

# Contents

<b>Preface and Introduction</b> .....	vii
<b>1 Vector Algebra I: Scalars and Vectors</b> .....	1
1.1 Scalars and Vectors .....	1
1.2 Addition of Vectors .....	4
1.2.1 Sum of Two Vectors: Geometrical Addition .....	4
1.3 Subtraction of Vectors .....	6
1.4 Components and Projection of a Vector .....	7
1.5 Component Representation in Coordinate Systems .....	9
1.5.1 Position Vector .....	9
1.5.2 Unit Vectors .....	10
1.5.3 Component Representation of a Vector .....	11
1.5.4 Representation of the Sum of Two Vectors in Terms of Their Components .....	12
1.5.5 Subtraction of Vectors in Terms of their Components .....	13
1.6 Multiplication of a Vector by a Scalar .....	14
1.7 Magnitude of a Vector .....	15
<b>2 Vector Algebra II: Scalar and Vector Products</b> .....	23
2.1 Scalar Product .....	23
2.1.1 Application: Equation of a Line and a Plane .....	26
2.1.2 Special Cases .....	26
2.1.3 Commutative and Distributive Laws .....	27
2.1.4 Scalar Product in Terms of the Components of the Vectors ..	27
2.2 Vector Product .....	30
2.2.1 Torque .....	30
2.2.2 Torque as a Vector .....	31
2.2.3 Definition of the Vector Product .....	32
2.2.4 Special Cases .....	33
2.2.5 Anti-Commutative Law for Vector Products .....	33
2.2.6 Components of the Vector Product .....	34

<b>3</b>	<b>Functions</b>	39
3.1	The Mathematical Concept of Functions and its Meaning in Physics and Engineering	39
3.1.1	Introduction	39
3.1.2	The Concept of a Function	40
3.2	Graphical Representation of Functions	42
3.2.1	Coordinate System, Position Vector	42
3.2.2	The Linear Function: The Straight Line	43
3.2.3	Graph Plotting	44
3.3	Quadratic Equations	47
3.4	Parametric Changes of Functions and Their Graphs	49
3.5	Inverse Functions	50
3.6	Trigonometric or Circular Functions	52
3.6.1	Unit Circle	52
3.6.2	Sine Function	53
3.6.3	Cosine Function	58
3.6.4	Relationships Between the Sine and Cosine Functions	59
3.6.5	Tangent and Cotangent	61
3.6.6	Addition Formulae	62
3.7	Inverse Trigonometric Functions	64
3.8	Function of a Function (Composition)	66
<b>4</b>	<b>Exponential, Logarithmic and Hyperbolic Functions</b>	71
4.1	Powers, Exponential Function	71
4.1.1	Powers	71
4.1.2	Laws of Indices or Exponents	72
4.1.3	Binomial Theorem	73
4.1.4	Exponential Function	73
4.2	Logarithm, Logarithmic Function	76
4.2.1	Logarithm	76
4.2.2	Operations with Logarithms	78
4.2.3	Logarithmic Functions	79
4.3	Hyperbolic Functions and Inverse Hyperbolic Functions	80
4.3.1	Hyperbolic Functions	80
4.3.2	Inverse Hyperbolic Functions	83
<b>5</b>	<b>Differential Calculus</b>	87
5.1	Sequences and Limits	87
5.1.1	The Concept of Sequence	87
5.1.2	Limit of a Sequence	88
5.1.3	Limit of a Function	91
5.1.4	Examples for the Practical Determination of Limits	91
5.2	Continuity	93

5.3	Series .....	94
5.3.1	Geometric Series .....	95
5.4	Differentiation of a Function .....	96
5.4.1	Gradient or Slope of a Line .....	96
5.4.2	Gradient of an Arbitrary Curve .....	97
5.4.3	Derivative of a Function .....	99
5.4.4	Physical Application: Velocity .....	100
5.4.5	The Differential .....	101
5.5	Calculating Differential Coefficients .....	102
5.5.1	Derivatives of Power Functions; Constant Factors .....	103
5.5.2	Rules for Differentiation .....	104
5.5.3	Differentiation of Fundamental Functions .....	108
5.6	Higher Derivatives .....	114
5.7	Extreme Values and Points of Inflexion; Curve Sketching .....	115
5.7.1	Maximum and Minimum Values of a Function .....	115
5.7.2	Further Remarks on Points of Inflexion (Contraflexure) ...	119
5.7.3	Curve Sketching .....	120
5.8	Applications of Differential Calculus .....	123
5.8.1	Extreme Values .....	123
5.8.2	Increments .....	124
5.8.3	Curvature .....	125
5.8.4	Determination of Limits by Differentiation: L'Hôpital's Rule .....	127
5.9	Further Methods for Calculating Differential Coefficients .....	129
5.9.1	Implicit Functions and their Derivatives .....	129
5.9.2	Logarithmic Differentiation .....	130
5.10	Parametric Functions and their Derivatives .....	131
5.10.1	Parametric Form of an Equation .....	131
5.10.2	Derivatives of Parametric Functions .....	136
<b>6</b>	<b>Integral Calculus</b> .....	<b>147</b>
6.1	The Primitive Function .....	147
6.1.1	Fundamental Problem of Integral Calculus .....	147
6.2	The Area Problem: The Definite Integral .....	149
6.3	Fundamental Theorem of the Differential and Integral Calculus .....	151
6.4	The Definite Integral .....	155
6.4.1	Calculation of Definite Integrals from Indefinite Integrals ..	155
6.4.2	Examples of Definite Integrals .....	158
6.5	Methods of Integration .....	161
6.5.1	Principle of Verification .....	161
6.5.2	Standard Integrals .....	161

6.5.3	Constant Factor and the Sum of Functions	162
6.5.4	Integration by Parts: Product of Two Functions	163
6.5.5	Integration by Substitution	166
6.5.6	Substitution in Particular Cases	168
6.5.7	Integration by Partial Fractions	172
6.6	Rules for Solving Definite Integrals	177
6.7	Mean Value Theorem	180
6.8	Improper Integrals	181
6.9	Line Integrals	183
<b>7</b>	<b>Applications of Integration</b>	<b>193</b>
7.1	Areas	193
7.1.1	Areas for Parametric Functions	196
7.1.2	Areas in Polar Coordinates	197
7.1.3	Areas of Closed Curves	199
7.2	Lengths of Curves	200
7.2.1	Lengths of Curves in Polar Coordinates	203
7.3	Surface Area and Volume of a Solid of Revolution	204
7.4	Applications to Mechanics	210
7.4.1	Basic Concepts of Mechanics	210
7.4.2	Center of Mass and Centroid	210
7.4.3	The Theorems of Pappus	213
7.4.4	Moments of Inertia; Second Moment of Area	215
<b>8</b>	<b>Taylor Series and Power Series</b>	<b>229</b>
8.1	Introduction	229
8.2	Expansion of a Function in a Power Series	230
8.3	Interval of Convergence of Power Series	234
8.4	Approximate Values of Functions	235
8.5	Expansion of a Function $f(x)$ at an Arbitrary Position	237
8.6	Applications of Series	239
8.6.1	Polynomials as Approximations	239
8.6.2	Integration of Functions when Expressed as Power Series	242
8.6.3	Expansion in a Series by Integrating	244
<b>9</b>	<b>Complex Numbers</b>	<b>249</b>
9.1	Definition and Properties of Complex Numbers	249
9.1.1	Imaginary Numbers	249
9.1.2	Complex Numbers	250
9.1.3	Fields of Application	250
9.1.4	Operations with Complex Numbers	251
9.2	Graphical Representation of Complex Numbers	252
9.2.1	Gauss Complex Number Plane: Argand Diagram	252
9.2.2	Polar Form of a Complex Number	253

9.3	Exponential Form of Complex Numbers	256
9.3.1	Euler's Formula	256
9.3.2	Exponential Form of the Sine and Cosine Functions	257
9.3.3	Complex Numbers as Powers	257
9.3.4	Multiplication and Division in Exponential Form	260
9.3.5	Raising to a Power, Exponential Form	261
9.3.6	Periodicity of $re^{j\alpha}$	261
9.3.7	Transformation of a Complex Number From One Form into Another	262
9.4	Operations with Complex Numbers Expressed in Polar Form	263
9.4.1	Multiplication and Division	263
9.4.2	Raising to a Power	265
9.4.3	Roots of a Complex Number	265
<b>10</b>	<b>Differential Equations</b>	<b>275</b>
10.1	Concept and Classification of Differential Equations	275
10.2	Preliminary Remarks	279
10.3	General Solution of First- and Second-Order DEs with Constant Coefficients	281
10.3.1	Homogeneous Linear DE	281
10.3.2	Non-Homogeneous Linear DE	287
10.4	Boundary Value Problems	293
10.4.1	First-Order DEs	293
10.4.2	Second-Order DEs	293
10.5	Some Applications of DEs	295
10.5.1	Radioactive Decay	295
10.5.2	The Harmonic Oscillator	296
10.6	General Linear First-Order DEs	304
10.6.1	Solution by Variation of the Constant	304
10.6.2	A Straightforward Method Involving the Integrating Factor	306
10.7	Some Remarks on General First-Order DEs	308
10.7.1	Bernoulli's Equations	308
10.7.2	Separation of Variables	309
10.7.3	Exact Equations	310
10.7.4	The Integrating Factor – General Case	313
10.8	Simultaneous DEs	315
10.9	Higher-Order DEs Interpreted as Systems of First-Order Simultaneous DEs	319
10.10	Some Advice on Intractable DEs	319
<b>11</b>	<b>Laplace Transforms</b>	<b>323</b>
11.1	Introduction	323
11.2	The Laplace Transform Definition	323
11.3	Laplace Transform of Standard Functions	324

11.4	Solution of Linear DEs with Constant Coefficients . . . . .	330
11.5	Solution of Simultaneous DEs with Constant Coefficients . . . . .	332
<b>12</b>	<b>Functions of Several Variables; Partial Differentiation; and Total Differentiation . . . . .</b>	<b>339</b>
12.1	Introduction . . . . .	339
12.2	Functions of Several Variables . . . . .	340
12.2.1	Representing the Surface by Establishing a Table of Z-Values . . . . .	341
12.2.2	Representing the Surface by Establishing Intersecting Curves . . . . .	342
12.2.3	Obtaining a Functional Expression for a Given Surface . . . . .	345
12.3	Partial Differentiation . . . . .	346
12.3.1	Higher Partial Derivatives . . . . .	350
12.4	Total Differential . . . . .	352
12.4.1	Total Differential of Functions . . . . .	352
12.4.2	Application: Small Tolerances . . . . .	356
12.4.3	Gradient . . . . .	358
12.5	Total Derivative . . . . .	360
12.5.1	Explicit Functions . . . . .	360
12.5.2	Implicit Functions . . . . .	362
12.6	Maxima and Minima of Functions of Two or More Variables . . . . .	363
12.7	Applications: Wave Function and Wave Equation . . . . .	369
12.7.1	Wave Function . . . . .	369
12.7.2	Wave Equation . . . . .	373
<b>13</b>	<b>Multiple Integrals; Coordinate Systems . . . . .</b>	<b>379</b>
13.1	Multiple Integrals . . . . .	379
13.2	Multiple Integrals with Constant Limits . . . . .	381
13.2.1	Decomposition of a Multiple Integral into a Product of Integrals . . . . .	383
13.3	Multiple Integrals with Variable Limits . . . . .	384
13.4	Coordinate Systems . . . . .	388
13.4.1	Polar Coordinates . . . . .	389
13.4.2	Cylindrical Coordinates . . . . .	391
13.4.3	Spherical Coordinates . . . . .	393
13.5	Application: Moments of Inertia of a Solid . . . . .	397
<b>14</b>	<b>Transformation of Coordinates; Matrices . . . . .</b>	<b>403</b>
14.1	Introduction . . . . .	403
14.2	Parallel Shift of Coordinates: Translation . . . . .	406
14.3	Rotation . . . . .	409
14.3.1	Rotation in a Plane . . . . .	409
14.3.2	Successive Rotations . . . . .	412
14.3.3	Rotations in Three-Dimensional Space . . . . .	413

14.4	Matrix Algebra	415
14.4.1	Addition and Subtraction of Matrices	417
14.4.2	Multiplication of a Matrix by a Scalar	418
14.4.3	Product of a Matrix and a Vector	418
14.4.4	Multiplication of Two Matrices	419
14.5	Rotations Expressed in Matrix Form	421
14.5.1	Rotation in Two-Dimensional Space	421
14.5.2	Special Rotation in Three-Dimensional Space	422
14.6	Special Matrices	423
14.7	Inverse Matrix	426
<b>15</b>	<b>Sets of Linear Equations; Determinants</b>	<b>431</b>
15.1	Introduction	431
15.2	Sets of Linear Equations	431
15.2.1	Gaussian Elimination: Successive Elimination of Variables	431
15.2.2	Gauss–Jordan Elimination	433
15.2.3	Matrix Notation of Sets of Equations and Determination of the Inverse Matrix	434
15.2.4	Existence of Solutions	437
15.3	Determinants	440
15.3.1	Preliminary Remarks on Determinants	440
15.3.2	Definition and Properties of an $n$ -Row Determinant	441
15.3.3	Rank of a Determinant and Rank of a Matrix	446
15.3.4	Applications of Determinants	447
<b>16</b>	<b>Eigenvalues and Eigenvectors of Real Matrices</b>	<b>453</b>
16.1	Two Case Studies: Eigenvalues of $2 \times 2$ Matrices	453
16.2	General Method for Finding Eigenvalues	456
16.3	Worked Example: Eigenvalues of a $3 \times 3$ Matrix	458
16.4	Important Facts on Eigenvalues and Eigenvectors	460
<b>17</b>	<b>Vector Analysis: Surface Integrals, Divergence, Curl and Potential</b>	<b>463</b>
17.1	Flow of a Vector Field Through a Surface Element	463
17.2	Surface Integral	466
17.3	Special Cases of Surface Integrals	468
17.3.1	Flow of a Homogeneous Vector Field Through a Cuboid	468
17.3.2	Flow of a Spherically Symmetrical Field Through a Sphere	470
17.3.3	Application: The Electrical Field of a Point Charge	472
17.4	General Case of Computing Surface Integrals	472
17.5	Divergence of a Vector Field	477
17.6	Gauss's Theorem	480
17.7	Curl of a Vector Field	482
17.8	Stokes' Theorem	486

17.9	Potential of a Vector Field .....	487
17.10	Short Reference on Vector Derivatives .....	490
<b>18</b>	<b>Fourier Series; Harmonic Analysis .....</b>	<b>493</b>
18.1	Expansion of a Periodic Function into a Fourier Series .....	493
18.1.1	Evaluation of the Coefficients .....	494
18.1.2	Odd and Even Functions .....	497
18.2	Examples of Fourier Series .....	498
18.3	Expansion of Functions of Period $2L$ .....	503
18.4	Fourier Spectrum .....	504
<b>19</b>	<b>Fourier Integrals and Fourier Transforms .....</b>	<b>509</b>
19.1	Transition from Fourier Series to Fourier Integral .....	509
19.2	Fourier Transforms .....	511
19.2.1	Fourier Cosine Transform .....	511
19.2.2	Fourier Sine Transform, General Fourier Transform .....	512
19.2.3	Complex Representation of the Fourier Transform .....	514
19.3	Shift Theorem .....	515
19.4	Discrete Fourier Transform, Sampling Theorems .....	516
19.5	Fourier Transform of the Gaussian Function .....	516
<b>20</b>	<b>Probability Calculus .....</b>	<b>519</b>
20.1	Introduction .....	519
20.2	Concept of Probability .....	520
20.2.1	Random Experiment, Outcome Space and Events .....	520
20.2.2	The Classical Definition of Probability .....	521
20.2.3	The Statistical Definition of Probability .....	521
20.2.4	General Properties of Probabilities .....	523
20.2.5	Probability of Statistically Independent Events. Compound Probability .....	525
20.3	Permutations and Combinations .....	527
20.3.1	Permutations .....	527
20.3.2	Combinations .....	528
<b>21</b>	<b>Probability Distributions .....</b>	<b>531</b>
21.1	Discrete and Continuous Probability Distributions .....	531
21.1.1	Discrete Probability Distributions .....	531
21.1.2	Continuous Probability Distributions .....	534
21.2	Mean Values of Discrete and Continuous Variables .....	537
21.3	The Normal Distribution as the Limiting Value of the Binomial Distribution .....	539
21.3.1	Properties of the Normal Distribution .....	542
21.3.2	Derivation of the Binomial Distribution .....	544



<b>22 Theory of Errors</b> .....	549
22.1 Purpose of the Theory of Errors .....	549
22.2 Mean Value and Variance .....	550
22.2.1 Mean Value .....	550
22.2.2 Variance and Standard Deviation .....	551
22.2.3 Mean Value and Variance in a Random Sample and Parent Population .....	552
22.3 Mean Value and Variance of Continuous Distributions .....	554
22.4 Error in Mean Value .....	556
22.5 Normal Distribution: Distribution of Random Errors .....	557
22.6 Law of Error Propagation .....	558
22.7 Weighted Average .....	560
22.8 Curve Fitting: Method of Least Squares, Regression Line .....	561
22.9 Correlation and Correlation Coefficient .....	564
<b>Answers</b> .....	569
<b>Index</b> .....	595

<http://www.springer.com/978-3-642-54123-0>

Mathematics for Physicists and Engineers

Fundamentals and Interactive Study Guide

Weltner, K.; John, S.T.; Weber, W.J.; Schuster, P.;

Grosjean, J.

2014, XVII, 602 p. 385 illus. With online files/update.,

Hardcover

ISBN: 978-3-642-54123-0