Chapter 2
Basic Field-Solver Techniques for RC Extraction

Because 3-D numerical methods accurately model the realistic geometry, they possess the highest precision. The field solver based on 3-D numerical methods is not only used as a library-building tool in industrial RC extraction but also of increasing importance for the modeling and analysis of critical nets or key signal integrity issues. The major challenge for applying field solver is due to its low computational speed. Therefore, a lot of research has been devoted to improve the computational efficiency of 3-D capacitance extraction methods. With these works and the widely spreading parallel computing techniques, it is now possible to directly use the field solver in chip-scale extraction tasks. This is strongly demanded because of the increased accuracy requirement of parasitic extraction under the nanometer process technologies.

In this chapter, the principle and basic methods of 3-D field solver are to be introduced, with the emphasis on capacitance extraction. They are the basis of other RC extraction problems and more cutting-edge techniques for capacitance extraction. Those problems and techniques will be presented in detail, with the following chapters of this book.

2.1 Problem Formulation

As it is well known, the capacitor is a kind of circuit elements commonly used in electric or electronic equipments. It is usually composed of two conductors insulated from each other. When charged, the two surfaces of the conductors facing each other carry equal and opposite charges: Q and $-Q$, respectively (see Fig. 2.1). The electric potential difference between the two conductors $\phi_1 - \phi_2$ is called the voltage of the capacitor and is always denoted by V. Experiments, and theoretical analyses show that, for a capacitor, Q is always proportional to V and thus the ratio Q/V is a constant determined by the structure of the capacitor. This ratio is called the *capacitance* of the capacitor and denoted by C: $C = Q/V$.

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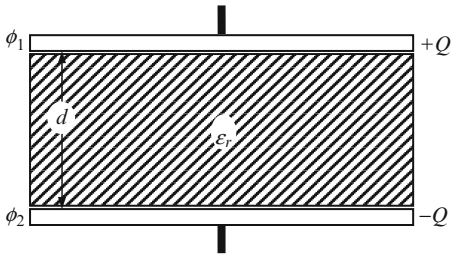

Fig. 2.1 A parallel plate capacitor

The SI unit of capacitance is faraday (F). It is the capacitance of a capacitor that has one coulomb on one of its conductor polar when the potential difference is 1 V. Other commonly used units of capacitance are μF (10^{-6} F), pF (10^{-12} F), and fF (10^{-15} F).

The capacitance of some simple capacitor can be calculated easily. For example, for the parallel plate capacitor shown in Fig. 2.1, we have

$$C = \frac{\varepsilon_0 \varepsilon_r S}{d}, \tag{2.1}$$

where ε_0 is the dielectric constant of free space (supposing that the plate's dimension is much larger than d). The dielectric constant takes the value

$$\varepsilon_0 = \frac{1}{4\pi \times 9 \times 10^9} = 8.85 \times 10^{-12}\ C^2/N \cdot m^2. \tag{2.2}$$

ε_r is the relative permittivity of the insulating material, S is the area of the facing plate, and d the distance between two parallel plates.

Actually, the capacitor has more generalized forms than that described above, which consists of two insulated conductors. The capacitance of a single conductor (conductor 1) is defined as if another conductor (conductor 2) was located at an infinite distance away to form a joint capacitor (conductors $1 + 2$). For example, the capacitance of an isolated conductor sphere with radius of R can be calculated as $C = 4\pi\varepsilon_0 R$.

There are many conductor interconnect wires in an integrated circuit (IC). The wires are insulated by some dielectric such as oxide SiO_2. The capacitance between any two wires reflects the electrostatic coupling between them. Calculating these capacitances with high accuracy is very important for the analysis of the circuit's performance.

For an N-conductor system, such as the interconnect wires in an IC, an $N \times N$ capacitance matrix $[C_{ij}]_{N\times N}$ is defined by

$$Q_i = \sum_{j=1}^{N} C_{ij}\phi_j, \quad i = 1, 2, \ldots, N, \tag{2.3}$$

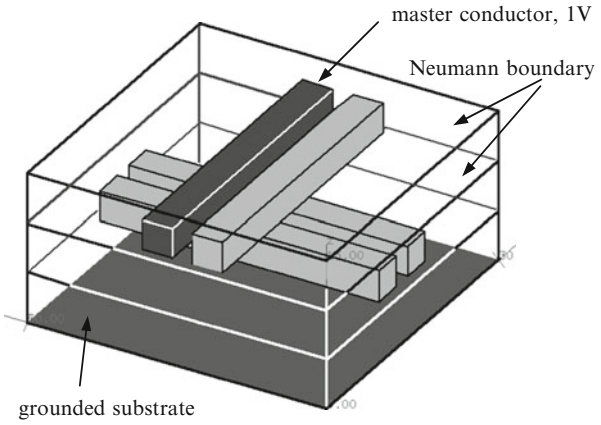

Fig. 2.2 A structure involving 2 × 2 crossover interconnect wires (Reprinted with permission from Yu and Wang [180] © 2005 John Wiley & Sons)

where C_{ij} $(i \neq j)$ is the coupling capacitance between conductor i and j and C_{ii} is called the self-capacitance or total capacitance of conductor i. Q_i is the induced charge on conductor i; ϕ_j is the electric potential of conductor j (usually the known bias voltage). Figure 2.2 shows typical crossover wires in VLSI circuit, where the coupling capacitance between any two conductors needs to be calculated.

Accurate modeling of the wire capacitances in a state-of-the-art IC is not a trivial task. The capacitance of a interconnect wire is a function of its shape, environment, distance from the substrate, and distance to surrounding wires. This usually calls for solving the electrostatic field in a region involving multiple dielectrics. Take the structure with three dielectric layers in Fig. 2.2 as an example. With one conductor (called *master conductor*) setting 1 V and others (called *environment conductors*) 0 V, the electric potential ϕ is governed by the *Laplace equation* for each homogenous dielectric region [70]:

$$\nabla^2 \phi = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0. \tag{2.4}$$

Taking into account the boundary conditions of the whole simulated region and those at the dielectric interfaces, the electrostatic field (potential and field intensity) can be solved. After that, we can obtain the charges of conductors. Note that the charge of an environment conductor equals to the capacitance between it and the master conductor, due to (2.3).

According to the boundary assumptions for the whole simulated region, there are several different models for capacitance extraction:

- The first one is called the infinite-domain model, because in it the electrostatic field spreads to the infinite, resulting in an infinite problem space.

- The second is called finite-domain model, where the electrostatic field is restricted within a finite domain, with the Neumann condition on the outer boundary [164]: $\partial\phi/\partial n = 0$. This means the electric field is not able to spread out of the finite simulated region. The Neumann condition is also called reflective boundary condition and was introduced as the "magnetic wall" in Hong et al. [61].
- The third boundary assumption employed in capacitance extraction problem is with a Dirichlet condition at the outer boundary [83]: $\phi = 0$. This is the default setting when employing the floating random walk method (see Sect. 2.5).

It should be pointed out that the infinite-domain model is ideal for simulating isolated structures, but for the IC capacitance extraction, it is not the case due to the influence of neighboring conductors. On the other hand, the finite-domain model considers a part cut from actual layout of IC. However, it assumes a condition not generally existing. Now, the three models of capacitance extraction are all used, for different extraction scenarios using different numerical methods. Although they produce different capacitance results for a given conductor system, the difference would be negligible if suitable size of the outer boundary is set. In the following discussions, the numerical methods will be introduced along with their suitable extraction models.

The background and problem formulations for resistance extraction, substrate parasitic extraction, and variation-aware capacitance extraction will be presented in Chaps. 5, 6, and 8, respectively.

2.2 Overview of the Numerical Methods

Basically, the methods for 3-D field solver are classified as the domain discretization method, the boundary integral equation method, semi-analytical approaches, and the stochastic method. The domain discretization method includes the finite difference method (FDM) [129, 144], the finite element method (FEM) [29, 31, 118, 137, 149, 169], and the method of the measured equation of invariance (MEI) [73, 88, 142]. The boundary integral equation method includes the method of moment [60, 124], indirect boundary element method (BEM) [13, 14, 27, 28, 54, 77, 78, 86, 98–100, 105, 106, 111, 132, 133, 136, 146, 164, 170, 171, 193–196], and direct BEM [4–6, 37, 46, 58, 63, 72, 89, 179, 181]. The semi-analytical approaches are combination of the analytical formulas and some traditional numerical methods [21, 61, 151, 152, 199, 200]. The stochastic method is based on the theory of random walk method [22, 83].

FDM and FEM discretize the whole 3-D simulated domain, thus producing a linear algebra system with large order. Hence, the computational speed of these methods is largely limited. However, since both methods are relatively well established, they are still used in the industry as a reference tool with accurate values calculated under fine grids. For example, the famous software of 2-D/3-D capacitance extraction "Raphael" utilizes FDM, and the "Q3D" of Ansoft Corp. is based on FEM.

Since the 1990s, the boundary integral equation method (i.e., BEM) has begun to replace the domain discretization method, thanks to its high efficiency. In both indirect and direct BEMs, only boundary of 3-D domain is discretized, and a smaller system of linear equations is obtained. Problems encountered with complex boundary can also be effectively handled with BEM, with accuracy comparable to FEM. Thus, BEM with rapid computing techniques has become the focus in the research of 3-D capacitance field solver.

Below, the basic method of indirect BEM is introduced. The principles of direct BEM and FRW method are discussed in the following two subsections, respectively. As for other field-solver methods, the reader is referred to the relevant references or [180].

2.3 Indirect Boundary Element Method

The indirect boundary method can be regarded as a variation of the method of moment (MoM). Because only domain boundary needs to be discretized, the indirect BEM involves much fewer unknowns than FDM and FEM. However, it leads to a dense coefficient matrix, whose formulation and solution introduce many difficulties. The innovation of the fast multipole method (FMM), the low-rank matrix compression method, and the hierarchical method had made the indirect BEM more applicable. Now, indirect BEM with a fast computing technique has become a major choice for 3-D field solver. Several commercial capacitance solvers have been developed based on the advanced indirect BEM approaches [78, 147].

The indirect BEM is also called the *equivalent charge method*, whose boundary integral equation involves the surface charge density $\sigma(\boldsymbol{r}')$ as an unknown function:

$$\phi(\boldsymbol{r}) = \int_{\Gamma} G(\boldsymbol{r}, \boldsymbol{r}')\, \sigma(\boldsymbol{r}')\, \mathrm{d}a', \quad (\boldsymbol{r} \in \Gamma), \tag{2.5}$$

where $G(\boldsymbol{r}, \boldsymbol{r}')$ is the Green's function and $\phi(\boldsymbol{r})$ stands for the electrostatic potential at point $\boldsymbol{r}$. For free space, $G(\boldsymbol{r}, \boldsymbol{r}') = 1/\|\boldsymbol{r} - \boldsymbol{r}'\|$; Γ is the boundary surface. After solving the surface charge density $\sigma(\boldsymbol{r}')$, the charge on conductor i can be calculated with

$$Q_i = \int_{S_d(i)} \sigma(\boldsymbol{r}')\, \mathrm{d}a', \tag{2.6}$$

where $S_d(i)$ is the surface of conductor i. We discretize the surfaces of m conductors into n constant elements (or panels), where charge density is assumed to be element-wise constant. Then, the potential at the center of the kth panel $\boldsymbol{r}_k$ can be expressed as a sum of the contributions of all panels:

$$\phi_k = \sum_{j=1}^{n} \int_{\Gamma_j} \frac{\sigma_j(\boldsymbol{r}')}{\|\boldsymbol{r}' - \boldsymbol{r}_k\|} \mathrm{d}a', \tag{2.7}$$

where $\sigma_j(\boldsymbol{r}')$ is the surface charge density of panel j (Γ_j). Substituting the known boundary conditions, we obtain a dense linear equation system:

$$\boldsymbol{Pq} = \boldsymbol{b}, \tag{2.8}$$

where the coefficient matrix $\boldsymbol{P}$ is dense and nonsymmetric. The Krylov subspace iterative method, such as GMRES algorithm [126], is usually used to solve the equation.

For a problem involving multiple dielectrics, the polarization charge density on dielectric interface needs to be introduced, which contributes to the potential distribution together with the free charge density on conductor surfaces. Therefore, the problem becomes equivalent to that in the free space, and the simple free-space Green's function is used to form Eq. (2.8). Except for Eq. (2.5) on each conductor panel, the normal derivative of the potential satisfies

$$\varepsilon_a \frac{\partial \phi_+(\boldsymbol{r})}{\partial \boldsymbol{n}_a} = \varepsilon_b \frac{\partial \phi_-(\boldsymbol{r})}{\partial \boldsymbol{n}_a}, \tag{2.9}$$

where $\boldsymbol{r}$ is on the interface of two dielectrics with permittivities ε_a and ε_b, respectively. Here $\boldsymbol{n}_a$ denotes the normal to the dielectric interface at $\boldsymbol{r}$ that points into dielectric a. $\phi_+(\boldsymbol{r})$ is the potential at $\boldsymbol{r}$ approached from the side of the interface ε_a, and $\phi_-(\boldsymbol{r})$ is the analogous potential for the b side. With (2.9), a discretized linear equation with variables of charge densities on conductors and dielectric interfaces can be formed for each dielectric interface panel. They are combined with (2.8) to obtain a linear equation system with the same number of equations and unknown variables.

For the multi-dielectric problem, the so-called *total-charge Green's function approach* presented above involves more unknowns at dielectric interfaces. Another choice to deal with the problem is to employ the multilayered Green's function. Then, only the free charge density on conductor surfaces needs to be considered as unknown function. However, to evaluate the Green's function for the multilayered medium, infinite summations are involved, which are computationally expensive. Oh et al. [105] derived a closed-form expression of the Green's function for the multilayered medium by approximating the Green's function using a finite number of images in the spectral domain. This greatly reduces the computational task. Li et al. [86] presented for the first time general analytical formulas for the static Green's functions for shielded and open arbitrarily multilayered media. Zhao et al. [193] proposed an efficient scheme for the generation of multilayered Green's functions using a generalized image method. The multilayered Green's function is much more complicated than the free-space Green's function and only applicable to the simple stratified structure of multiple dielectrics. While for more complex structures, such as the conformal dielectric, the deduction of the Green's function may be impossible.

More research work has been conducted to accelerate the capacitance extraction using the total-charge Green's function approach. In 1991, Nabors et al. successfully

applied the multipole accelerated method, proposed earlier by Greengard and Rokhlin [57], to 3-D capacitance extraction with the indirect BEM. In the MPA method, calculation of the interaction between charges [i.e., the coefficients in (2.8)] is divided into two parts: the near-field computation and far-field computation. For the near-field computation, the coefficients are calculated directly; for the far-field computation, the multipole expansion and local expansion are used to accelerate the computation. Therefore, the CPU time of forming and solving (2.8) with iterative equation solver is greatly reduced. In the successive works [98, 100], the adaptive, preconditioned MPA method was developed. Corresponding software prototype FastCap is shared on the MIT's website [97] and has become a popular academic tool of capacitance extraction. Other works on the capacitance extraction using the multipole accelerated indirect BEM can be found in Beattie and Pileggi [14] and Pan et al. [106].

In 1998, a fast hierarchical algorithm for 3-D capacitance extraction was presented on the *Design Automation Conference* [132] and was later presented in a journal article [133]. Similar to the fast multipole algorithm, it is based on indirect BEM and also originated from the fast computation of the *N-body problem* [9]. For the singular integral kernel of $1/\|\boldsymbol{r}-\boldsymbol{r}'\|$, it can achieve high speedup of computation. And only $O(N)$ operations are needed for each iteration while solver the linear equation system. For other weaker singular kernel, the efficiency of this method may be reduced. In 1997, Kapur et al. proposed an accelerating approach based on the *singular-value decomposition* (SVD) [77]. It is independent on the kernel and based on the Galerkin method using the pulse function as the base function. It needs $O(N)$ times operation to construct the coefficient matrix and $O(N\log N)$ operations to perform an iteration in the equation solution. Besides, the precorrected fast Fourier transform (pFFT) algorithm [111] has the same computational complexity, while it is based on the collocation method for discretization. Other fast computing approaches for indirect BEM include those based on wavelet transformation [136], multiscale method [146], and the second kind of integral equation [110].

The aforementioned works on capacitance extraction with indirect BEM all handle the infinite-domain model. In 1996, Wang et al. improved the multipole accelerated indirect BEM to make it suitable to handle the finite-domain problem [164]. And a parallel multipole accelerated 3-D capacitance simulator based on nonuniformed cube partition was proposed. In Beattie and Pileggi [13], a window technique is combined with the indirect BEM to reduce its computational expense, along with an error bound analysis for the capacitance results.

The SVD-based fast approach [77] employs a low-rank matrix compression technique. This idea has been further developed [15, 16, 78] and evolved to several commercial capacitance solvers. The recently developed $\boldsymbol{H}$-matrix-based fast approaches [27, 28] can be regarded as another way for matrix compression, which however enable solving the obtained linear system with direct linear equation solver. Several enhancements have also been proposed for the fast hierarchical algorithm [133]. In Yan et al. [170], a sparsification technique was proposed for the coefficient matrix generated from indirect BEM, which also facilitates an efficient

preconditioning technique for the iterative solution of the linear equation system. Then, a matrix reduction technique was proposed in Yan et al. [171] to gain further computational speedup. In Zhou et al. [196], the total-charge Green's function approach was combined with the multilayered Green's function approach to efficiently handle the realistic interconnect structure with multilayer and conformal dielectrics. To extend the application of capacitance field solver, a divide-and-conquer method was proposed in Shi and Yu [134]. It invokes the fast hierarchical BEM solver and extends it to the extraction task with a whole critical net.

More recently, the parallel computing techniques have been proposed for the multipole accelerated fast capacitance solver, on platforms of multi-core CPU [54] or single-instruction-multiple-data (SIMD) graphic processing units (GPUs) [194].

In Chap. 3, the technical details of fast multipole method and low-rank matrix compression in the indirect BEM capacitance solvers will be presented.

2.4 Direct Boundary Element Method

The direct BEM is based on the *direct boundary integral equation (direct BIE)* and suitable for solving 3-D Laplace equation with various boundary conditions [24]. Specifically, the direct BEM is very efficient for handling the finite-domain model of capacitance extraction.

Within the finite domain that is involved in capacitance extraction (see Fig. 2.2), electric potential ϕ fulfills the following Laplace equation with mixed boundary conditions [181]:

$$\begin{cases} \varepsilon_i \nabla^2 \phi = 0, & \text{in } \Omega_i \quad (i = 1, \ldots, M) \\ \phi = \phi_0, & \text{on } \Gamma_u \\ q = \partial\phi / \partial \boldsymbol{n} = 0, & \text{on } \Gamma_q, \end{cases} \tag{2.10}$$

where the whole domain $\Omega = \underset{M}{\cup} \Omega_i$ and Ω_i stands for the space possessed by the ith dielectric. Γ_u stands for the Dirichlet boundary (conductor surfaces), where ϕ is known as the bias voltages; Γ_q represents the Neumann boundary (outer boundary of the simulated region), where electric flux q is supposed to be zero. Here $\boldsymbol{n}$ denotes the unit vector outward normal to the boundary. At the dielectric interface, the compatibility equation (2.9) holds.

With the fundamental solution as the weighting function, the Laplace equations in (2.4) are transformed into following direct BIEs by the Green's identity [24]:

$$c_s u_s^i + \int_{\partial\Omega_i} q^* u^i \, \mathrm{d}\Gamma = \int_{\partial\Omega_i} u^* q^i \, \mathrm{d}\Gamma, \quad (i = 1, \ldots, M), \tag{2.11}$$

where u_s^i stands for the electric potential at collocation point s (in dielectric region i) and c_s is a constant dependent on the boundary geometry near to the point s.

$u^* = 1/4\pi r$ is the fundamental solution of 3-D Laplace equation, whose derivative along the outward normal direction $\boldsymbol{n}$ is $q^* = \partial u^*/\partial \boldsymbol{n} = -\left(\overrightarrow{\boldsymbol{r}}, \overrightarrow{\boldsymbol{n}}\right)/4\pi r^3$. r is the distance from the collocation point s to the point on Γ. $\partial\Omega_i$ is the boundary that surrounds dielectric region i. Note that the fundamental solution u^* is actually the free-space Green's function in (2.5).

Employing the collocation method after discretizing the boundary, like that in the indirect BEM, we get a system of linear equations [181]:

$$\boldsymbol{A}\boldsymbol{x} = \boldsymbol{f}. \tag{2.12}$$

Finally, with the preconditioned Krylov iterative equation solver, the normal electric field intensity on the conductor surface is obtained [181].

In the direct BEM, both variables of potential and field intensity are involved. So, two kinds of integral kernels are dealt with. Though this is more complex than the indirect BEM, the direct BEM has its own advantages. Firstly, it is suitable for the capacitance extraction within the finite domain since two variables are included. On the second, because the variables in each BIE are within a same dielectric region, it has a so-called *localization character*. This character leads to a sparse linear system for problem with multiple dielectrics.

In the direct BEM, a great deal of time and memory are consumed in forming and solving the system of discretized BEM equations. Fukuda et al. [46] solved the problem of 2-D capacitance extraction using the direct BEM. In 1997, Bachtold et al. extended the multipole method to handle the "potential boundary integral" (whose kernel is $1/r^3$) in the direct BEM [4]. An adaptive boundary meshing scheme with an error indictor was also proposed. What they discussed was the model of multiple dielectrics within infinite domain. Hou et al. proposed an adaptive 3-D BEM solver for capacitance extraction [62], whose idea is to automatically meet the specified accuracy requirement by gradually refining the boundary meshes. In 2000, Gu et al. extended the fast hierarchical method used in the indirect BEM, making it feasible to the direct BEM-based capacitance extraction [58].

Yu et al. proposed a quasi-multiple medium (QMM) method, based on the localization character of direct BEM [181]. The QMM method exploits the sparsity of the resulting coefficient matrix when handling the multi-dielectric problem. Together with the efficient equation organization and iterative solving technology, the QMM-accelerated method greatly reduced the computing time and memory usage. In Yu et al. [181], an efficient analytical/semi-analytical integration scheme was also proposed to accurately calculate the boundary integrals under the VLSI planar process technology. The QMM method has been successfully applied to the actual 3-D multi-dielectric capacitance extraction [179, 181]. For finite-domain and multi-dielectric problems, the QMM-based method has shown 10× speedup and memory saving over the multipole accelerated indirect BEM (FastCap 2.0) with comparable accuracy [179]. Based on the direct BEM, techniques were recently proposed for fast 2-D capacitance extraction of nanometer VLSI interconnects [188].

There is another kind of approach for capacitance field solver, called *global approach*. It does not solve the resulting linear system in the usual way. The global approach discretizes the field equation and converts them to a circuit network of resistors or capacitors. Finally, with circuit reduction or matrix computation, the whole resistance or capacitance matrix can be obtained directly. In 1997, Dengi et al. proposed a global approach (called *macromodel method*) for 2-D interconnect capacitance extraction based on direct BEM [36, 37]. Lu et al. successfully extended the idea of boundary element macromodel to the 3-D case and developed a rapid *hierarchical block boundary element method (HBBEM)* for interconnect capacitance extraction [89]. HBBEM is suitable for extracting the whole capacitance matrix of an N-conductor system, with faster computational speed than the QMM-accelerated BEM. It have been extended and applied for the capacitance extraction problem in analog integrated circuits [176]. Besides, based on HBBEM, an IBM in-house tool (*CSurf*) has been developed for extracting the capacitances of interconnect and packaging structures [72].

In Chap. 4, the major techniques of QMM-accelerated BEM for capacitance extraction will be presented. Moreover, the fast direct BEM techniques have been developed for the resistance extraction and substrate extraction problems, which will be presented in Chaps. 5, 6, and 7.

2.5 Floating Random Walk Method

The FRW algorithm for capacitance extraction, presented in a 2-D version, was firstly proposed in 1992 [83]. Its basic idea is to convert the calculation of conductor charge to the Monte Carlo (MC) integration performed with floating random walks. This can be illustrated with the following equation:

$$\phi(\boldsymbol{r}) = \oint_S P\left(\boldsymbol{r}, \boldsymbol{r}^{(1)}\right)\phi\left(\boldsymbol{r}^{(1)}\right) \mathrm{d}\boldsymbol{r}^{(1)}, \tag{2.13}$$

where $\phi(\boldsymbol{r})$ is the electric potential at point $\boldsymbol{r}$ and S is a closed surface surrounding $\boldsymbol{r}$. $P(\boldsymbol{r}, \boldsymbol{r}^{(1)})$ is called the surface Green's function. For a fixed $\boldsymbol{r}$, $P(\boldsymbol{r}, \boldsymbol{r}^{(1)})$ can be regarded as the probability density function (PDF) for selecting a random point $\boldsymbol{r}^{(1)}$ on S. In this sense, $\phi(\boldsymbol{r})$ can be estimated by the mean value of $\phi(\boldsymbol{r}^{(1)})$, providing sufficient large number of sample points $\boldsymbol{r}^{(1)}$ on S are evaluated. If S is the surface of a homogeneous 3-D cube centered at $\boldsymbol{r}$, $P(\boldsymbol{r}, \boldsymbol{r}^{(1)})$ only depends on the relative position of $\boldsymbol{r}^{(1)}$ and is not related to the size of cube [69, 83]. More importantly, this surface Green's function can be derived analytically [69] and pre-calculated and stored as the discrete probabilities for jumping to the discretized cells of the cube surface.

In the situation that $\phi(\boldsymbol{r}^{(1)})$ is unknown, we apply (2.13) recursively to obtain the following nested integral formula:

$$\phi(\boldsymbol{r}) = \oint_{S^{(1)}} P^{(1)}\left(\boldsymbol{r}, \boldsymbol{r}^{(1)}\right) \oint_{S^{(2)}} P^{(2)}\left(\boldsymbol{r}^{(1)}, \boldsymbol{r}^{(2)}\right) \cdots \oint_{S^{(k+1)}} P^{(k+1)}\left(\boldsymbol{r}^{(k)}, \boldsymbol{r}^{(k+1)}\right) \phi\left(\boldsymbol{r}^{(k+1)}\right) \mathrm{d}\boldsymbol{r}^{(k+1)} \cdots \mathrm{d}\boldsymbol{r}^{(2)} \mathrm{d}\boldsymbol{r}^{(1)}, \tag{2.14}$$

where $S^{(i)}$, $(i = 1, \ldots, k+1)$ is the surface of the ith cube centered at $\boldsymbol{r}^{(i-1)}$. $P^{(i)}(\boldsymbol{r}^{(i-1)}, \boldsymbol{r}^{(i)})$, $(i = 1, \ldots, k+1)$, are the surface Green's functions relating the potentials at $\boldsymbol{r}^{(i-1)}$ to $\boldsymbol{r}^{(i)}$. This can be interpreted as a floating random walk procedure: for the ith hop of a walk, the maximum conductor-free cube centered at $\boldsymbol{r}^{(i-1)}$ is constructed, and then a point $\boldsymbol{r}^{(i)}$ is randomly selected on the cube surface according to the discrete probabilities obtained with $P^{(i)}(\boldsymbol{r}^{(i-1)}, \boldsymbol{r}^{(i)})$. Note that, to obtain the probabilities, we only need to consider the normalized unit-size cube for the ith cube and the corresponding positions of $\boldsymbol{r}^{(i-1)}$ and $\boldsymbol{r}^{(i)}$ in the unit-size cube. The walk terminates after k hops if the potential at point $\boldsymbol{r}^{(k)}$ is known, e.g., it is on a conductor surface in the problem of capacitance extraction. With the surface Green's function and derived sampling probabilities for a unit-size cube calculated in advance [69], the major cost of random walk is for geometric operations. After performing many walks, the mean value of these estimates approximates $\phi(\boldsymbol{r})$ very well.

For extracting the capacitance, each walk starts from a point on a Gaussian surface enclosing the master conductor and terminates on a conductor surface after some successive hops. For each hop of a walk, a conductor-free cube centered at current location is constructed, and the hop reaches a random point on the cube's boundary. The FRW algorithm does not rely on assembling any linear equation system and has several computational advantages over the deterministic methods: lower memory usage, more scalability for large structures, tunable accuracy, and better parallelism.

The 3-D FRW algorithm for capacitance extraction has been developed and applied to the design and analysis of VLSI circuits [69, 75, 82]. In 2005, Batterywala et al. proposed several techniques to reduce the variance of MC procedure in the FRW-based capacitance extraction [12] and further reduce the total computing time. The FRW algorithm was also extended to handle the floating dummy-fills [11]. In 2008, a technique based on the FRW algorithm was proposed to enable fast incremental variational capacitance extraction [43]. A general FRW algorithm was also proposed in El-Moselhy et al. [43] for arbitrary dielectric configuration, where the whole problem domain was covered by a set of cubic transition subdomains for which the transition probability is numerically calculated online, rather than offline. This technique largely reduces the number of hops for a FRW walk, with the overhead of calculating and storing the transition probabilities for a lot of transition domains. The general FRW algorithm can be very time-consuming for a large-scale 3-D problem. A hierarchical FRW (HFRW) algorithm was later proposed for a fabric-aware extraction problem [44, 45], where the topological variation rather than the common non-topological variation was considered. Note that the HFRW is not suitable for the general problem of capacitance extraction, because an arbitrary structure cannot be regarded as the composition of predefined "motif" structures. Different from most of FRW-based capacitance solvers which employ

the cubic transition domain to suit the Manhattan geometry of VLSI interconnects, the technique using spherical transition domains was investigated in Brambilla et al. [22, 23].

Compared with the BEM-based techniques, there are much fewer literatures devoted to the 3-D FRW algorithms for multi-dielectric capacitance extraction. To fill in the gap between the theory and application of the FRW-based capacitance solver, a program set called RWCap [125, 187] has been developed since 2012. An approach to precisely handle the multilayered dielectrics, a novel variance reduction scheme for acceleration, and a parallel implementation on the multi-core/multi-CPU platform were presented in Yu et al. [187]. Efficient techniques were also proposed to parallelize the FRW-based capacitance extraction on GPUs [189]. The RWCap was further enhanced with a comprehensive space management technique, which facilitates efficient capacitance extraction of chip-scale large interconnect structures [190].

The advanced FRW techniques in RWCap will be introduced in the last two chapters of this book.

2.6 Summary

In this chapter, the problem formulation for the 3-D capacitance field solver is firstly described. Then, we present a survey on the numerical methods for capacitance extraction. The principles of indirect BEM and direct BEM are introduced, which form the basis for the techniques presented in Chaps. 3 and 4. They also provide a necessary background for the materials in Chaps. 5, 6, 7, 8, and 9. Finally, the FRW method and its state of the art are briefly introduced. Detailed treatment of this method will be given in the last two chapters of this book.

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