Preface

Probability theory was at its origin, almost solely, centered around games of chance (cards and dice). With the *Ars Conjectandi*, Jakob Bernoulli, while obtaining the first version of the theorem now known as the Law of Large Numbers (LLN), moved the theory of probability away from being primarily a vehicle for calculating gambling odds. This step has been crucial by showing that the probability theory might have an important role in the understanding of a variety of problems in many areas of the natural sciences and human experiences.

In 1913, when the tsar Nicholas II called for celebrations of the 300th Anniversary of the Romanov rule, the great Russian mathematician Andrei Andreyevich Markov responded by organizing a symposium aimed at commemorating a different anniversary. Markov took the occasion to celebrate the bicentenary of Bernoulli's *Ars conjectandi*: Bernoulli actually completed his book by 1690, but the book was only published posthumously in 1713 by his nephew Niklaus because of family quarrels.

Nowadays, one century after Markov, the autocratic tsarist government is over and we can take the occasion to celebrate the Law of Large Numbers with no need of extra scientific pretexts.

The LLN is at the base of a scientific legacy whose relevance cannot be overestimated. We can start mentioning the great visionary idea of the ergodic hypothesis by Ludwig Boltzmann. The ergodicity issue, originally introduced in the context of the statistical mechanics and then developed as an autonomous branch of measure theory, can be seen as the generalization of the LLN to non-independent variables. This topic is still an active research field in mathematical physics. In addition, it constitutes the starting point of the numerical methods used in statistical mechanics, namely the molecular dynamics and the Monte Carlo methods.

The most important physical properties of macroscopic objects are determined by mean values, whose mathematical base is guaranteed by the LLN. But, in many cases, also the fluctuations can be important. The control of "small" fluctuations around the mean value is provided by the Central Limit Theorem (CLT), whose general relevance was established for the first time in 1812 with the book *Théorie Analytique des Probabilités* by Pierre Simon Laplace. From a physical point of view, however, even very small fluctuations can be dramatically important. As a paradigmatic example, we can mention the treatment of the Brownian motion, which among the many still in progress applications brought conclusive evidence for the atomic hypothesis.

Beyond their conceptual importance, thanks to their link to response functions via the fluctuation-dissipation theorem, fluctuations are becoming more and more important in present-day applications, especially via the recently established fluctuation relations. Their relevance is amplified in small (micro- and nano-) systems, and in materials (as granular matter) where the number of effective elementary constituents is not as large as in gases or liquids. In such systems large excursions from the average cannot be neglected, therefore it is necessary to go beyond the Gaussian approximation, i.e. beyond the realm of validity of the CLT. The proper technical tool to study such strong fluctuations is the Large Deviation Theory (LDT), which generalizes the CLT.

The first general mathematical formulation of the Large Deviation Theory is due to Harald Cramér in the 1930s. However, the very first application of LDT can be ascribed to Boltzmann who, using combinatorial arguments, had been able to show the relevance of the entropy as a bridge between microscopic and macroscopic levels.

This book encompasses some recent developments of the fundamental limit theorems – LLN, CLT and LDT – of the probability theory in statistical physics, in particular: ergodicity breaking, non-equilibrium and fluctuation relations, disordered systems, computational methods, systems with long-range interactions, Brownian motors, chaotic dynamics, anomalous diffusion and turbulence.

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