Then $V_{i,0}(s^{i*}, Q^*, \mathbf{m}^*)$ is differentiable in $m_0^*$ and*

$$\frac{\partial V_{i,0}(s^{i*}, Q^*, \mathbf{m}^*)}{\partial m_0^*} \gtreqqless 0 \Leftrightarrow \beta_i L v'\big(Q^*\big) \gtreqqless \mu u'\big(c^{i*}\big)\big(\Phi'\big(Q^*\big) - \beta_i\big). \tag{21}$$

*Proof* See Sect. 8.2. □

The interpretation of Lemma 1 runs as follows. Consider the first inequality in (21) at a given maintenance $m_0^*$ and suppose that the left-hand side is higher than the right-hand side. In this case, out of a marginal change in maintenance, the induced marginal utility of environmental quality (i.e. the LHS of (21)), is larger than the induced marginal utility of consumption (i.e. the RHS of (21)). This is likely to happen when the given maintenance level $m_0^*$ is low. In such a case the consumer will be in favor of an increase in maintenance. In the opposite case, the given maintenance $m_0^*$ is likely to be large so that the induced marginal utility of consumption is higher than the induced marginal utility of quality and the consumer is in favor of decreasing maintenance.

To check whether a couple $(m^*, E^{m^*})$ is a voting steady-state equilibrium or not, let us consider the following problem $\mathcal{P}_2$ in which household $i$ is free to determine her preferred level of maintenance $m_t$:

$$\max_{(c_t)_{t=0}^{+\infty},(s_t)_{t=0}^{+\infty},(m_t)_{t=0}^{+\infty},(Q_t)_{t=0}^{+\infty}} \sum_{t=0}^{\infty} \beta_i^t \big[u(c_t) + v(Q_t)\big],$$

subject to:

$$\begin{aligned}
&\mu\Phi(Q_{t+1}) \le \mu\big(Q_t - \kappa P^*\big) + Lm_t, \quad t = 0, 1,\\
&c_t + s_t + m_t \le w^* + \big(1 + r^*\big)s_{t-1}, \quad t = 0, 1,\\
&\quad s_{-1} = \hat{s}_{-1}, Q_0 = \hat{Q}_0,\\
&\quad c_t \ge 0, s_t \ge 0, m_t \ge 0, Q_t \ge 0, t = 0, 1.
\end{aligned}$$

Let $(\tilde{s}, \tilde{c}, \tilde{m}, \tilde{Q}) \in \mathrm{R}_+^4$ *determine a steady-state solution* to this problem if the sequence $(\tilde{s}_{t-1}, \tilde{c}_t, \tilde{m}_t, \tilde{Q}_t)_{t=0}^{\infty}$ given by

$$\tilde{s}_{t-1} = \tilde{s}, \qquad \tilde{c}_t = \tilde{c}, \qquad \tilde{m}_t = \tilde{m}, \qquad \tilde{Q}_t = \tilde{Q} \tag{22}$$

is its solution at $\hat{s}_{-1} = \tilde{s}$ and $\hat{Q}_0 = \tilde{Q}$.

Prior to formulating the following lemma, remind that $\beta^h(1 + r^*) = 1$ and hence that $\beta_i(1 + r^*) < 1, \forall i \in H_l$, and $\beta_i(1 + r^*) = 1, \forall i \in H_h$.

**Lemma 2** (Characterization of steady state solution to $\mathcal{P}_2$) *The tuple* $(\tilde{s}, \tilde{c}, \tilde{m}, \tilde{Q}) \in \mathrm{R}_+^4$ *determines a steady-state solution to* $\mathcal{P}_2$ *if and only if*

$$\beta_i\big(1 + r^*\big) < 1 \Rightarrow \tilde{s} = 0, \tag{23}$$

$$\beta_i L v'(\tilde{Q}) \le \mu u'(\tilde{c})\big(\Phi'(\tilde{Q}) - \beta_i\big) \ (= \textit{if } \tilde{m} > 0), \tag{24}$$

$$\tilde{c} = w^* + r^*\tilde{s} - \tilde{m}, \tag{25}$$

$$\mu\big(\Phi(\tilde{Q}) - \tilde{Q} + \kappa P^*\big) = L\tilde{m}. \tag{26}$$

*Proof* See Sect. 8.3. □

For the sake of simplicity we can get rid of $\tilde{m}$ by noticing that $\tilde{m} > 0 \Leftrightarrow \tilde{c} < w^* + r^*\tilde{s}$. We can thus rewrite conditions (24)–(25) as follows:

$$\tilde{c} = \left(w^* + r^*\tilde{s} - \frac{\mu\kappa}{L}P^*\right) + \frac{\mu}{L}\big(\tilde{Q} - \Phi(\tilde{Q})\big), \tag{27}$$

$$\tilde{c} \le w^* + r^*\tilde{s}, \tag{28}$$

$$\beta_i L v'(\tilde{Q}) \le \mu u'(\tilde{c})\big(\Phi'(\tilde{Q}) - \beta_i\big) \ \big(= \text{if } \tilde{c} < w^* + r^*\tilde{s}\big). \tag{29}$$

Equation $\beta_i L v'(Q) = \mu u'(c)(\Phi'(Q) - \beta_i)$ shows that $c$ in increasing in $Q$. As for equation $c = (w^* + r^*\tilde{s} - \frac{\mu\kappa}{L}P^*) + \frac{\mu}{L}(Q - \Phi(Q))$, it shows that, for any given $\tilde{s}$, the relationship between $c$ and $Q$ is either always decreasing, or first increasing ($\Phi'(Q) < 1$) and then decreasing ($\Phi'(Q) > 1$). Suppose we are given $m^* \in [0, w^*)$, where $w^*$ is given by (8). Let $E^{m^*} = \{k^*, 1 + r^*, w^*, (s^{i*}, c^{i*})_{i=1}^{L}, P^*, Q^*\}$ be a

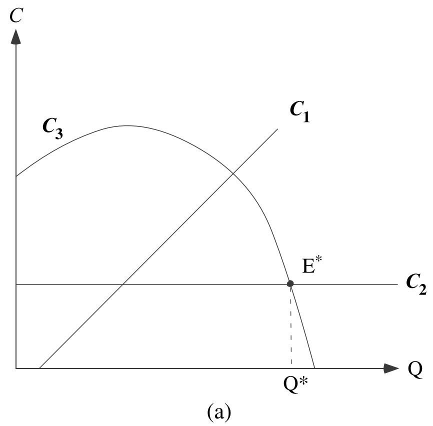


**Fig. 1** *Left*: zero-maintenance equilibrium (Regime 1)—*right*: positive maintenance equilibrium (Regime 2)

steady-state equilibrium constructed at the maintenance policy $\mathbf{m}^* = (m^*, m^*, \ldots)$. Put all households in ascending order of their savings and take the median one, $i_m$.[5] Lemmas 1 and 2 lead to the following theorem.

**Theorem 1** (Steady state voting equilibrium and median voter) *The couple* $(m^*, E^{m^*})$ *is a steady-state voting equilibrium if and only if for* $i = i_m$, *the tuple* $(s^{i*}, c^{i*}, m^*, Q^*)$ *is a steady-state solution to problem* $\mathcal{P}_2$.

The economic interpretation of the theorem runs as follows. We know from Proposition 1 that the *per capita* stock of capital in a steady-state voting equilibrium ($k^*$) and the wage and interest rates ($w^*$ and $r^*$) are determined by the discount factor of patient households ($\beta^h$). In the meantime, Theorem 1 says that maintenance and environmental quality do depend on the median discount factor and the median savings. Combining the two yields the following outcome. If the median value of the discount factor is $\beta^l$, and so $\beta_{i_m} = \beta^l$, then maintenance and environmental quality are determined by $\beta^l$ and $w^*$, because the savings of agent $i_m$ are unambiguously zero. But if the median value of the discount factor is $\beta^h$ and hence $\beta_{i_m} = \beta^h$, then they are determined by $\beta^h$, $w^*$, $r^*$ and the savings of agent $i_m$, $s^{i_m*}$, which can be either zero or positive.

It follows from this theorem that there exist two possible equilibria depending on whether $c^{i_m*} = w^* + r^* s^{i_m*}$ ($\Leftrightarrow m^* = 0$) or $c^{i_m*} < w^* + r^* s^{i_m*}$ ($\Leftrightarrow m^* > 0$). They are illustrated by the left and right panel of Fig. 1, in which we take $s^{i_m*}$ as given. On these graphs the three curves $\mathcal{C}_1$, $\mathcal{C}_2$ and $\mathcal{C}_3$ are defined as follows:

[5] More formally, we can put the set of households in an order such that, if $\beta_i < \beta_j$ and if $s^{i*} < s^{j*}$, then $i$ precedes $j$. Such an order exists because the impatient consumers do not save in a steady-state equilibrium. Now take the household median in the sense of the introduced order, $i_m$.

Curve $\mathcal{C}_1$: $\beta_{i_m} L v'(Q) = \mu u'(c)(\Phi'(Q) - \beta_{i_m})$
Curve $\mathcal{C}_2$: $c = w^* + r^* s^{i_m *}$
Curve $\mathcal{C}_3$: $c = (w^* + r^* s^{i_m *} - \frac{\mu\kappa}{L} P^*) + \frac{\mu}{L}(Q - \Phi(Q))$.

Let us describe these two regimes.

Regime 1 Zero-maintenance. The equilibrium point $(Q^*, c^{i_m *})$ is at the intersection of the $\mathcal{C}_2$ curve and the $\mathcal{C}_3$ curve (see Fig. 1a) and, as far as curve $\mathcal{C}_1$ is concerned, we have $\beta_{i_m} L v'(Q^*) < \mu u'(c^{i_m *})(\Phi'(Q^*) - \beta_{i_m})$.

Regime 2 Positive-maintenance. The equilibrium point $(Q^*, c^{i_m *})$ is at the intersection of the $\mathcal{C}_1$ curve with the $\mathcal{C}_3$ curve (see Fig. 1b) and, as far as curve $\mathcal{C}_2$ is concerned, we have $c^{i_m *} < w^* + r^* s^{i_m *}$ .

In combination with the above-mentioned two regimes, two cases must be distinguished:

Case 1 Impatient median voter. $\beta_{i_m} = \beta^l$ and savings of the median voter are uniquely determined, $s^{i_m *} = 0$.

Case 2 Patient median voter. $\beta_{i_m} = \beta^h$ and the savings of the median voter, $s^{i_m *}$, are not unique: they can take any value in the interval $[0, \frac{2}{L+1} L k^*]$.

In both cases, the regime of equilibrium maintenance can be nil or positive. In Case 1, the equilibrium level of maintenance and environmental quality are uniquely determined. As for Case 2, if there exists at least one equilibrium with positive maintenance, the equilibrium levels of maintenance and environmental quality are indeterminate since there is a continuum of these.

The very existence of steady-state voting equilibria deserves some comments. It is clear that if the majority of consumers is impatient, then steady-state voting equilibria exist for any distribution of savings among patient consumers because, in this case, the solution to problem $\mathcal{P}_2$ for the median voter, $(\tilde{s}, \tilde{c}, \tilde{m}, \tilde{Q})$, unconditionally satisfies $\tilde{m} < w^*$. But if the majority of consumers is patient, then steady-state voting equilibria do exist for any distribution of savings among patient consumers, where the savings of the median voter are nil or small enough.

# 4 Some Comparative Statics on Preferences, Income Inequality, and Technology

In our model, comparative statics requires some caution. As stressed above, if the median voter is patient, in a steady state the savings of the median voter are not determined uniquely. They can take any value in the interval $[0, \frac{2}{L+1} L k^*]$. Therefore, when making a comparative statics exercise, a change in a parameter will have an indeterminate effect on the savings of the median voter. To circumvent this problem we will assume that $k^*$ is does not change with $s^{i_m *}$. We will also assume that the ratio $s^{i_m *}/(\sum_{i=1}^{L} s^{i *})$, and hence the ratio $s^{i_m *}/k^*$, remain unchanged when a parameter changes (notice that since $k^* = (\sum_{i=1}^{L} s^{i *})/L$ shows the mean savings, $s^{i_m *}/k^*$ shows the proportion between the median and mean savings).

## 4.1 An Increase in $s^{i_m*}$ Other Things Equal

Let us first carry out a comparative statics exercise relevant in Case 2, when the median voter is patient and his savings can be positive. Assume that $k^*$ is kept unchanged and $s^{i_m*}$ increases. The increase in $s^{i_m*}$ translates a change in the distribution of savings among the patient consumers only. Consequently, it leads to a different income distribution (an increase in the median income relative to the mean).

- Under *Zero-Maintenance Equilibrium* (Regime 1), a small increase in $s^{i_m*}$, other things equal, will shift $\mathcal{C}_2$ and $\mathcal{C}_3$ upwards by the same magnitude. Hence, consumption of the median voter $c^{i_m*}$ will increase, but environmental quality $Q^*$ will remain unchanged. A larger increase in $s^{i_m*}$ may shift the economy to Regime 2.
- Under *Positive-Maintenance Equilibrium* (Regime 2), a small increase in $s^{i_m*}$, other things equal, will shift $\mathcal{C}_3$ upwards, while letting $\mathcal{C}_1$ untouched. Hence the environmental quality $Q^*$ will increase.

Following the literature in political economy and income inequality (see e.g. Meltzer and Richard 1981), the "more equal" the income distribution, the higher the median income relative to the mean (this is only reasonable in the case where the median income does not exceed the mean, which is considered as a typical situation). In our model, it means that, in developed economies where maintenance is positive, lower inequality has a positive effect on environmental quality. Conversely, in developing economies where there is no maintenance, inequality itself does not effect the environmental quality.

## 4.2 An Increase in Total Factor Productivity

In the following sub-sections of this section, we consider a Cobb-Douglas production function, $f(k) = k^\alpha, 0 < \alpha < 1$. We also assume that the fraction of output necessary to remove all emissions is lower than the labor share in output, $1 - \alpha > \mu\lambda$. Geometrically, the latter assumption implies that the curve $\mathcal{C}_3$ shifts upwards after an increase in capital intensity.

Let us first consider an increase in the total factor productivity by introducing a scale parameter $a$ in the production function, $aF(K, L) = Laf(k)$. The impact of an increase in total factor productivity will depend on the regime the economy follows in equilibrium.

**Regime 1. Zero-Maintenance Equilibrium** In this regime, a small increase in $a$ leads to an increase in $k^*$, $w^*$ and $w^* + r^*s^{i_m*}$. It will also increase the output $Lf(k^*)$ and pollution $P^*$ levels, but it cannot make maintenance positive. As a consequence, the environmental quality $Q^*$ decreases. Graphically (see Fig. 2,

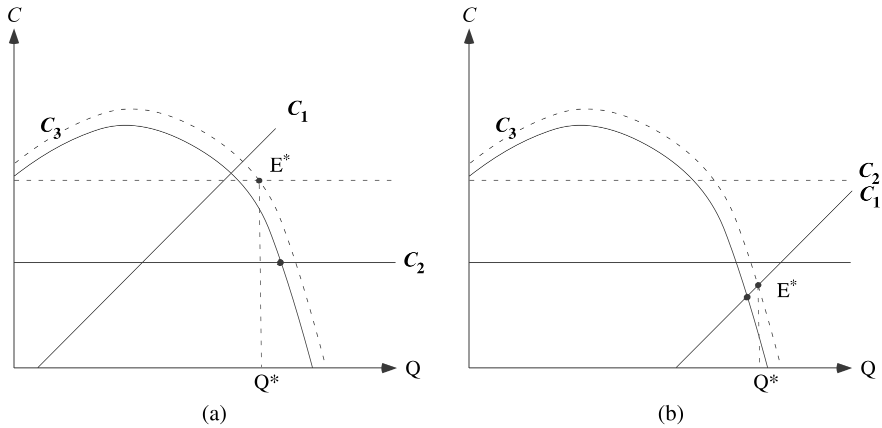


**Fig. 2** An increase in total factor productivity in Regime 1 (*left*) and Regime 2 (*right*)

left panel), $\mathcal{C}_2$ shifts upwards due to the increase in $w^* + r^* s^{i_m *}$. $\mathcal{C}_3$ also shifts upwards, but to a smaller extent because both $w^*$ and $P^*$ increase. If the increase in $a$ becomes large enough, then the economy switches to Regime 2, namely the Positive-maintenance Equilibrium.

**Regime 2. Positive-Maintenance Equilibrium** In this regime, an *increase* in $a$ will shift $\mathcal{C}_3$ upwards, as shown in Fig. 2, right panel, and hence to an *increase* in $Q^*$.

To sum up, if the economy starts in Regime 1, then an increase in $a$ from 0 to $+\infty$ first leads to a decrease in environmental quality $Q^*$, and then to an increase, as shown in Fig. 2. If one considers that developing countries most likely correspond to Regime 1 and rich countries to Regime 2, then this conclusion means that technological progress first goes with a decrease in environmental quality, and after some stage of development with an increase in environmental quality. This result provides a new rationale for an Environmental Kuznets Curve (see e.g. Stokey 1998; Dasgupta et al. 2002 or Prieur 2009) with heterogeneous agents and voting.

## *4.3 Patient Agents Become More Patient: An Increase in $\beta^h$*

We first consider an increase in $\beta^h$, which means that patient agents become even more patient. The effects on the environmental quality will depend on which regime the economy experiences.

Under Zero-Maintenance Equilibrium (Regime 1), a small increase in $\beta^h$ leads to an increase in capital intensity $k^*$, wage rate $w^*$, output $Lf(k^*)$ and pollution $P^*$, but it cannot make maintenance positive. Hence $Q^*$ decreases as $\beta^h$ increases under Regime 1. Graphically (see Fig. 2, left panel), $\mathcal{C}_2$ shifts upwards due to the increase in $w^*$; $\mathcal{C}_3$ also shifts upwards, but to a smaller extent ($w^*$ will increase but $P^*$ will

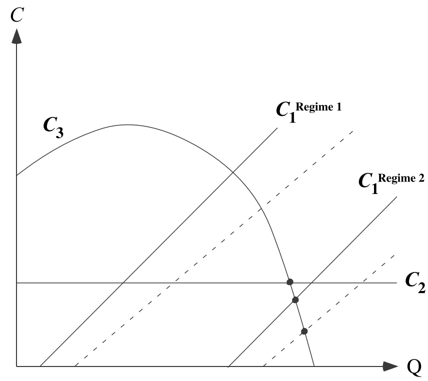


**Fig. 3** Impatient agents become less impatient

also increase). If the median voter is patient (Case 2) then, $\mathcal{C}_1$ shifts to the right. As a consequence the economy may well switch to the Positive-maintenance regime (Regime 2).

Under Positive-Maintenance Equilibrium (Regime 2, see Fig. 2b) an increase in $\beta^h$ will lead to an upward shift of $\mathcal{C}_3$ and, in Case 2, to a shift of $\mathcal{C}_1$ to the right. Hence $Q^*$ will increase.

### *4.4 Impatient Agents Become Less Impatient: An Increase in $\beta^l$*

Let us now consider the case where impatient agents become less impatient, i.e. an increase in $\beta^l$. The effect on $Q^*$ will depend on whether the median consumer is impatient or patient, what we referred to as Case 1 and Case 2, respectively. In the case where the median voter is impatient (Case 1, $\beta_{i_m} = \beta^l$), then the two regimes have to be considered.

- Under Zero-Maintenance Equilibrium (Regime 1), a small increase in $\beta^l$ does not change $k^*$, $w^*$, $Lf(k^*)$ or $P^*$. It neither changes $Q^*$. This case results in a shift of $\mathcal{C}_1$ to the right, as illustrated in Fig. 3. Still, if the increase in $\beta^l$ becomes large enough, then the economy switches to Regime 2.
- Under Positive-Maintenance Equilibrium (Regime 2), a small increase in $\beta^l$ does not change $k^*$, $w^*$, $Lf(k^*)$ or $P^*$, but it does increase $Q^*$, as illustrated in Fig. 3.

In the case where the median voter is patient (Case 2, $\beta_{i_m} = \beta^h$), then it is clear that changing $\beta^l$ has no effect on $Q^*$.

## 5 How Agents' Heterogeneity Shapes Environmental Maintenance

In this section we compare the level of environmental quality in voting steady-state equilibria with that in steady-state equilibria of a similar economy, but populated with symmetric agents. We constrain our consideration to the case where the equilibrium values of the capital stock, and hence the output level, are the same in both models. The question we address is the following: how does agents' heterogeneity in discount factor and wealth shape environmental maintenance when agents are asked to vote?[6]

How to reduce our heterogeneous agent model to an homogeneous agent one is not straightforward. We have one alternative: either to assume that all agents have the same high discount factor ($\beta^h$), or the same low discount factor ($\beta^l$). The important issue is that these options do not yield the same outcome. Actually, the latter option cannot be considered because it will be associated to different levels of macroeconomic variables in equilibrium. What we are interesting in is the analysis of the effect of heterogeneity on pollution, so we need to keep all other macroeconomic variables unchanged. Thus, the only solution is to assume that all agents have the same—high discount factor.

Moreover, the steady-state equilibria in the homogenous population model coincides with the symmetric voting steady-state equilibria in this particular case, i.e. equilibria where the level of savings for all consumers is the same and, consequently, the level of consumption is also the same. To be more precise, voting is irrelevant in symmetric equilibria, because it is unanimous.

Let $\{k_S^*, 1 + r_S^*, w_S^*, (s_S^{i*}, c_S^{i*})_{i=1}^L, P_S^*, Q_S^*\}$ be a symmetric steady-state voting equilibrium with $\beta_i = \beta^h,\ i = 1, \ldots, L$, and $\{k^*, 1 + r^*, w^*, (s^{i*}, c^{i*})_{i=1}^L, P^*, Q^*\}$ be a steady-state voting equilibrium with arbitrary discount factors. By "symmetric" we mean that $s_S^{1*} = \cdots = s_S^{L*}$. It must be noticed that

$$k_S^* = k^*, \qquad r_S^* = r^*, \qquad w_S^* = w^*,$$

and that, by assumption,

$$s_S^{i*} = k^*, \quad i = 1, \ldots, L.$$

The last equation says that the savings of agents in the symmetric steady-state voting equilibrium with $\beta_i = \beta^h, i = 1, \ldots, L$, are equal to the mean of the savings in the heterogeneous-agent economy. We assume that the discount factor shared by all consumers in the former model is $\beta^h$, and not $\beta^l$. This is simply because, otherwise, the equilibrium level of capital stock and output would differ between the two models.

Let

$$\begin{aligned} m^* &= w^* + r^* s^{i_m *} - c^{i_m *}, \\ m_S^* &= w_S^* + r_S^* k_S^* - c_S^* \big(= w^* + r^* k^* - c_S^*\big), \end{aligned}$$

[6] Note that this is different from the question raised by Caselli and Ventura (2000): under which condition does a model with heterogenous agents "admits" a representative agent model, namely a model with homogenous agents displaying the same aggregate and average behavior. Indeed, in our case, by assumption, we assume capital intensity to be the same in both models. On the other hand we do not fix the maintenance level, nor do we look at the representative agent version of the model which would yield the same maintenance level.

where $c_S^* = c_S^{1*}(= \cdots = c_S^{L*})$. The following proposition can be proved with the same argument as in the previous section.

**Proposition 2** (Homogeneous *vs.* heterogeneous population equilibria) (1) *Suppose that $\beta_{i_m} = \beta^l$ and hence $s^{i_m*} = 0$ in the heterogenous-agent economy. Then*:

1. *if $m_S^* = 0$, then $m^* = 0$ and $Q^* = Q_S^*$;*
2. *if $m_S^* > 0$, then $m^* < m_S^*$ and $Q^* < Q_S^*$.*

(2) *Suppose that $\beta_{i_m} = \beta^h$ in the heterogenous-agent economy. Then*:

1. *if $s^{i_m*} \leq s_S^{i*} = k^*$, then*:
   (a) *if $m_S^* = 0$, $m^* = 0$ and $Q^* = Q_S^*$;*
   (b) *if $m_S^* > 0$, $m^* < m_S^*$ and $Q^* < Q_S^*$;*
2. *if $s^{i_m*} \geq s_S^{i*} = k^*$, then*:
   (a) *if $m^{i*} = 0$, $m_S^* = 0$ and $Q^* = Q_S^*$;*
   (b) *if $m^* > 0$, then $m^* > m_S^*$ and $Q^* > Q_S^*$.*

In the last section we have seen why developed countries are likely to vote in favor of environmental maintenance, and developing countries against. According to the proposition, the predictions of the two models coincide for developing countries: no maintenance in steady-state equilibria irrespective of whether the median voter is patient or impatient. However, the models' predictions differ for developed countries. If the majority of agents is impatient, then the maintenance equilibrium levels and environmental quality are lower than those predicted by the homogenous-agent model. If the majority of agents is patient, then the median saving or income must be compared with the mean ones. If the median savings are lower than the mean (or, equivalently, if the median income is lower then the mean income) then the maintenance equilibrium levels and environmental quality are lower in the heterogenous-agent model that in the homogenous-agent model. Otherwise, the opposite outcome holds.[7]

To sum up, our comparison shows that, because of heterogeneity, in most cases in the real world the observed levels of environmental maintenance and quality will be lower than what the homogenous-agent model would predict. Taking heterogeneity into account is key, even when interested in macroeconomic outcomes.

## 6 Debt-Financed Versus Tax-Financed Maintenance Policy

Until now we have assumed that the maintenance policy was financed by a pay-as-you-go tax ($\tau_t$), a so-called *tax-financed* scheme. An alternative scheme could

[7]The case where the median income is lower than the mean is usually considered as typical on empirical grounds.

be a *debt-finance* one. It would consist for the government to issue public bonds to finance the environmental maintenance. Comparing the two schemes makes sense in our setting because heterogenous households are likely to be hit differently by the taxes needed to finance the public debt and by the interest earned on public bonds. The median voter may well depend on the funding scheme. On the government side, reducing the funding of environmental maintenance by taxes could improve its political acceptability. Finally, because introducing a public debt in our infinitely-lived agent model does not impact on the equilibrium steady state capital intensity, we can focus on its impact on environmental quality.

In this section we assume that the government is able to raise the voted tax $\tau_t$ or to issue one-period public bonds $d_{t+1}$ to finance environmental maintenance $m_t$. The repayment of interests and principal of public bonds will appear in its budget constraint. It is assumed that public bonds and physical capital are perfect substitute and bear the same market interest rate $r_t$.

Let $d_t \geq 0$ be the per capita public debt and $\tau_t \geq 0$ be the lump-sum tax at time $t$. The government budget constraint reads:

$$\tau_t + d_{t+1} = m_t + (1 + r_t)d_t,$$

and the consumer's budget constraint (see (4)) becomes:

$$c_t + s_t + \tau_t \leq w_t + (1 + r_t)s_{t-1}, \quad s_t \geq 0.$$

One can easily update the definitions of competitive equilibrium path consequently. The only thing that deserves attention is that condition 3 (equilibrium in the capital market) now turns to:

$$\sum_{i=1}^{L} s_{t-1}^{i*} = L\big(k_t^* + d_t\big), \quad t = 0, 1, \ldots.$$

Suppose that the public debt is constant over time, $d_t = d, t = 0, 1, \ldots$. Then we can define the competitive steady-state equilibrium. Consider such an equilibrium, $(m^*, E^{m^*})$, where $E^{m^*} = \{k^*, 1 + r^*, w^*, (s^{i*}, c^{i*})_{i=1}^{L}, P^*, Q^*\}$. As in Mankiw (2000), government debt does not affect the steady-state capital stock and national income. So, as in the case with no governmental debt, we have:

$$\beta^h = \frac{1}{1 + r^*}, \quad 1 + r^* = f'\big(k^*\big), \qquad w^* = f\big(k^*\big) - f'\big(k^*\big)k^*.$$

In the meantime, the governmental debt does influence the distribution of income among households. The higher the debt, the higher the level of taxation to pay for the interest payments on that debt. The tax falls on both patient and impatient consumers, but the interest payments entirely go to the patient consumers, just because only patient consumers have positive savings in a steady-state equilibrium. In the steady-state equilibrium the budget constraint of the government becomes

$$\tau_t + d = m^* + \big(1 + r^*\big)d.$$

Hence, $\tau_t = \tau(d), t = 0, 1, \ldots$, where

$$\tau(d) = m^* + r^* d.$$

Therefore, the budget constraint of a consumer in the steady-state equilibrium is as follows:

$$c_t + s_t \le w^* - \tau(d) + \big(1 + r^*\big)s_{t-1}, \quad s_t \ge 0.$$

If the median voter is impatient, then we have $s^{i_m*} = 0$ in a steady-state equilibrium, and hence

$$c^{i_m*} + m^* = w^* - r^* d.$$

As a result, an increase in $d$ is equivalent for the median voter to a decrease in the post-tax wage rate. It follows that (in the case where maintenance is positive) an increase in public debt unambiguously leads to a decrease in maintenance and environmental quality in the voting steady-state equilibrium if the majority of agents is impatient.

If the median voter is patient, in a steady state the savings of the median voter are not determined uniquely. Hence a change in $d$ will have an indeterminate effect on the savings of the median voter. Let us assume that the ratio $s^{i_m*}/(\sum_{i=1}^{L} s^{i*})$ does not change. Since, in equilibrium, $(\sum_{i=1}^{L} s^{i*})/L = k^* + d$, it implies that the ratio $\gamma = s^{i_m*}/(k^* + d)$ (which is the proportion between the median and the mean savings) remains unchanged. Under this assumption, the key parameter becomes $\gamma$ because we now have:

$$c^{i_m*} + m^* = w^* + r^* s^{i_m*} - r^* d = w^* + r^*\big(\gamma k^* + (\gamma - 1)d\big).$$

It is clear from the previous equation that an increase in $d$ leads to a decrease in $c^{i_m*} + m^*$, if $\gamma < 1$, and to an increase in $c^{i_m*} + m^*$, if $\gamma > 1$.

Thus, in the case where maintenance is positive, $m^* > 0$, if the median savings and income are lower than the mean ($\gamma < 1$), then an increase in public debt leads to a decrease in maintenance and environmental quality. But if the median savings and income are higher than the mean ($\gamma > 1$), then an increase in public debt increases environmental maintenance quality. As noticed above, the case where the median savings and income are lower than the mean is usually considered as common.

## 7 Conclusion

In this paper we assumed that the population is exogenously divided into two groups: one with patient households and the other with impatient households. The environmental maintenance is voted by the households. We introduce the notion of voting equilibrium, look for steady state voting equilibria and find that the median voter theorem applies to them. If the majority of households is impatient, then the equilibrium level of maintenance and environmental quality is determined uniquely, but if the majority of households is patient, there can exist a continuum of these. We also fulfill comparative statics analysis and we show that (i) an increase in total factor productivity may produce a so-called Environmental Kuznets Curve, (ii) an increase in the patience of impatient households may improve the environmental quality if the median voter is impatient and maintenance positive, (iii) in the case where the median voter is patient and maintenance positive, and in the case where the median income is lower than the mean one (which is empirically grounded), then a shrink in inequality can lead to an increase in the environmental quality.

We also compare our model with a representative agent model, which is defined as a particular case of our model where all consumers are patient and savings are distributed evenly across them. We show that, in the case of impatient median voter, the level of environmental quality predicted by the heterogeneous-agent model is lower than the one predicted by the representative agent model. The same holds true if the median voter is patient but the median income lower that the mean, which is the common case.

Finally, some policy implications of our model are discussed. In this purpose we introduce public debt as an alternative source of financing environmental maintenance. We show that, if the median income is lower than the mean, then an increase in public debt leads to a lower environmental quality in the long run.

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## Appendix

### *8.1 Proof of Proposition 1*

It is sufficient to notice that since in a steady-state equilibrium we have $\mu\Phi(Q^*) = \mu(Q^* - \kappa P^*) + L\bar{m}$ and for each $i$, the sequence $(\tilde{s}^i_{t-1}, \tilde{c}^i_t)_{t=0}^{\infty}$ given by $\tilde{s}^i_{t-1} = s^{i*}$, $\tilde{c}^i_t = c^{i*}$ is a solution to

$$\max \sum_{t=0}^{\infty} \beta_i^t u(c_t), \quad c_t + s_t \leq \big(w^* - \bar{m}\big) + \big(1 + r^*\big)s_{t-1},\ s^i_{-1} = s^{i*},$$
$$c_t \geq 0,\ s_t \geq 0$$

and to refer to Becker (1980, 2006).

## 8.2 Proof of Lemma 1

We have:

$$\frac{\partial V_{i,0}(s^{i*}, Q^*, \mathbf{m}^*)}{\partial m_0^*} = \frac{\partial \Lambda_{i,0}(Q^*, \mathbf{m}^*)}{\partial m_0^*} + \frac{\partial \Gamma_{i,t}(s^{i*}, \mathbf{m}^*)}{\partial m_0^*},$$

where the functions $\Lambda_{i,0}$ and $\Gamma_{i,0}$ are defined as follows:

$$\Lambda_{i,0}\big(Q_0, \mathbf{m}^*\big) = \max_{(Q_t)_{t=1}^{\infty}} \left\{ \sum_{t=0}^{\infty} \beta_i^t v(Q_t) \mid \mu \Phi(Q_{t+1}) \le \mu\big(Q_t - \kappa P^*\big) + L m_t^*, \right.$$
$$\left. Q_{t+1} \ge 0,\ t = 0, 1, \ldots \right\},$$

$$\Gamma_{i,0}\big(s_{-1}, \mathbf{m}^*\big) = \max_{(c_t)_{t=0}^{\infty}, (s_t)_{t=0}^{\infty}} \left\{ \sum_{t=0}^{\infty} \beta_i^t u(c_t) \mid c_t + s_t + m_t^* \le w^* + \big(1 + r^*\big) s_{t-1}, \right.$$
$$\left. c_t \ge 0,\ s_t \ge 0,\ t = 0, 1, \ldots \right\}.$$

It is not difficult to check that

$$\frac{\partial \Lambda_{i,0}(Q^*, \mathbf{m}_t^*)}{\partial m_t^*} = \beta_i \frac{L v'(Q^*)}{\mu(\Phi'(Q^*) - \beta_i)},$$
$$\frac{\partial \Gamma_{i,0}(s^{i*}, \mathbf{m}_t^*)}{\partial m_t^*} = -u'\big(c^*\big).$$

Therefore,

$$\frac{\partial V_{i,0}(s^{i*}, Q^*, \mathbf{m}^*)}{\partial m_t^*} = \beta_i \frac{L v'(Q^*)}{\mu(\Phi'(Q^*) - \beta_i)} - u'\big(c^*\big),$$

which implies (21).

## 8.3 Proof of Lemma 2

Using a traditional argument (see e.g. McKenzie 1986) we can prove that a sequence $(\tilde{s}_{t-1}, \tilde{c}_t, \tilde{m}_t, \tilde{Q}_t)_{t=0}^{\infty}$ given by (22) is a steady-state solution to problem $\mathcal{P}_2$ if and only if there exist $q$ and $p$ such that for $p_t = \beta_i p_{t-1} = \cdots = \beta_i^t p$ and $q_{t+1} = \beta_i q_t = \cdots = \beta_i^{t+1} q$ the following relationships hold:

$$\beta_i^t u'(\tilde{c}_t) = p_t,$$
$$\beta_i^t v'(\tilde{Q}_t) + q_{t+1}\mu - q_t \mu \Phi'(\tilde{Q}_t) = 0,$$
$$\big(1 + r^*\big) p_t \le p_{t-1} \ (= \text{if } \tilde{s}_{t-1} > 0),$$
$$q_{t+1} L - p_t \ge 0 \ (= \text{if } \tilde{m}_t > 0),$$
$$q_{t+1} \tilde{Q}_t + p_t \tilde{s}_{t-1} \to_{t \to \infty} 0,$$

or, equivalently,

$$\begin{aligned}
&u'(\tilde{c}) = p,\\
&v'(\tilde{Q}) = \mu q\big(\Phi'(\tilde{Q}) - \beta_i\big),\\
&\beta_i \leq \frac{1}{1+r^*} \ (= \text{if } \tilde{s} > 0),\\
&\beta_i L q - p \geq 0 \ (= \text{if } \tilde{m} > 0).
\end{aligned}$$

The existence of such $q$ and $p$ is equivalent to conditions (23)–(24).

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