Chapter 2
Vector Algebra II: Scalar and Vector Products

We saw in the previous chapter how vector quantities may be added and subtracted. In this chapter we consider the products of vectors and define rules for them. First we will examine two cases frequently encountered in practice.

1. In science we define the work done by a force as the magnitude of the force multiplied by the distance it moves along its line of action, or by the component of the magnitude of the force in a given direction multiplied by the distance moved in that direction. Work is a scalar quantity and the product obtained when force is multiplied by displacement is called the *scalar product*.
2. The torque on a body produced by a force $\boldsymbol{F}$ (Fig. 2.1) is defined as the product of the force and the length of the lever arm OA, the line of action of the force being perpendicular to the lever arm. Such a product is called a *vector product* or *cross product* and the result is a vector in the direction of the axis of rotation, i.e. perpendicular to both the force and the lever arm.

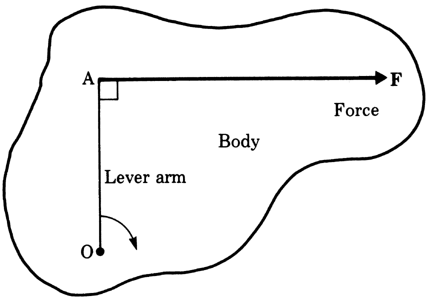

Fig. 2.1

2.1 Scalar Product

Consider a carriage running on rails. It moves in the s-direction (Fig. 2.2) under the application of a force $\boldsymbol{F}$ which acts at an angle α to the direction of travel. We

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require the work done by the force when the carriage moves through a distance s in the s-direction (Fig. 2.3).

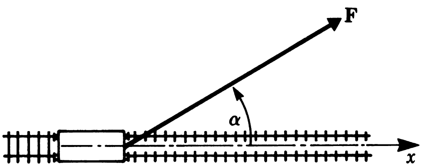

Fig. 2.2

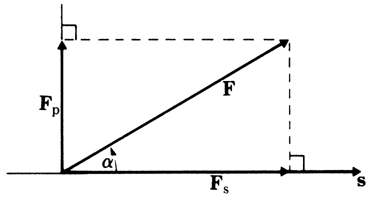

Fig. 2.3

In order to study the action of the force $\boldsymbol{F}$ on the carriage we resolve it into two components: one along the rails (in the s-direction), and one perpendicular to the rails, i.e. $\boldsymbol{F}_\text{s}$ and $\boldsymbol{F}_\text{p}$ respectively. $\boldsymbol{F}_\text{s}, \boldsymbol{F}_\text{p}$ and $\boldsymbol{s}$ are vector quantities; the work is, by definition, the product of the force along the direction of motion and the distance moved. In this case, it is the product of $\boldsymbol{F}_\text{s}$ and $\boldsymbol{s}$. It follows also from the definition that the work done by $\boldsymbol{F}_\text{p}$ is zero since there is no displacement in that direction. Furthermore, if the rails are horizontal then the motion of the carriage and the work done is not influenced by gravity, since it acts in a direction perpendicular to the rails.

If W is the work done then $W = F \cdot \cos\alpha \cdot s$ or $F \cdot s \cdot \cos\alpha$ in magnitude.

Since work is a scalar quantity the product of the two vectors is called a *scalar product* or *dot product*, because one way of writing it is with a dot between the two vectors:

$$W = \boldsymbol{F}_\text{s} \cdot \boldsymbol{s}$$

where

$$|\boldsymbol{F}_\text{s}| = |\boldsymbol{F}| \cos\alpha$$

It is also referred to as the *inner product* of two vectors. Generally, if $\boldsymbol{a}$ and $\boldsymbol{b}$ are two vectors their inner product is written $\boldsymbol{a} \cdot \boldsymbol{b}$.

Definition The *inner* or *scalar product* of two vectors is equal to the product of their magnitude and the cosine of the angle between their directions:

$$\boldsymbol{a} \cdot \boldsymbol{b} = ab \cos\alpha \tag{2.1}$$

Geometrical interpretation. The scalar product of two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ is equal to the product of the magnitude of vector $\boldsymbol{a}$ with the projection of $\boldsymbol{b}$ on $\boldsymbol{a}$ (Fig. 2.4a):

$$\boldsymbol{a} \cdot \boldsymbol{b} = ab \cos\alpha$$

Or it is the product of the magnitude of $\boldsymbol{b}$ with the projection of $\boldsymbol{a}$ on $\boldsymbol{b}$ (Fig. 2.4b):

$$\boldsymbol{a} \cdot \boldsymbol{b} = ba \cos \alpha$$

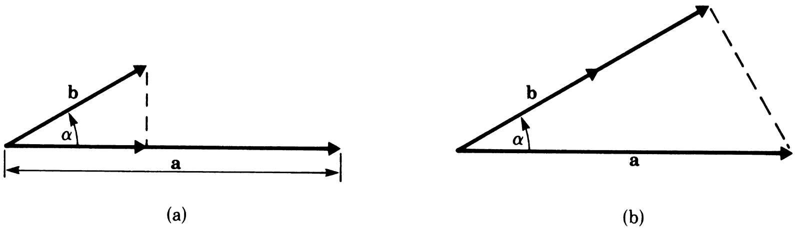

Fig. 2.4

In the case of the carriage, we can also evaluate the work done by the product of the magnitude of the force and the component of the displacement along the direction of the force (Fig. 2.5).

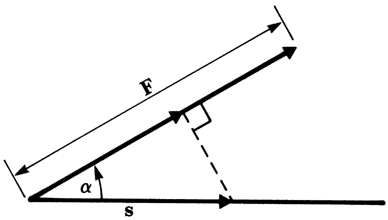

Fig. 2.5

Example A force of 5 N is applied to a body. The body is moved through a distance of 10 m in a direction which subtends an angle of 60° with the line of action of the force.

The mechanical work done is

$$\begin{aligned} U &= \boldsymbol{F} \cdot \boldsymbol{s} = Fs \cos \alpha \\ &= 5 \times 10 \times \cos 60^\circ = 25\,\text{Nm} \end{aligned}$$

The unit of work should be noted: it is newtons × metres = Nm or joules (J). This example could be considered to represent the force of gravity acting on a body which slides down a chute through a distance s; the force $\boldsymbol{F} = m\boldsymbol{g}$ where m is the mass of the body and $\boldsymbol{g}$ the acceleration due to gravity (Fig. 2.6).

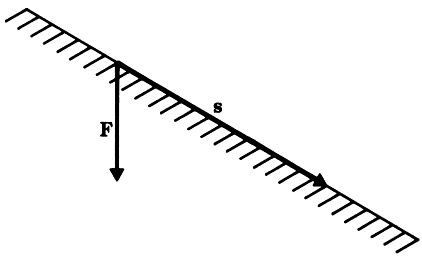

Fig. 2.6

2.1.1 Application: Equation of a Line and a Plane

The scalar product can be used to obtain the *equation of a line* in an $x-y$ plane if the normal from the origin to the line is given (Fig. 2.7). In this case the scalar product of $\boldsymbol{n}$ with any position vector $\boldsymbol{r}$ to a point on the line is constant and equal to n^2. Thus

$$\begin{aligned} n^2 &= \boldsymbol{n}\cdot\boldsymbol{r} = (n_x, n_y)\cdot(x,y) \\ n^2 &= xn_x + yn_y \\ y &= \frac{n_x}{n_y}x + \frac{n^2}{n_y} \end{aligned} \tag{2.2a}$$

If we extend the procedure to three dimensions we obtain the *equation of a plane* in an $x-y-z$ coordinate system:

$$n^2 = xn_x + yn_y + zn_z \tag{2.2b}$$

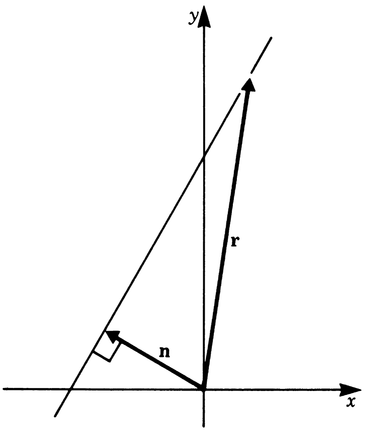

Fig. 2.7

2.1.2 Special Cases

Scalar Product of Perpendicular Vectors

If two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are perpendicular to each other so that $\alpha = \frac{\pi}{2}$ and hence $\cos\alpha = 0$, it follows that the scalar product is zero, i.e. $\boldsymbol{a}\cdot\boldsymbol{b} = 0$.

The converse of this statement is important. If it is known that the scalar product of two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ vanishes, then it follows that the two vectors are perpendicular to each other, provided that $\boldsymbol{a} \neq \boldsymbol{0}$ and $\boldsymbol{b} \neq \boldsymbol{0}$.

Scalar Product of Parallel Vectors

If two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are parallel to each other so that $\alpha = 0$ and hence $\cos\alpha = 1$, it follows that their scalar product $\boldsymbol{a}\cdot\boldsymbol{b} = ab$.

2.1.3 Commutative and Distributive Laws

The scalar product obeys the commutative and distributive laws. These are given without proof.

Commutative law	$\boldsymbol{a}\cdot\boldsymbol{b} = \boldsymbol{b}\cdot\boldsymbol{a}$	(2.3)
Distributive law	$\boldsymbol{a}\cdot(\boldsymbol{b}+\boldsymbol{c}) = \boldsymbol{a}\cdot\boldsymbol{b}+\boldsymbol{a}\cdot\boldsymbol{c}$	(2.4)

As an example of the scalar product let us derive the *cosine rule*. Figure 2.8 shows three vectors; α is the angle between the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$.

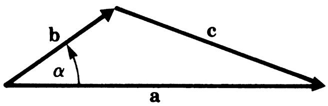

Fig. 2.8

We have
$$\boldsymbol{b}+\boldsymbol{c} = \boldsymbol{a}$$
$$\boldsymbol{c} = \boldsymbol{a}-\boldsymbol{b}$$

We now form the scalar product of the vectors with themselves, giving

$$\begin{aligned}\boldsymbol{c}\cdot\boldsymbol{c} &= \boldsymbol{c}^2 = (\boldsymbol{a}-\boldsymbol{b})^2\\ \boldsymbol{c}^2 &= \boldsymbol{a}\cdot\boldsymbol{a}+\boldsymbol{b}\cdot\boldsymbol{b}-2\boldsymbol{a}\cdot\boldsymbol{b}\\ \boldsymbol{c}^2 &= \boldsymbol{a}^2+\boldsymbol{b}^2-2ab\cos\alpha \qquad (2.5)\end{aligned}$$

If $\alpha = \frac{\pi}{2}$, we have Pythagoras' theorem for a right-angled triangle.

2.1.4 Scalar Product in Terms of the Components of the Vectors

If the components of two vectors are known, their scalar product can be evaluated. It is useful to consider the scalar product of the unit vectors $\boldsymbol{i}$ along the x-axis and $\boldsymbol{j}$ along the y-axis, as shown in Fig. 2.9.

From the definition of the scalar product we deduce the following:

$$
\begin{aligned}
\mathbf{i}\cdot\mathbf{i} &= 1\\
\mathbf{j}\cdot\mathbf{j} &= 1\\
\mathbf{i}\cdot\mathbf{j} &= 0\\
\mathbf{j}\cdot\mathbf{i} &= 0
\end{aligned}
$$

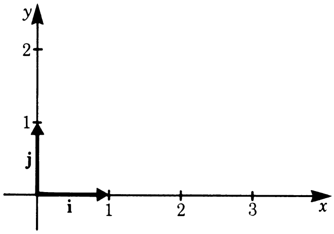

Fig. 2.9

Figure 2.10 shows two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ that issue from the origin of a Cartesian coordinate system. If a_x, b_x, a_y and b_y are the components of these vectors along the x-axis and y-axis, respectively, then

$$
\begin{aligned}
\mathbf{a} &= a_x\mathbf{i} + a_y\mathbf{j}\\
\mathbf{b} &= b_x\mathbf{i} + b_y\mathbf{j}
\end{aligned}
$$

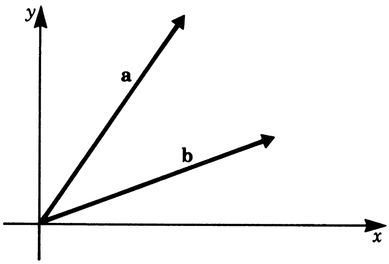

Fig. 2.10

The scalar product is

$$
\begin{aligned}
\boldsymbol{a}\cdot\boldsymbol{b} &= (a_x\boldsymbol{i} + a_y\boldsymbol{j})\cdot(b_x\boldsymbol{i} + b_y\boldsymbol{j})\\
&= a_xb_x\boldsymbol{i}\cdot\boldsymbol{i} + a_xb_y\boldsymbol{i}\cdot\boldsymbol{j} + a_yb_x\boldsymbol{j}\cdot\boldsymbol{i} + a_yb_y\boldsymbol{j}\cdot\boldsymbol{j}\\
\boldsymbol{a}\cdot\boldsymbol{b} &= a_xb_x + a_yb_y
\end{aligned}
$$

Thus the scalar product is obtained by adding the products of the components of the vectors along each axis (Fig. 2.11).

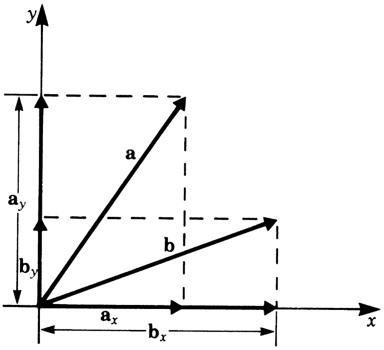

Fig. 2.11

In the case of three-dimensional vectors it is easily demonstrated that the following rule holds true:

$$\boldsymbol{a} \cdot \boldsymbol{b} = a_x b_x + a_y b_y + a_z b_z \quad \text{(scalar product)} \tag{2.6}$$

It is also an easy matter to calculate the magnitude of a vector in terms of its components. Thus

$$\begin{aligned} a^2 &= \boldsymbol{a} \cdot \boldsymbol{a} \\ &= a_x a_x + a_y a_y + a_z a_z \\ &= {a_x}^2 + {a_y}^2 + {a_z}^2 \\ a &= |\boldsymbol{a}| = \sqrt{{a_x}^2 + {a_y}^2 + {a_z}^2} \end{aligned}$$

(In Sect. 1.7, (1.5b))

Example Given that $\boldsymbol{a} = (2, 3, 1), \boldsymbol{b} = (-1, 0, 4)$, calculate the scalar product.

$$\begin{aligned} \boldsymbol{a} \cdot \boldsymbol{b} &= a_x b_x + a_y b_y + a_z b_z \\ &= 2 \times (-1) + 3 \times 0 + 1 \times 4 = 2 \end{aligned}$$

The magnitude of each vector is

$$\begin{aligned} a &= \sqrt{2^2 + 3^2 + 1} = \sqrt{14} \approx 3.74 \\ b &= \sqrt{1 + 4^2} = \sqrt{17} \approx 4.12 \end{aligned}$$

2.2 Vector Product

2.2.1 Torque

At the beginning of this chapter we defined the torque C, resulting from a force $\boldsymbol{F}$ applied to a body at a point P (Fig. 2.12), to be the product of that force and the position vector $\boldsymbol{r}$ from the axis of rotation O to the point P, the directions of the force and the position vector being perpendicular.

The magnitude of the torque is therefore $C = |\boldsymbol{r}||\boldsymbol{F}|$ or, more simply, $C = rF$.

This is known as the *lever law*.

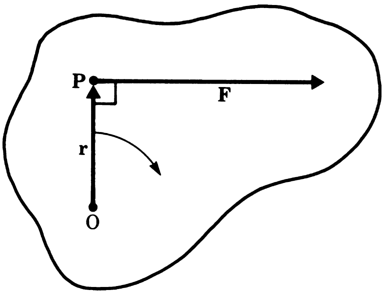

Fig. 2.12

A special case is illustrated in Fig. 2.13 where the line of action of the force $\boldsymbol{F}$ is in line with the axis (the angle between force and position vector $\boldsymbol{r}$ is zero). In this situation, the force cannot produce a turning effect on the body and consequently $C = 0$.

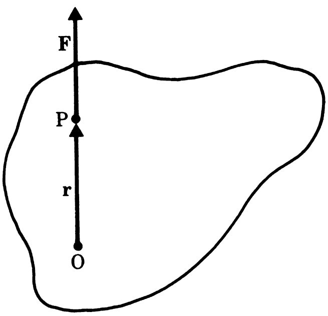

Fig. 2.13

The general case is when the force $\boldsymbol{F}$ and the radius $\boldsymbol{r}$ are inclined to each other at an angle α, as shown in Fig. 2.14. To calculate the torque C applied to the body we resolve the force into two components: one perpendicular to $\boldsymbol{r}$, $\boldsymbol{F}_{\perp}$, and one in the direction of $\boldsymbol{r}$, $\boldsymbol{F}_{\parallel}$.

The first component is the only one that will produce a turning effect on the body. Now $\boldsymbol{F}_{\perp} = F \sin\alpha$ in magnitude; hence $C = rF \sin\alpha$.

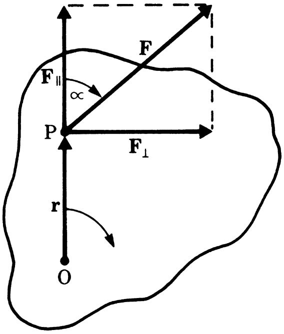

Fig. 2.14

Definition Magnitude of torque C

$$C = rF \sin\alpha$$

2.2.2 *Torque as a Vector*

Physically, torque is a vector quantity since its direction is taken onto account. The following convention is generally accepted.

The torque vector $\boldsymbol{C}$ is perpendicular to the plane containing the force $\boldsymbol{F}$ and the radius vector $\boldsymbol{r}$. The direction of $\boldsymbol{C}$ is that of a screw turned in a way that brings $\boldsymbol{r}$

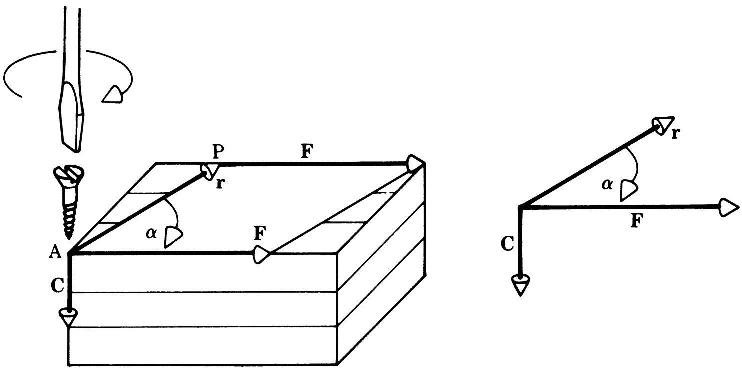

Fig. 2.15

by the shortest route into the direction of $\boldsymbol{F}$. This is called the right-hand rule. To illustrate this statement let us consider the block of wood shown in Fig. 2.15 where the axis of rotation is at A and a force $\boldsymbol{F}$ is applied at $\boldsymbol{P}$ at a distance $\boldsymbol{r}$. The two vectors $\boldsymbol{r}$ and $\boldsymbol{F}$ define a plane in space. $\boldsymbol{F}$ is then moved parallel to itself to act at A; as the screw is turned it rotates the radius vector $\boldsymbol{r}$ towards $\boldsymbol{F}$ through an angle α. Hence the direction of the torque $\boldsymbol{C}$ coincides with the penetration of the screw.

2.2.3 Definition of the Vector Product

The *vector product* of two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ (Fig. 2.16) is defined as a vector $\boldsymbol{c}$ of magnitude $ab\sin\alpha$, where α is the angle between the two vectors. It acts in a direction perpendicular to the plane of the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ in accordance with the right-hand rule.

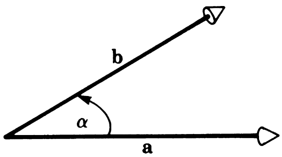

Fig. 2.16

This product, sometimes referred to as the *outer product* or *cross product*, is written

$$\boldsymbol{c} = \boldsymbol{a} \times \boldsymbol{b} \quad \text{or} \quad \boldsymbol{c} = \boldsymbol{a} \wedge \boldsymbol{b} \tag{2.7}$$

It is pronounced ‘a cross b’ or ‘a wedge b’. Its magnitude is $c = ab\sin\alpha$. Note that $\boldsymbol{a} \times \boldsymbol{b} = -\boldsymbol{b} \times \boldsymbol{a}$.

This definition is quite independent of any physical interpretation. It has geometrical significance in that the vector $\boldsymbol{c}$ represents the area of a parallelogram having sides $\boldsymbol{a}$ and $\boldsymbol{b}$, as shown in Fig. 2.17. $\boldsymbol{c}$ is perpendicular to the plane containing $\boldsymbol{a}$ and $\boldsymbol{b}$, direction given by the right-hand rule.

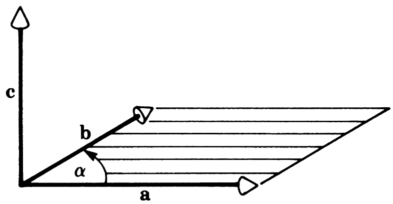

Fig. 2.17

The *distributive laws* for vector products are given here without proof.

$$\boldsymbol{a} \times (\boldsymbol{b} + \boldsymbol{c}) = \boldsymbol{a} \times \boldsymbol{b} + \boldsymbol{a} \times \boldsymbol{c} \tag{2.8}$$

and

$$(\boldsymbol{a} + \boldsymbol{b}) \times \boldsymbol{c} = \boldsymbol{a} \times \boldsymbol{c} + \boldsymbol{b} \times \boldsymbol{c} \tag{2.9}$$

Further, we note with respect to a scalar λ that

$$\lambda \boldsymbol{a} \times \boldsymbol{b} = \boldsymbol{a} \times \lambda \boldsymbol{b} = \lambda(\boldsymbol{a} \times \boldsymbol{b}) \tag{2.10}$$

Example Given two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ of magnitudes $a = 4$ and $b = 3$ and with an angle $\alpha = \frac{\pi}{6} = 30°$ between them, determine the magnitude of $\boldsymbol{c} = \boldsymbol{a} \times \boldsymbol{b}$.

$$c = ab \sin 30° = 4 \times 3 \times 0.5 = 6$$

2.2.4 *Special Cases*

Vector Product of Parallel Vectors

The angle between two parallel vectors is zero. Hence the vector product is $\mathbf{0}$ and the parallelogram degenerates into a line. In particular

$$\boldsymbol{a} \times \boldsymbol{a} = \mathbf{0}$$

It is important to note that the converse of this statement is also true. Thus, if the vector product of two vectors is zero, we can conclude that they are parallel, provided that $\boldsymbol{a} \neq \mathbf{0}$ and $\boldsymbol{b} \neq \mathbf{0}$.

Vector Product of Perpendicular Vectors

The angle between perpendicular vectors is $90°$, i.e. $\sin\alpha = 1$. Hence

$$|\boldsymbol{a} \times \boldsymbol{b}| = ab$$

2.2.5 *Anti-Commutative Law for Vector Products*

If $\boldsymbol{a}$ and $\boldsymbol{b}$ are two vectors then

$$\boldsymbol{a} \times \boldsymbol{b} = -\boldsymbol{b} \times \boldsymbol{a} \tag{2.11}$$

Proof Figure 2.18 shows the formation of the vector product. The vector product is $\boldsymbol{c} = \boldsymbol{a} \times \boldsymbol{b}$ and $\boldsymbol{c}$ points upwards. In Fig. 2.19, $\boldsymbol{c}$ is now obtained by turning $\boldsymbol{b}$ towards $\boldsymbol{a}$, then, by our definition, the vector $\boldsymbol{b} \times \boldsymbol{a}$ points downwards. It follows therefore that $\boldsymbol{a} \times \boldsymbol{b} = -\boldsymbol{b} \times \boldsymbol{a}$. The magnitude is the same, i.e. $ab \sin\alpha$.

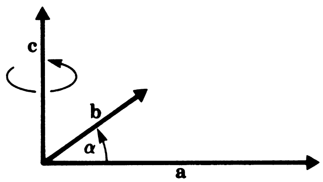

Fig. 2.18

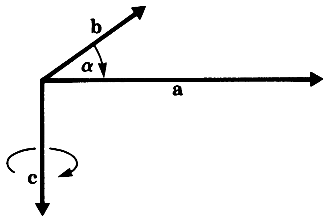

Fig. 2.19

2.2.6 Components of the Vector Product

Let us first consider the vector products of the unit vectors $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$ (Fig. 2.20). According to our definition the following relationships hold:

$$
\begin{aligned}
\mathbf{i} \times \mathbf{i} &= 0\\
\mathbf{i} \times \mathbf{j} &= \mathbf{k}\\
\mathbf{i} \times \mathbf{k} &= -\mathbf{j}\\[4pt]
\mathbf{j} \times \mathbf{j} &= 0\\
\mathbf{j} \times \mathbf{k} &= \mathbf{i}\\
\mathbf{j} \times \mathbf{i} &= -\mathbf{k}\\[4pt]
\mathbf{k} \times \mathbf{k} &= 0\\
\mathbf{k} \times \mathbf{i} &= \mathbf{j}\\
\mathbf{k} \times \mathbf{j} &= -\mathbf{i}
\end{aligned}
$$

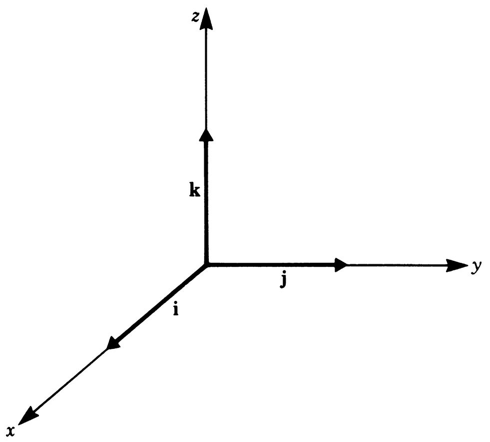

Fig. 2.20

Let us now try to express the vector product in terms of components. The vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ expressed in terms of their components are

$$\boldsymbol{a} = a_x \boldsymbol{i} + a_y \boldsymbol{j} + a_z \boldsymbol{k}$$
$$\boldsymbol{b} = b_x \boldsymbol{i} + b_y \boldsymbol{j} + b_z \boldsymbol{k}$$

The vector product is

$$\boldsymbol{a} \times \boldsymbol{b} = (a_x \boldsymbol{i} + a_y \boldsymbol{j} + a_z \boldsymbol{k}) \times (b_x \boldsymbol{i} + b_y \boldsymbol{j} + b_z \boldsymbol{k})$$

Expanding in accordance with the distributive law gives

$$\begin{aligned} \boldsymbol{a} \times \boldsymbol{b} = {} & (a_x b_x \boldsymbol{i} \times \boldsymbol{i}) + (a_x b_y \boldsymbol{i} \times \boldsymbol{j}) + (a_x b_z \boldsymbol{i} \times \boldsymbol{k}) \\ & + (a_y b_x \boldsymbol{j} \times \boldsymbol{i}) + (a_y b_y \boldsymbol{j} \times \boldsymbol{j}) + (a_y b_z \boldsymbol{j} \times \boldsymbol{k}) \\ & + (a_z b_x \boldsymbol{k} \times \boldsymbol{i}) + (a_z b_y \boldsymbol{k} \times \boldsymbol{j}) + (a_z b_z \boldsymbol{k} \times \boldsymbol{k}) \end{aligned}$$

Using the relationships for the vector products of unit vectors we obtain

$$\boldsymbol{a} \times \boldsymbol{b} = (a_y b_z - a_z b_y)\boldsymbol{i} + (a_z b_x - a_x b_z)\boldsymbol{j} + (a_x b_y - a_y b_x)\boldsymbol{k} \tag{2.12a}$$

The vector product may conveniently be written in determinant form. A detailed treatment of determinants can be found in Chap. 15.

$$\boldsymbol{a} \times \boldsymbol{b} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} \tag{2.12b}$$

Example The velocity of a point P on a rotating body is given by the vector product of the angular velocity and the position vector of the point from the axis of rotation. In Fig. 2.21, if the z-axis is the axis of rotation, the angular velocity $\boldsymbol{\omega}$ is a vector along this axis. If the position vector of a point P is $\boldsymbol{r} = (0, r_y, r_z)$ and the angular velocity $\boldsymbol{\omega} = (0, 0, \boldsymbol{\omega}_z)$, as shown in the figure, then the velocity $\boldsymbol{v}$ of P is

$$\boldsymbol{v} = \boldsymbol{\omega} \times \boldsymbol{r} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 0 & 0 & \omega_z \\ 0 & r_y & r_z \end{vmatrix} = -r_y \omega_z \boldsymbol{i}$$

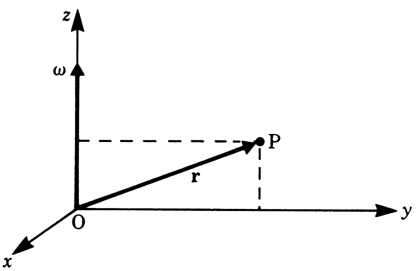

Fig. 2.21

Exercises

2.1 Scalar Product

1. Calculate the scalar products of the vectors **a** and $\boldsymbol{b}$ given below:

(a) $\boldsymbol{a} = 3$ $\boldsymbol{b} = 2$ $\alpha = \pi/3$ (b) $\boldsymbol{a} = 2$ $\boldsymbol{b} = 5$ $\alpha = 0$

(c) $\boldsymbol{a} = 1$ $\boldsymbol{b} = 4$ $\alpha = \pi/4$ (d) $\boldsymbol{a} = 2.5$ $\boldsymbol{b} = 3$ $\alpha = 120°$

2. Considering the scalar products, what can you say about the angle between the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$?

(a) $\boldsymbol{a} \cdot \boldsymbol{b} = 0$ (b) $\boldsymbol{a} \cdot \boldsymbol{b} = ab$

(c) $\boldsymbol{a} \cdot \boldsymbol{b} = \frac{ab}{2}$ (d) $\boldsymbol{a} \cdot \boldsymbol{b} < 0$

3. Calculate the scalar product of the following vectors:

(a) $\boldsymbol{a} = (3,\ -1,\ 4)$, $\boldsymbol{b} = (-1,\ 2,\ 5)$ (b) $\boldsymbol{a} = (3/2,\ 1/4,\ -1/3)$, $\boldsymbol{b} = (1/6,\ -2,\ 3)$

(c) $\boldsymbol{a} = (-1/4,\ 2,\ -1)$, $\boldsymbol{b} = (1,\ 1/2,\ 5/3)$ (d) $\boldsymbol{a} = (1,\ -6,\ 1)$, $\boldsymbol{b} = (-1,\ -1,\ -1)$

2. Which of the following vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are perpendicular?

(a) $\boldsymbol{a} = (0,\ -1,\ 1)$, $\boldsymbol{b} = (1,\ 0,\ 0)$ (b) $\boldsymbol{a} = (2,\ -3,\ 1)$, $\boldsymbol{b} = (-1,\ 4,\ 2)$

(c) $\boldsymbol{a} = (-1,\ 2,\ -5)$, $\boldsymbol{b} = (-8,\ 1,\ 2)$ (d) $\boldsymbol{a} = (4,\ -3,\ 1)$, $\boldsymbol{b} = (-1,\ -2,\ -2)$

(e) $\boldsymbol{a} = (2,\ 1,\ 1)$, $\boldsymbol{b} = (-1,\ 3,\ -2)$ (f) $\boldsymbol{a} = (4,\ 2,\ 2)$, $\boldsymbol{b} = (1,\ -4,\ 2)$

5. Calculate the angle between the two vectors **a** and $\boldsymbol{b}$:

(a) $\boldsymbol{a} = (1,\ -1,\ 1)$, $\boldsymbol{b} = (-1,\ 1,\ -1)$ (b) $\boldsymbol{a} = (-2,\ 2,\ -1)$, $\boldsymbol{b} = (0,\ 3,\ 0)$

6. A force $\mathbf{F} = (0\,\text{N},\ 5\,\text{N})$ is applied to a body and moves it through a distance **s**. Calculate the work done by the force.

(a) $\mathbf{s_1} = (3\,\text{m},\ 3\,\text{m})$ (b) $\mathbf{s_2} = (2\,\text{m},\ 1\,\text{m})$ (c) $\mathbf{s_3} = (2\,\text{m},\ 0\,\text{m})$

2.2 Vector Product

7. Indicate in figures 2.22 and 2.23 the direction of the vector $\boldsymbol{c}$ if $\mathbf{c} = \boldsymbol{a} \times \boldsymbol{b}$

(a) when $\boldsymbol{a}$ and $\boldsymbol{b}$ lie in the $x-y$ plane

(b) when $\boldsymbol{a}$ and $\boldsymbol{b}$ lie in the $y-z$ plane

8. Calculate the magnitude of the vector product of the following vectors:

(a) $a = 2$ $b = 3$ $\alpha = 60°$ (b) $a = 1/2$ $b = 4$ $\alpha = 0°$

(c) $a = 8$ $b = 3/4$ $\alpha = 90°$

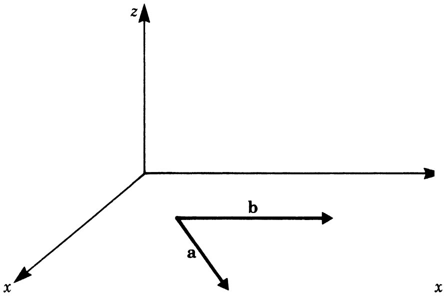

Fig. 2.22

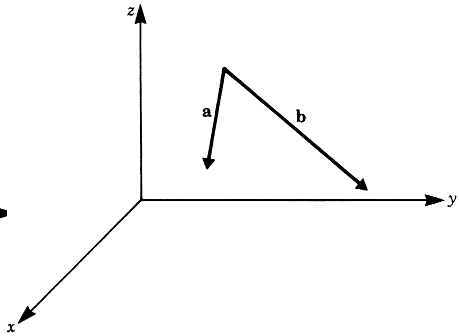

Fig. 2.23

9. In figure 2.24 $\boldsymbol{a} = 2\mathbf{i}$, $\mathbf{b} = 4\boldsymbol{j}$, $\boldsymbol{c} = -3\boldsymbol{k}$ ($\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$ are the unit vectors along the x-, y- and z-axes, respectively). Calculate

(a) $\boldsymbol{a} \times \boldsymbol{b}$ (b) $\boldsymbol{a} \times \boldsymbol{c}$ (c) $\boldsymbol{c} \times \boldsymbol{a}$

(d) $\boldsymbol{b} \times \boldsymbol{c}$ (e) $\boldsymbol{b} \times \boldsymbol{b}$ (f) $\boldsymbol{c} \times \boldsymbol{b}$

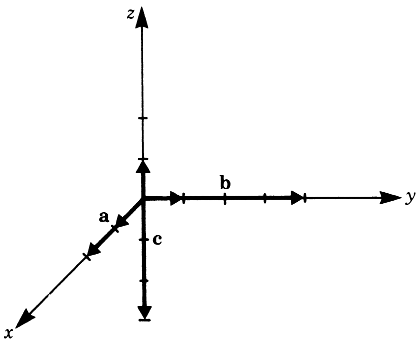

Fig. 2.24

10. Calculate $\boldsymbol{c} = \boldsymbol{a} \times \boldsymbol{b}$ when

(a) $\boldsymbol{a} = (2,\ 3,\ 1)$
$\boldsymbol{b} = (-1,\ 2,\ 4)$

(b) $\boldsymbol{a} = (-2,\ 1,\ 0)$
$\boldsymbol{b} = (1,\ 4,\ 3)$

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