

Chapter 2

Thermal Analysis of Friction Welding

Abstract Thermal energy is generated during the friction welding process. In this case, the solid surfaces rub against each other and heat is generated as a result of friction. Since the process is transient and involves with axis-symmetric heating situation, formulation of the thermal analysis becomes essential. In this chapter, thermal analysis based on Fourier heat conduction is introduced and the solution of conduction equation is obtained for appropriate boundary conditions.

Keywords Friction welding • Heating analysis • Temperature

2.1 Introduction

Welding of solid materials is achieved by providing thermal energy in the form of heat for melting or softening the interface between the two materials and bringing or pressing them together. Friction is one of the methods of generating the required thermal energy for welding process. As the solid surfaces rub against each other heat is generated as a result of friction. The heat generated due to friction subsequently diffuses through the bulk of the contacting solid materials. As the heat is necessary for obtaining sound welds, it also affects the mechanical as well as the micro-structural properties of the welded materials in the vicinity of the welding interface.

Thermal analysis of friction welding is carried out to determine the resulting temperature distribution around the welding interface and thus allows determination of the high temperature effects on the micro-structure of the materials as well as the quality of the weld. The thermal analysis related to the friction welding is carried out in line with the previous studies [1–12].

As an elementary example, consider two solid objects with flat surfaces pressed together with a force F and sliding against each other with a relative velocity of V . The power consumed against the frictional force $F_f = \mu F$ is converted to thermal energy generation at the interface. The rate of thermal energy generation is given by

$$\dot{Q} = F_f V = \mu F V \quad (2.1)$$

where μ is the coefficient of friction. If the contact area is A then the rate of thermal energy generation per unit area becomes

$$q = \frac{\dot{Q}}{A} = \frac{\mu F V}{A}. \quad (2.2)$$

Example 1 A solid block of 100 kg mass slides along a horizontal concrete pavement with a speed of 2 m/s. The coefficient of friction between the block and the pavement is estimated to be 0.4. Determine the rate of thermal energy generation in W.

Solution:

The force F acting on the pavement due to the weight of the block is

$$F = m g = 100 \times 9.81 = 981 \text{ N}$$

Therefore, the thermal energy generation as a result of friction between the solid block and the pavement is obtained by using Eq. (2.1)

$$\dot{Q} = \mu F V = 0.4 \times 981 \times 2 = 784.8 \text{ W}$$

The thermal energy generated at the interface diffuses through the solid objects. The amount of thermal energy diffusion in each of the solids depends on the thermal conductivity of the solids.

The temperature distribution in solids can be determined by solving the heat conduction equation. If the thermo-physical properties are assumed constant and no phase change (i.e. no melting of solids) is considered the heat conduction equation is given by

$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T \quad (2.3)$$

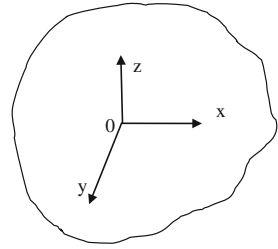
where $\alpha = k/\rho C_p$ is the thermal diffusivity, k is the thermal conductivity, ρ is the density and C_p is the specific heat. Solution of Eq. (2.3) with appropriate initial and boundary conditions yields the transient temperature distribution inside the solids.

2.2 Infinite Medium

2.2.1 Instantaneous Release of Heat

One of the fundamental solutions of the heat conduction equation in relation to the welding process is for the instantaneous point source in an infinite medium (Fig. 2.1). In this case the heat liberated at a point diffuses in all directions in the medium. The solution for the transient temperature distribution in this case is given by

Fig. 2.1 Coordinate system in an infinite medium



$$T(r, t) = T_i + \frac{q}{\rho C_P (4\pi\alpha t)^{3/2}} \exp\left(-\frac{r^2}{4\alpha t}\right) \quad (2.4)$$

where q (J) is the amount of heat suddenly released at time $t = 0$ at the origin ($r = 0$), the radial coordinate is $r = \sqrt{x^2 + y^2 + z^2}$ and the initial temperature is T_i . This solution is useful to study the thermal explosion that occurs in a homogeneous medium and the resulting temperature distribution in the medium as function of time and distance. Figure 2.2 shows the temperature distribution in a large steel sample as a result of instantaneous heat release of 1000 J of thermal energy at origin at different times after the initial release of the energy. The initial temperature of the steel material is 300 K. The density, the specific heat and the thermal diffusivity of the steel sample are 7800 kg/m^3 , 473 J/kg K and $1.172 \times 10^{-5} \text{ m}^2/\text{s}$, respectively. Each of the curves represent the temperature variation in the material at times 0.25, 0.5, 0.75, 1.0, and 1.25 s after the thermal energy release. As a result of heat diffusion in the material the temperature profiles are flattened in an exponential manner as given in Eq. (2.4).

Example 2 1 kJ of point energy is released in a large block of steel at time $t = 0$ that is initially at 300 K. Determine the temperature at a location 5 mm away from the location of the heat release after 2 s.

Solution:

Using Eq. (2.4) and substituting $q = 1000 \text{ J}$, $r = 0.005 \text{ m}$ and $t = 2 \text{ s}$

$$\begin{aligned} T(0.005, 2) &= 300 + \frac{1000}{7800 \times 473 \times (4\pi \times 1.172 \times 10^{-5} \times 2)^{3/2}} \exp\left(-\frac{0.005^2}{4 \times 1.172 \times 10^{-5} \times 2}\right) \\ &= 341 \text{ K} \end{aligned}$$

If the instantaneous heat release occurs along the z -axis rather than a point, then the solution of the heat conduction equation yields

$$T(r, t) = T_i + \frac{q'}{\rho C_P (4\pi\alpha t)} \exp\left(-\frac{r^2}{4\alpha t}\right) \quad (2.5)$$

where q' (J/m) is the amount of heat released instantaneously along the z -axis per unit length at time $t = 0$ and $r = \sqrt{x^2 + y^2}$ is the radial distance from the z -axis.

Fig. 2.2 The temperature distribution in a large size steel material after an instantaneous thermal energy release of 1 kJ at the origin. ($t = 0.25, 0.5, 0.75, 1.0$ and 1.25 s)

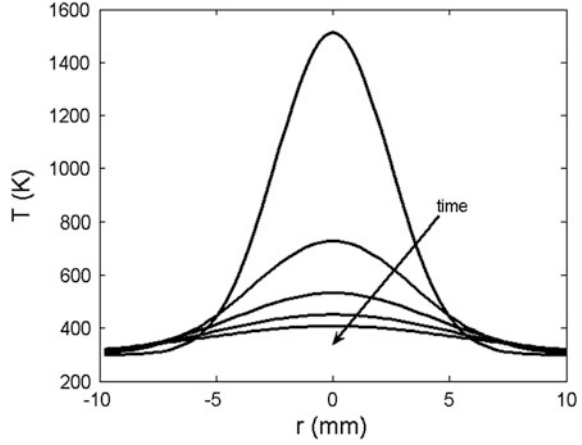


Fig. 2.3 The temperature distribution in a large size steel material after a thermal energy release of 100 kJ/m along the z-axis at time $t = 0$. ($t = 0.25, 0.5, 0.75, 1.0$ and 1.25 s)

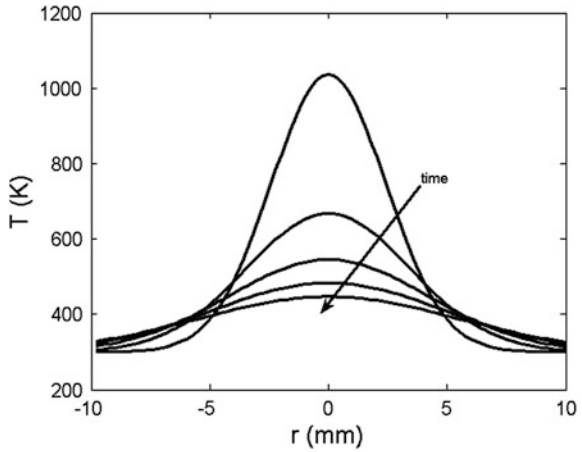


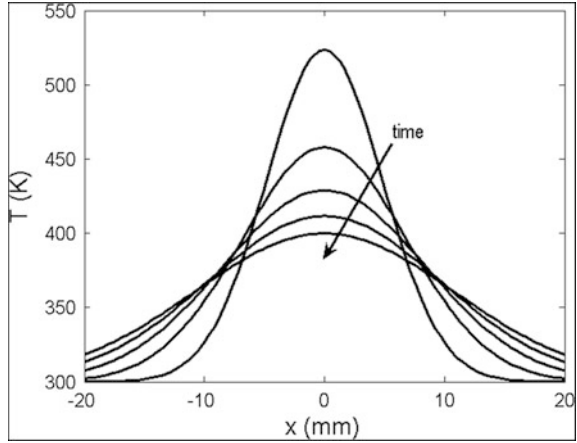
Figure 2.3 shows the temperature profiles in a large size steel material after an instantaneous release of 100 kJ/m of heat along the z-axis at time $t = 0$. Each of the curves in Fig. 2.3 represents the temperature profile at different times after the release of the heat, namely, at $t = 0.25, 0.5, 0.75, 1.0$, and 1.25 s. The temperature on the z axis at $t = 0.25$ s is more than 1000 K and it decreases sharply as a result of heat diffusion.

Similarly, if the instantaneous heat release occurs on the entire y-z plane, then the heat diffuses along the x direction. Thus the solution of the heat conduction equation yields a transient one-dimensional temperature distribution in the form

$$T(x, t) = T_i + \frac{q''}{\rho C_P (4\pi\alpha t)^{1/2}} \exp\left(-\frac{x^2}{4\alpha t}\right) \quad (2.6)$$

where q'' (J/m^2) is the amount of heat released instantaneously over the y-z plane per unit area at time $t = 0$. Figure 2.4 shows the temperature profiles in this case

Fig. 2.4 The temperature distribution in a large size steel material after a thermal energy release of 10 MJ/m^2 on the y-z plane at time $t = 0$. ($t = 0.25, 0.5, 0.75, 1.0$ and 1.25 s)



for the instantaneous heat release of 10 MJ/m^2 on the y-z plane at time $t = 0$. The material considered in this case is also steel with the same thermophysical characteristics mentioned above. The temperature profiles show similar behavior to the cases mentioned above, however, the peak values of temperature are different. This is because the instantaneous heat release takes place in a larger area and the diffusion of heat occurs in a larger volume. In this case the change (or decrease) of peak temperature during the time period $t = 0.25$ to 1.25 s is around 120 K .

The fundamental solutions given by the above three equations provide information on the diffusion of heat in the solid medium with no boundaries. However, they do not adequately describe the thermal energy diffusing during the welding process. This is because the thermal energy generation in a typical welding process is not spontaneous and the domain is normally finite.

2.2.2 Continuous Release of Heat

Now, consider a steady (continuous) case of thermal energy generation in an infinite medium. In this case there is no transient term in the governing heat conduction equation and therefore Eq. (2.3) simplifies to

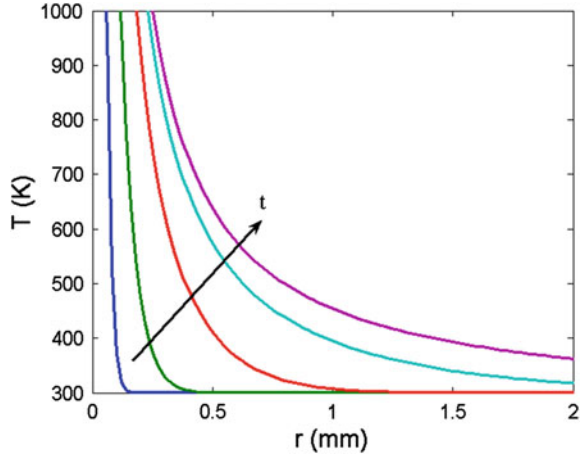
$$\nabla^2 T = 0 \quad (2.7)$$

If the thermal energy generation occurs at a rate \dot{q} (W) at the origin ($r = 0$) the temperature distribution in the medium is obtained to be

$$T(r, t) = T_i + \frac{\dot{q}}{\rho C_p (4\pi\alpha r)} \operatorname{erfc}\left(\frac{r}{2\sqrt{\alpha t}}\right) \quad (2.8)$$

where $r = \sqrt{x^2 + y^2 + z^2}$. It should be noted that as $t \rightarrow \infty$ the term $\operatorname{erfc}(0) = 1$ and therefore the steady-state temperature distribution in the medium is obtained to be

Fig. 2.5 The radial temperature distribution at different times in a steel material for a continuous release of 100 W rate of heat at the origin ($t = 10^{-4}$, 10^{-3} , 10^{-2} , 10^{-1} and 1 s)



$$T(r) = T_i + \frac{\dot{q}}{\rho C_p (4\pi\alpha r)}. \quad (2.9)$$

Figure 2.5 shows the temperature distribution in a steel material for the case of a continuous thermal energy release of 100 W at the origin. The steady temperature distribution in the material varies inversely with the radial distance as given in Eq. (2.9).

Example 3 Consider the point energy released in the large steel material as given in Example 1 to be continuous, i.e. 1 kW. Determine the temperature at a location 5 mm away from the location of the heat release after 2 s.

Solution:

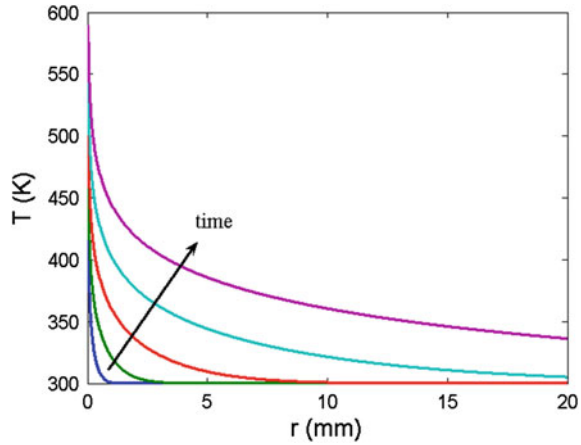
In this case the solution is given by Eq. (2.8)

$$T(r, t) = T_i + \frac{\dot{q}}{\rho C_p (4\pi\alpha r)} \operatorname{erfc}\left(\frac{r}{2\sqrt{\alpha t}}\right)$$

$$\begin{aligned} T(0.005, 2) &= 300 + \frac{1000}{7800 \times 473 \times (4\pi \times 1.172 \times 10^{-5} \times 2)} \operatorname{erfc}\left(-\frac{0.005}{2 \times \sqrt{1.172 \times 10^{-5} \times 2}}\right) \\ &= 317 \text{ K} \end{aligned}$$

The temperature in this case is lower when compared with the solution given in Example 2. The reason for this is because the heat release in the current case is not instantaneous and therefore it is slower than the case in Example 2. Therefore, the temperature of the material around the spot where the heat is released continues to increase and approaches a steady value. This value for $t \rightarrow \infty$ can be shown to be 336.8 K. For the case of instantaneous heat release, however, the temperature at the given location increases initially and then decreases as the wave of heat passes through that point.

Fig. 2.6 The radial temperature distribution at different times in a steel material for a continuous release of 10 kW/m rate of heat along the z-axis ($t = 0.01, 0.1, 1, 10$ and 100 s)



In case of continuous heat release along the z-axis at a rate of \dot{q}' (W/m) the temperature distribution in the infinite medium becomes

$$T(r, t) = T_i + \frac{\dot{q}'}{4\pi\rho C_p\alpha} Ei\left(\frac{r^2}{4\alpha t}\right) \quad (2.10)$$

where $r = \sqrt{x^2 + y^2}$. Since for $t \rightarrow \infty$ the term $Ei(0) \rightarrow \infty$ there is no steady temperature distribution in this case. Figure 2.6 shows the temperature profiles at various times ($t = 0.01, 0.1, 1, 10$ and 100 s) in a steel material when a continuous release of 10 kW/m rate of heat takes place along the z-axis. The heat diffuses through the bulk of the material and therefore the temperature in the material continues to increase.

Example 4 A high resistance electric wire is located in a large steel block releasing 10 kW/m heat energy. The initial temperature of the block is 25 °C. Estimate the temperature 5 mm away from the wire after 1 min of heating.

Solution:

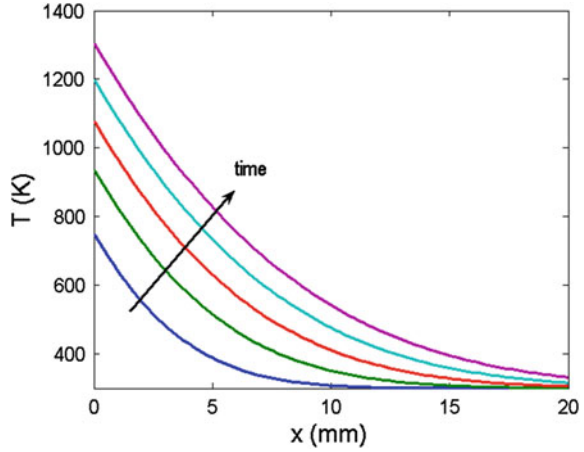
According to Eq. (2.10) the temperature is obtained after substituting the given information:

$$\begin{aligned} T(0.005, 60) &= 298 + \frac{10000}{4\pi \times 7800 \times 473 \times 1.172 \times 10^{-5}} Ei\left(\frac{0.005^2}{4 \times 1.172 \times 10^{-5} \times 60}\right) \\ &= 374.5 \text{ K} \end{aligned}$$

For the case of thermal energy generation over the y-z plane at a rate of \dot{q}'' (W/m²) the temperature distribution in the infinite medium is given by

$$T(x, t) = T_i + \frac{\dot{q}''}{2k} \left[\sqrt{\frac{4\alpha t}{\pi}} \exp\left(-\frac{x^2}{4\alpha t}\right) - x \operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) \right] \quad (2.11)$$

Fig. 2.7 The temperature distribution at different times in a steel material for a continuous release of 10 MW/m^2 rate of over the y-z plane ($t = 1, 2, 3, 4$ and 5 s)



It is clear that there is no steady-state solution for this case either since the first term in the equation is proportional to \sqrt{t} . The temperature anywhere in the medium including that on the y-z plane increases continuously with time proportional to \sqrt{t} . For $x = 0$ the temperature on the y-z plane is obtained to be

$$T(0, t) = T_i + \frac{\dot{q}''}{k} \sqrt{\frac{\alpha t}{\pi}} \quad (2.12)$$

Figure 2.7 shows the temperature distribution in the direction of x-axis for the case of continuous rate of heat release of 10 MW/m^2 on the y-z plane. Each of the curves in Fig. 2.7 represents the temperature distribution at time $t = 1, 2, 3, 4$, and 5 s , respectively. The temperature in the bulk of the material increases in a continuous manner as a result of diffusion of heat away from the y-z plane. The temperature increase at the y-z plane is proportional to \sqrt{t} as shown in Fig. 2.8.

2.2.3 Moving Sources of Heat

In most practical situations the heat source moves along the medium. Therefore, for accurate determination of the temperature distribution in the medium, moving heat sources must be considered. When the heat source or the medium moves along the x-direction the governing equation for conduction heat transfer becomes

$$V \frac{\partial T}{\partial x} = \alpha \nabla^2 T \quad (2.13)$$

Fig. 2.8 The temperature variation with time on the y-z plane in the steel material for a continuous release of 100 kW/m^2 rate of over the y-z plane

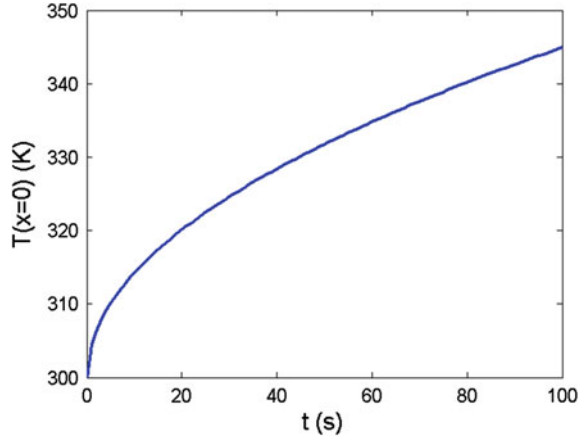
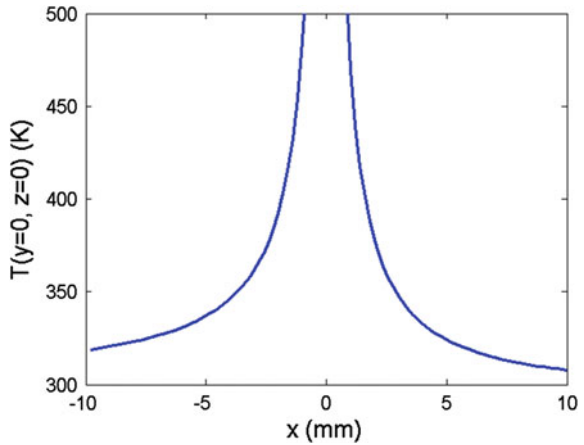


Fig. 2.9 Temperature distribution in a steel material along the x-axis subjected to the moving point source of 100 W along the x-axis with a speed of 0.01 m/s



2.2.3.1 Moving Point-Source

Now, consider a steadily moving point heat source in the x-direction with a speed of V in an infinite medium. The temperature distribution on the coordinates moving with the point source is named the quasi-steady state solution. This solution can be expressed as

$$T(x, y, z) = T_i + \frac{\dot{q}}{\rho C_P (4\pi\alpha r)} \exp\left(\frac{-V(r+x)}{2\alpha}\right) \quad (2.14)$$

where $r = \sqrt{x^2 + y^2 + z^2}$. It should be noted that when the velocity goes to zero the solution approaches the steady-state solution for the case of continuous point source given in Eq. (2.9). Figure 2.9 shows the temperature distribution along the x-axis in a steel material in which a moving point release of heat at a rate of 100 W occurs along the x-axis. The speed of moving point heat source along the x-direction is 0.01 m/s .

Example 5 A point source of 100 W moves in a large steel material with a speed of 10 mm/s. The temperature of the material far away from the source of heat release is 300 K. Determine the temperature 2 mm behind and 2 mm in front of the moving front.

Solution:

Equation (2.14) can be written in the one-dimensional form as:

$$T(x) = T_i + \frac{\dot{q}}{\rho C_P (4\pi\alpha|x|)} \exp\left(\frac{-V(|x| + x)}{2\alpha}\right)$$

2 mm in front:

$$\begin{aligned} T(0.002) &= 300 + \frac{100}{7800 \times 473 \times (4\pi \times 1.172 \times 10^{-5} \times 0.002)} \exp\left(\frac{-0.01 \times (0.002 + 0.002)}{2 \times 1.172 \times 10^{-5}}\right) \\ &= 316.8 \text{ K} \end{aligned}$$

2 mm behind:

$$\begin{aligned} T(-0.002) &= 300 + \frac{100}{7800 \times 473 \times (4\pi \times 1.172 \times 10^{-5} \times 0.002)} \exp\left(\frac{-0.01 \times (0.002 - 0.002)}{2 \times 1.172 \times 10^{-5}}\right) \\ &= 392.5 \text{ K} \end{aligned}$$

2.2.3.2 Moving Line-Source

If the moving source is a line heat source along the z direction and moving along the x-axis the quasi-steady state solution for the temperature on the moving coordinate system is obtained to be

$$T(x, y) = T_i + \frac{\dot{q}'}{2\pi k} \exp\left(-\frac{Vx}{2\alpha}\right) K_0\left(\frac{Vr}{2\alpha}\right) \quad (2.15)$$

Figure 2.10 shows the temperature distribution in the steel material along the x-axis when a line source of heat of 100 kW/m (along the z-axis) moves in the direction of x-axis. The speed of the line heat source is taken as 0.01 m/s.

2.2.3.3 Moving Plane-Source

Finally when the heat source is uniformly distributed on the y-z plane that is moving along the x-axis with a velocity V the quasi steady state solution of the temperature on the moving coordinate axis becomes

$$T(x) = T_i + \frac{\dot{q}''}{\rho C_P V} \exp\left(-\frac{Vx}{\alpha} \left(\frac{1 + \text{sgn}(x)}{2}\right)\right) \quad (2.16)$$

Fig. 2.10 Temperature distribution in a steel material along the x-axis subjected to the moving line source of 100 kW/m on the z-axis with a speed of 0.01 m/s along the x-axis

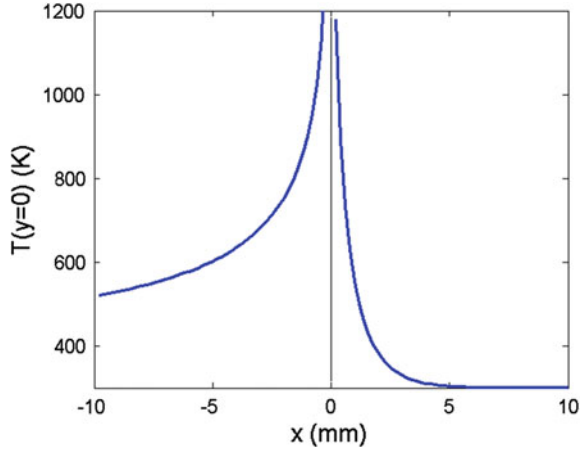
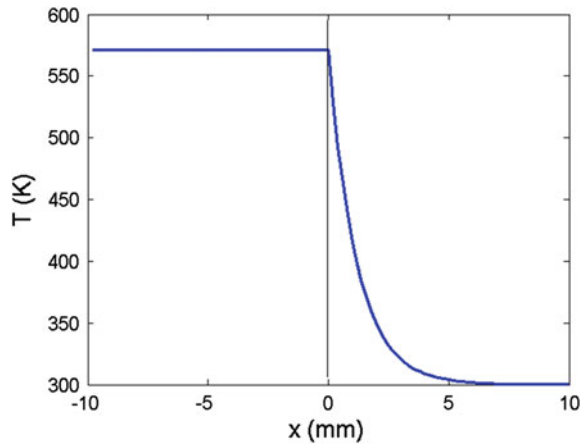


Fig. 2.11 Temperature variation along the x-direction in a steel material subjected to a plane moving source in the direction of x-axis



where $\text{sgn}(x)$ is the Signum function which is defined as

$$\text{sgn}(x) = \frac{x}{|x|} = \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ +1 & \text{if } x > 0 \end{cases} \quad (2.17)$$

Figure 2.11 shows the variation of temperature in the direction of x-axis when a plane heat source of 10 MW/m² (on the y-z plane) moves along the x direction with a velocity of 0.01 m/s.

On the other hand the solution on the stationary coordinates yields the transient temperature on the stationary plane at $x = 0$ as

$$T(0, t) = T_i + \frac{\dot{q}''}{2\rho C_p V} \left[1 - \exp\left(\frac{V^2 t}{\alpha}\right) \text{erfc}\left(\sqrt{\frac{V^2 t}{\alpha}}\right) \right] \quad (2.18)$$

In this case the plane heat source moves away from the origin in the direction of x and the temperature at the stationary y - z plane decreases with time.

The solutions for the infinite medium presented above are useful in analyzing local temperature variations far away from the boundaries of the physical medium. They also describe the early stages of the transient behavior in the finite domain before the time the heat diffusion reaches the boundaries of the physical medium. These solutions provide an insight of the expected temperature history during the actual welding process. However, in the actual welding process the intensity of the heat generation is high and there may be sufficient time for the diffusion of heat to reach the physical boundaries of the medium. Therefore, the finite size medium with its boundaries must be considered in order to make more accurate predictions for the temperature distribution in most of the welding processes. The first step to consider the physical boundaries is to study the semi-infinite medium that is considered in the following section.

2.3 Semi-Infinite Medium

When the surface of the semi-infinite medium is insulated, in other words, when the heat transfer from the surface is neglected the temperature distribution in the domain and its mirror image across the surface of the semi-infinite medium (Fig. 2.12) matches the solution of the temperature distribution in the infinite medium. Therefore, the solutions for the semi-infinite medium with insulated boundary condition can easily be obtained by using the solutions for the infinite medium.

All the heat released on the insulated surface of the semi-infinite medium will have to diffuse towards the depth of the semi-infinite medium as opposed to the infinite medium where the heat released diffuses in all directions.

2.3.1 Instantaneous Point Heat Release q (J) on the Surface

The solution for the temperature distribution in the case of instantaneous point heat release q (J) on the surface of the semi-infinite medium follows Eq. (2.4), i.e.

$$T(r, t) = T_i + \frac{2q}{\rho C_P (4\pi\alpha t)^{3/2}} \exp\left(-\frac{r^2}{4\alpha t}\right) \quad (2.19)$$

where q (J) is the amount of heat suddenly released at time $t = 0$ at the origin ($r = 0$), the radial coordinate is $r = \sqrt{x^2 + y^2 + z^2}$, the initial temperature is T_i , and the factor 2 in front of the second term on the right hand side is due to the fact that all the heat released has to diffuse only towards the depth of the semi-infinite medium ($z > 0$) and no heat loss is assumed from the surface. Figure 2.13 shows

Fig. 2.12 Coordinate system in semi-infinite medium

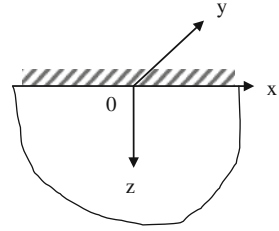
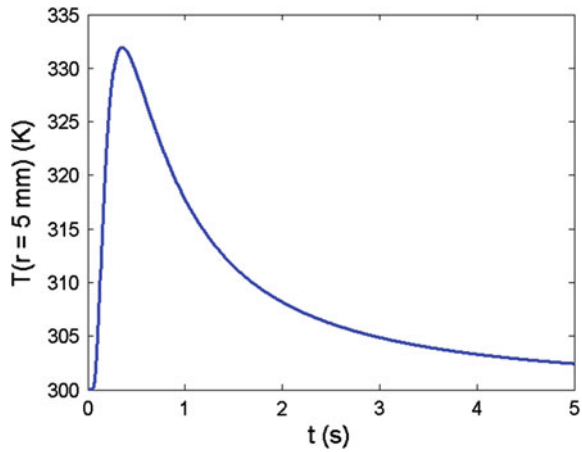


Fig. 2.13 Temperature variation with respect to time at a distance 5 mm away from the origin where an instantaneous heat release of 100 J takes place



the temporal variation of temperature in a steel material 5 mm away from the origin where an instantaneous heat release of 100 J occurs at time $t = 0$. As a result of heat diffusion the temperature at $r = 5$ mm initially rises and reaches at a peak value at around $t = 0.5$ s and then decreases afterwards.

Example 6 1 kJ of point energy is released on the surface of a large semi infinite block of steel at time $= 0$ that is initially at 300 K. Determine the temperature inside the steel 5 mm below the location of the heat release after 2 s.

Solution:

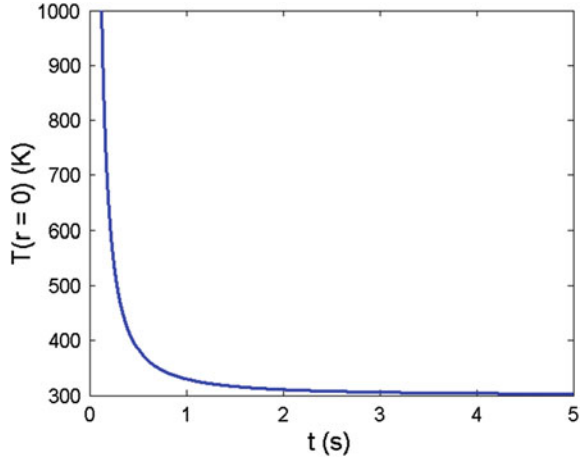
Equation (2.19) can be written in the z direction as:

$$T(z, t) = T_i + \frac{2q}{\rho C_P (4\pi\alpha t)^{3/2}} \exp\left(-\frac{z^2}{4\alpha t}\right)$$

Therefore, for $z = 5$ mm the temperature is

$$\begin{aligned} T(0.005, 2) &= 300 + \frac{2 \times 1000}{7800 \times 473 \times (4\pi \times 1.172 \times 10^{-5} \times 2)^{3/2}} \exp\left(-\frac{0.005^2}{4 \times 1.172 \times 10^{-5} \times 2}\right) \\ &= 382 \text{ K} \end{aligned}$$

Fig. 2.14 Temperature variation with time at the origin after 100 J instantaneous heat release at the origin



It can also be shown that the temperature at the spot where the heat is released is 407.2 K after the time the heat is released.

The temperature at the origin ($r = 0$) decreases with time (Fig. 2.14) as

$$T(0, t) = T_i + \frac{2q}{\rho C_p (4\pi\alpha t)^{3/2}}. \quad (2.20)$$

2.3.2 Instantaneous Line Heat Release q' (J/m) on the Surface

For the case of the instantaneous line heat release along the y-axis direction the transient temperature distribution is obtained from Eq. (2.5) as

$$T(r, t) = T_i + \frac{2q'}{\rho C_p (4\pi\alpha t)} \exp\left(-\frac{r^2}{4\alpha t}\right) \quad (2.21)$$

where q' (J/m) is the amount of heat released instantaneously along the y-axis per unit length at time $t = 0$, $r = \sqrt{x^2 + z^2}$ is the radial distance from the y-axis and $z > 0$. Figure 2.15 shows the temperature variation with time at a distance of 5 mm away from the y-axis after 100 kJ/m heat release along the y-axis. The temperature initially rises sharply and after reaching a peak value it starts decreasing.

The temperature along the y-axis ($r = 0$) decreases with time as

$$T(0, t) = T_i + \frac{q'}{\rho C_p (2\pi\alpha t)}. \quad (2.22)$$

In this case the temperature along the y-axis decreases inversely proportional with time after the sudden release of heat along the y-axis.

Fig. 2.15 Temperature variation with time at a location 5 mm away from the y-axis after an instantaneous heat release of 100 kJ/m along the y-axis

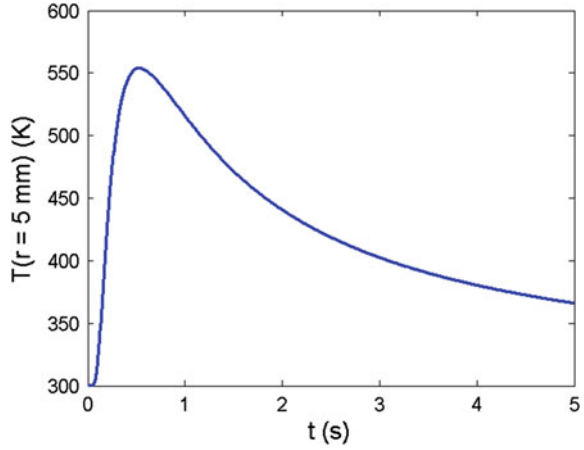
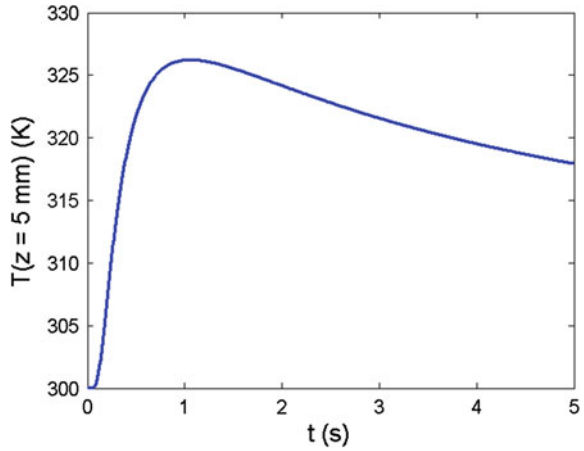


Fig. 2.16 Temporal variation of temperature 5 mm below the surface of the semi-infinite steel material after an instantaneous and uniform heat release of 1 MJ/m² at the surface of the material



2.3.3 Instantaneous Plane Heat Release q'' (J/m²) on the Surface

If the instantaneous heat release occurs uniformly on the surface of the semi-infinite medium ($z = 0$ plane) the solution for the temperature distribution can be written from Eq. (2.6) as

$$T(z, t) = T_i + \frac{2q''}{\rho C_p (4\pi\alpha t)^{1/2}} \exp\left(-\frac{z^2}{4\alpha t}\right) \quad (2.23)$$

where q'' (J/m²) is the amount of heat released instantaneously over the x-y plane per unit area at time $t = 0$. The temperature variation with time at a depth of 5 mm from the surface is shown in Fig. 2.16 after an instantaneous heat release of 1 MJ/m² on

the surface. As the heat is diffused through the depth of the steel material the temperature initially increases and then decreases after reaching a peak value.

The temperature on the insulated surface of the semi-infinite medium ($z = 0$) decreases with time as

$$T(0, t) = T_i + \frac{q''}{\rho C_P (\pi \alpha t)^{1/2}}. \quad (2.24)$$

In this case the temperature variation with time is inversely proportional with the square root of time.

Example 7 The surface of a semi infinite steel material is exposed to an instantaneous heat release of 1 MJ/m^2 . Determine the maximum temperature at a location 5 mm below the surface and the time at which this occurs.

Solution:

The time at which a maximum temperature is reached at a given z location in the material can be obtained by differentiating Eq. (2.23) with respect to t and setting the result equal to zero, i.e.

$$\frac{\partial T(z, t)}{\partial t} = \frac{\partial T}{\partial t} \left[T_i + \frac{2q''}{\rho C_P (4\pi \alpha t)^{1/2}} \exp\left(-\frac{z^2}{4\alpha t}\right) \right] = 0$$

Carrying out the differentiating the time for the maximum temperature is obtained as

$$t = \frac{z^2}{2\alpha}.$$

Substituting the values we get

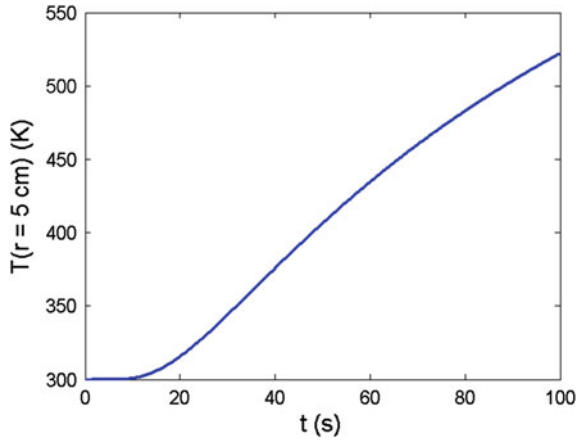
$$t = \frac{0.005^2}{2 \times 1.172 \times 10^{-5}} = 1.067 \text{ s}$$

Therefore the maximum temperature becomes

$$\begin{aligned} T(0.005, 1.067) &= 300 + \frac{2 \times 10^6}{7800 \times 473 \times (4\pi \times 1.172 \times 10^{-5} \times 1.067)^{1/2}} \exp\left(-\frac{0.005^2}{4 \times 1.172 \times 10^{-5} \times 1.067}\right) \\ &= 326.2 \text{ K} \end{aligned}$$

In most welding applications the heat release is continuous for a certain period of time rather than spontaneous. Therefore the case of continuous release of heat must be analyzed to describe the thermal behavior in such welding processes.

Fig. 2.17 Temperature variation in a semi infinite steel material at a location 5 cm away from the origin where a continuous heat release of 10 kW occurs



2.3.4 Continuous Point Heat Release \dot{q} (W) on the Surface

The most elementary example in this case is the continuous point heat release \dot{q} (W) on the insulated surface of the semi-infinite medium at the origin ($r = 0$). The temperature distribution in this case is obtained to be

$$T(r, t) = T_i + \frac{2\dot{q}}{\rho C_p (4\pi\alpha r)} \operatorname{erfc}\left(\frac{r}{2\sqrt{\alpha t}}\right) \quad (2.25)$$

where $r = \sqrt{x^2 + y^2 + z^2}$ and $z > 0$. The temperature variation with time in a semi-infinite steel material at a location 5 cm away from the origin is shown in Fig. 2.17. In this case a continuous heat release of 10 kW occurs at the origin. The temperature increases in a continuous manner as the heat diffusion occurs.

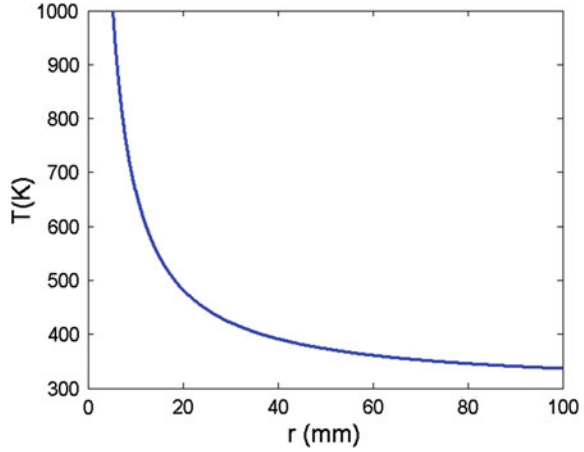
As $t \rightarrow \infty$ the steady-state temperature distribution in the medium becomes

$$T(r) = T_i + \frac{\dot{q}}{\rho C_p (2\pi\alpha r)}. \quad (2.26)$$

Figure 2.18 shows the steady temperature variation with r in a semi-infinite steel substance which is subjected to 1 kW continuous point heat release at the origin. The temperature varies inversely proportional with the distance from the origin. In this case all the heat released at the origin diffuses through the depth of the material and a steady temperature variation is obtained. In other words, the temperature at any point in the material starts increasing after the penetration of heat reaches at the point as shown in Fig. 2.17 and then the temperature approaches a constant steady value asymptotically.

Example 8 1 kW of continuous point heat release starts taking place on the surface of a large steel sample whose temperature before the heat release was 300 K.

Fig. 2.18 Steady temperature variation in a semi-infinite steel substance subjected to 1 kW of continuous point heat release at the origin on the surface



Determine the maximum temperature at a point 20 mm below the spot at which the heat release occurs.

Solution:

The maximum temperature is reached when $t \rightarrow \infty$, i.e. the steady state situation.

Therefore, using Eq. (2.26), we have

$$T(z) = T_i + \frac{\dot{q}}{\rho C_P (2\pi\alpha z)}$$

or

$$\begin{aligned} T(0.02) &= 300 + \frac{1000}{7800 \times 473 \times (2\pi \times 1.172 \times 10^{-5} \times 0.02)} \\ &= 484 \text{ K} \end{aligned}$$

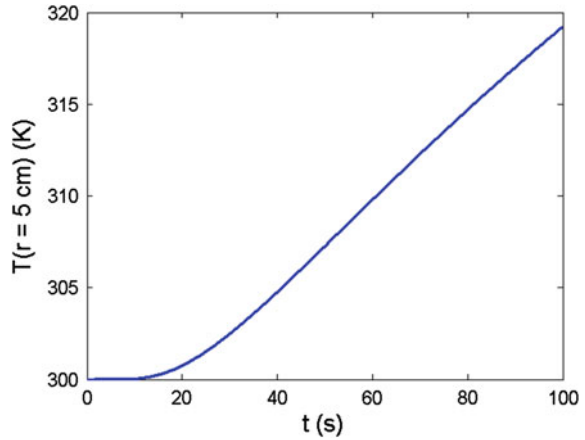
2.3.5 Continuous Line Heat Release \dot{q}' (W/m) on the Surface

In the case of continuous heat release \dot{q}' (W/m) along y-axis the temperature distribution in the semi-infinite medium is found to be

$$T(r, t) = T_i + \frac{2\dot{q}'}{4\pi\rho C_P\alpha} Ei\left(\frac{r^2}{4\alpha t}\right) \quad (2.27)$$

where $r = \sqrt{x^2 + z^2}$ and $z > 0$. The temperature in the medium increases with time t but decreases with distance r . Figure 2.19 shows the temperature variation with time in a semi-infinite steel material at a distance of 5 cm from the y-axis

Fig. 2.19 Temperature variation with time in a semi-infinite steel medium at a distance of 5 cm from the y-axis where a continuous line heat release of 10 kW/m occurs



where a continuous line heat release of 10 kW/m takes place. The temperature starts increasing after the penetration time of about 10 s. The increase of temperature continues and no steady temperature is reached.

2.3.6 Uniform Heat Flux \dot{q}'' (W/m²) on the Surface

For the case of uniform heat release \dot{q}'' (W/m²) on the insulated surface of the semi-infinite medium the temperature distribution along the depth $z > 0$ of the medium is found to be

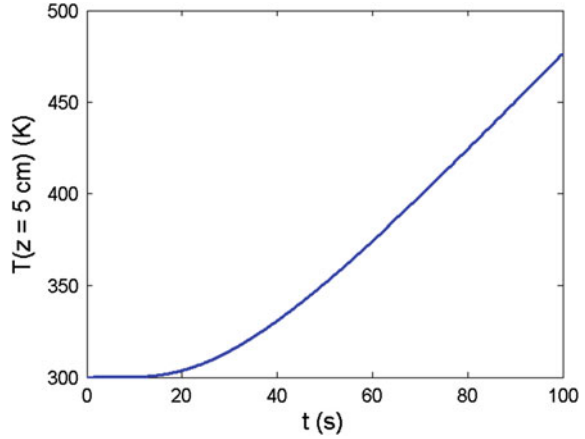
$$T(z, t) = T_i + \frac{\dot{q}''}{k} \left[\sqrt{\frac{4\alpha t}{\pi}} \exp\left(-\frac{z^2}{4\alpha t}\right) - z \operatorname{erfc}\left(\frac{z}{2\sqrt{\alpha t}}\right) \right] \quad (2.28)$$

Figure 2.20 shows the temperature variation with time at a depth of 5 cm of a semi-infinite steel medium subjected to 1 MW/m² uniform surface heat flux. The temperature at this depth starts increasing after the penetration time and the increase of temperature takes place continuously. Steady-state solution does not exist since the first term in the bracket is proportional to \sqrt{t} . The temperature on the surface of the semi-infinite medium $z = 0$ also varies with \sqrt{t} as

$$T(0, t) = T_i + \frac{2\dot{q}''}{k} \sqrt{\frac{\alpha t}{\pi}}. \quad (2.29)$$

Example 9 The surface of a large steel block which is at 300 K is suddenly exposed to 1 MW/m² heat flux. Determine the time needed for the surface temperature to reach 1500 K.

Fig. 2.20 Temperature rise inside a semi infinite steel medium at a depth of 5 cm that is subjected to a continuous heat flux of 1 MW/m^2 at the surface



Solution:

Solving t from Eq. (2.29)

$$T(0, t) = T_i + \frac{2\dot{q}''}{k} \sqrt{\frac{\alpha t}{\pi}}$$

$$t = \frac{\pi}{\alpha} \left[k \frac{T - T_i}{2\dot{q}''} \right]^2 = \frac{\pi}{1.172 \times 10^{-5}} \left[43 \times \frac{1500 - 300}{2 \times 10^6} \right]^2 = 178.4 \text{ s}$$

2.3.7 Continuous Strip (Along y-Axis) Heat Release on the Surface of Semi-Infinite Medium

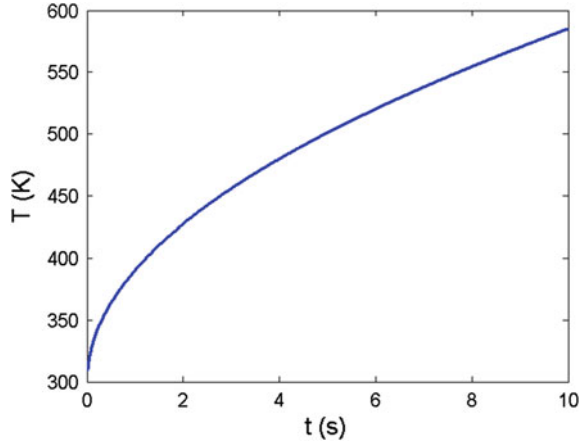
When the heat release \dot{q}'' (W/m^2) occurs continuously over the long strip of $2b$ width on the surface of the semi-infinite medium the initial and surface boundary condition can be expressed as

$$T = T_i \quad \text{at} \quad t = 0$$

$$-k \frac{\partial T}{\partial z} = \begin{cases} \dot{q}'' & z = 0, \quad -b < x < +b \\ 0 & z = 0, \quad x > b, \quad x < -b \end{cases} \quad (2.30)$$

In this case the transient temperature variation over the surface of the semi-infinite medium at $z = 0$ is given by

Fig. 2.21 Temperature variation with time on the surface at $x = 5$ cm



$$T(0, x, t) = T_i + \frac{\dot{q}'' b \sqrt{Fo_b}}{k \sqrt{\pi}} \left\{ \begin{aligned} & \left[\operatorname{erf}\left(\frac{1+x}{2\sqrt{Fo_b}}\right) + \operatorname{erf}\left(\frac{1-x}{2\sqrt{Fo_b}}\right) - \left(\frac{1+x}{2\sqrt{Fo_b}}\right)^2 \operatorname{Ei}\left[-\left(\frac{1+x}{2\sqrt{Fo_b}}\right)^2\right] \right] \\ & - \left(\frac{1-x}{2\sqrt{Fo_b}}\right)^2 \operatorname{Ei}\left[-\left(\frac{1-x}{2\sqrt{Fo_b}}\right)^2\right] \end{aligned} \right\} \quad (2.31)$$

where $Fo_b = \frac{at}{b^2}$. Figure 2.21 shows the temperature variation on the surface at $x = 5$ cm when a continuous strip heat release of 1 MW/m^2 takes place on the strip of 10 cm width.

2.3.8 Continuous Circular Disk Area (of Radius R) Heat Release on the Surface of Semi-Infinite Medium

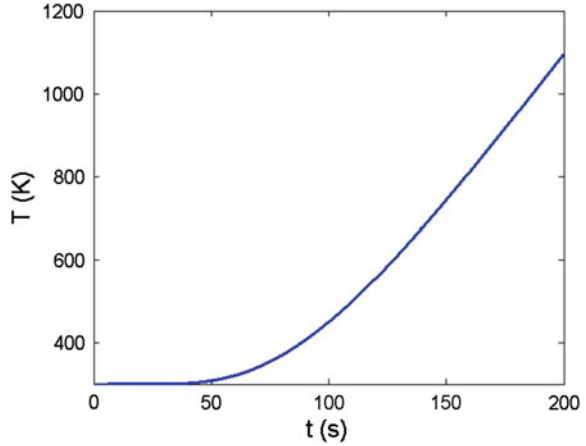
If the continuous heat release \dot{q}'' (W/m^2) occurs uniformly over the circular area of radius R over the surface of the semi-infinite medium the initial and boundary conditions are given by

$$\begin{aligned} T &= T_i \quad \text{at } t = 0 \\ -k \frac{\partial T}{\partial z} &= \begin{cases} \dot{q}'' & z = 0, \quad 0 < r < R \\ 0 & z = 0, \quad r > R \end{cases} \end{aligned} \quad (2.32)$$

In this case the transient temperature variation along the z -axis going through the center ($r = 0$) of the circular disk area is found to be

$$T(0, z, t) = T_i + 2 \frac{\dot{q}'' R}{k} \sqrt{Fo_R} \left\{ \operatorname{ierfc}\left(\frac{1}{2\sqrt{Fo_R}}\right) - \operatorname{ierfc}\left(\frac{\sqrt{1 + \left(\frac{R}{z}\right)^2}}{2\sqrt{Fo_R}}\right) \right\} \quad (2.33)$$

Fig. 2.22 Temperature variation with time in a semi-infinite steel substance at a depth of 10 cm below the origin for the case of continuous circular disk heat release of 10 MW/m^2 on the surface with the radius of disk 10 cm



where $For = \frac{\alpha t}{R^2}$ and $r = \sqrt{x^2 + y^2}$. Figure 2.22 shows the temperature variation with time inside the semi-infinite steel material 10 cm below the origin that is located on the surface. The continuous disk area heat release is considered to be 10 MW/m^2 on the disk of radius 10 cm.

As can be seen from the figure, the temperature starts increasing after a penetration time of about 45 s. The increase of temperature is continuous and no steady temperature is expected since the temperature variation is proportional to the square root of time as can be seen from Eq. (2.33).

Example 10 The flat surface of a large steel sample initially at 300 K is subjected to a circular disk heat release of 10 MW/m^2 with the radius of the disk being 10 cm. Determine the temperature 10 cm below the center of the disk after 2 min.

Solution:

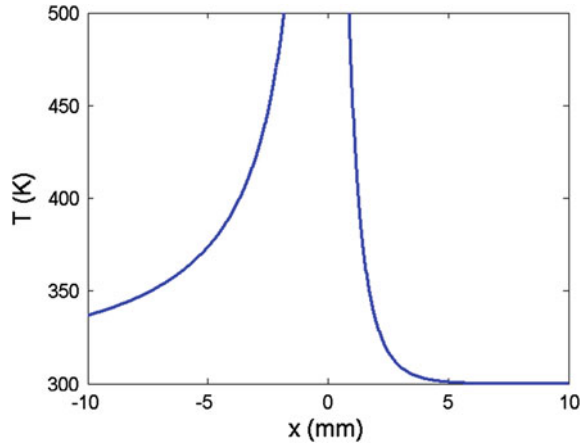
The Fourier number is

$$For = \frac{\alpha t}{R^2} = \frac{1.172 \times 10^{-5} \times 120}{0.1^2} = 0.14$$

Therefore the temperature is obtained using Eq. (2.33) as

$$T(0, 0.1, 120) = 300 + 2 \frac{10^6 \times 0.1}{43} \sqrt{0.14} \left\{ \text{ierfc} \left(\frac{1}{2\sqrt{0.14}} \right) - \text{ierfc} \left(\frac{\sqrt{1 + \left(\frac{0.1}{0.1} \right)^2}}{2\sqrt{0.14}} \right) \right\} = 554 \text{ K}$$

Fig. 2.23 Temperature distribution along x direction on the surface of steel substance with a moving point heat source of 100 W along the x-axis



2.3.9 Moving Point-Source on the Surface

If the heat source is not stationary but is moving along the x-axis with a velocity of V then the quasi-steady state solution on the moving coordinate system is obtained as

$$T(x, y, z) = T_i + \frac{2\dot{q}}{\rho C_P (4\pi\alpha r)} \exp\left(\frac{-V(r+x)}{2\alpha}\right) \quad (2.34)$$

where $r = \sqrt{x^2 + y^2 + z^2}$ and $z > 0$. Figure 2.23 shows the temperature distribution along the x-axis on the surface of a semi-infinite steel material with a moving point heat source of 100 W along the x direction. The speed of the moving heat source is taken as 10 mm/s.

As the velocity tends to zero ($V = 0$) the temperature distribution becomes

$$T(x, y, z) = T_i + \frac{\dot{q}}{\rho C_P (2\pi\alpha r)} \quad (2.35)$$

as shown in Fig. 2.24. The temperature distribution becomes symmetrical with respect to the origin and decreases with r in an inversely proportional manner.

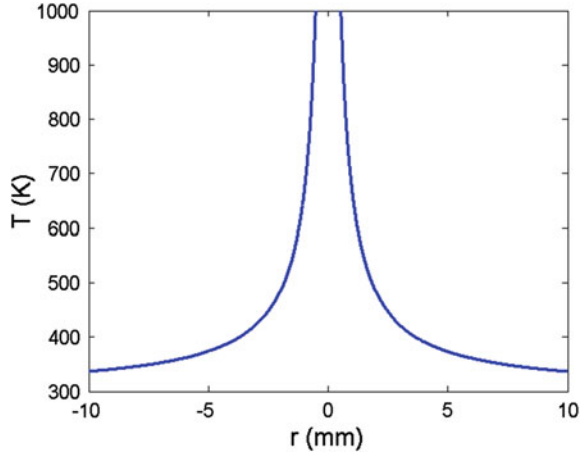
Example 11 A continuous point heat source of 100 W moves along the surface of a large steel sample with a velocity of 10 mm/s. The temperature of the sample far away from the heat source is 300 K. Determine the temperature on the surface 5 mm behind and 5 mm in front of the heat source location.

Solution:

Equation (2.34) can be written in one dimensional form as

$$T(x) = T_i + \frac{2\dot{q}}{\rho C_P (4\pi\alpha|x|)} \exp\left(\frac{-V(|x| + x)}{2\alpha}\right)$$

Fig. 2.24 Temperature distribution along the radial direction when the velocity of the moving heat source is zero



Therefore, the temperature 5 mm behind the heat source is

$$T(-0.005) = 300 + \frac{2 \times 100}{7800 \times 473 \times (4\pi \times 1.172 \times 10^{-5} \times 0.005)} \exp\left(\frac{-0.01 \times (0.005 - 0.005)}{2 \times 1.172 \times 10^{-5}}\right) = 373.6 \text{ K}$$

and the temperature 5 mm in front of the heat source is

$$T(0.005) = 300 + \frac{2 \times 100}{7800 \times 473 \times (4\pi \times 1.172 \times 10^{-5} \times 0.005)} \exp\left(\frac{-0.01 \times (0.005 + 0.005)}{2 \times 1.172 \times 10^{-5}}\right) = 301 \text{ K}$$

Note that the temperature in front of the heat source is very low since the penetration of heat has not yet occurred at that location. It can be shown using Eq. (2.35) that when the velocity of the heat source is set to zero, then the temperatures at both locations become 373.6 K.

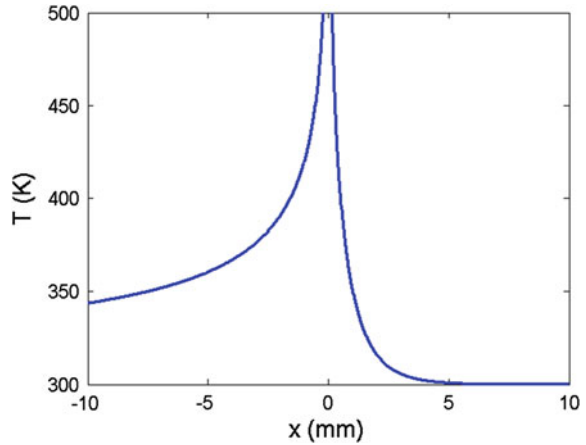
2.3.10 Moving Line-Source on the Surface

In the case of moving y-axis line source \dot{q}' (W/m) in the direction of x-axis with a velocity of V the quasi-steady state temperature variation on the moving coordinate system in the semi-infinite medium is obtained as

$$T(x, z) = T_i + \frac{\dot{q}'}{\pi k} \exp\left(-\frac{Vx}{2\alpha}\right) K_0\left(\frac{Vr}{2\alpha}\right) \quad (2.36)$$

where $r = \sqrt{x^2 + z^2}$ and $z > 0$. Figure 2.25 shows the temperature distribution on the surface of the semi-infinite steel material along the x-axis. The y-axis line heat

Fig. 2.25 Temperature distribution on the surface of a semi-infinite steel material along the x-axis subjected to a moving line heat source of 10 kW/m in the x direction



source is 10 kW/m and moves with a speed of 10 mm/s along the x-axis. Temperature gradient is higher on the right side because of the travel of the heat source in that direction.

2.3.11 Moving Infinite y-Strip Source (in x Direction) on the Surface of a Semi-Infinite Solid

If the infinite y-axis line source \dot{q}'' (W/m²) acts uniformly on a strip of finite width $2b$ and moves in the direction of x-axis with a velocity of V , then the initial and boundary conditions can be written as

$$T = T_i \quad \text{at} \quad t = 0$$

$$-k \frac{\partial T}{\partial z} = \begin{cases} \dot{q}'' & z = 0, \quad -b < x < +b \\ 0 & z = 0, \quad x > b, \quad x < -b \end{cases} \quad (2.37)$$

The quasi-steady state temperature distribution in the semi-infinite medium on the moving coordinate system is given by

$$T(x, z) = T_i + \frac{\dot{q}'' \alpha}{kV} \int_{X-B}^{X+B} \exp(\lambda) K_0(\sqrt{Z^2 + \lambda^2}) d\lambda \quad (2.38)$$

where $X = \frac{Vx}{2\alpha}$, $Z = \frac{Vz}{2\alpha}$, and $B = \frac{Vb}{2\alpha}$.

In all case considered above the convection heat transfer from the surface of the medium is neglected. The case of convective boundary condition is considered in the following.

2.3.12 Moving Infinite y-Strip Source (in x Direction) on the Surface of a Semi-Infinite Solid with Convection Boundary

If the convection heat transfer is taken into consideration in the above case, the initial and boundary conditions can be written as

$$\begin{aligned}
 T &= T_{\infty} \quad \text{at} \quad t = 0 \\
 -k \frac{\partial T}{\partial z} &= \begin{cases} \dot{q}'' & z = 0, \quad -b < x < +b \\ h(T_{\infty} - T) & z = 0, \quad x > b, \quad x < -b \end{cases} \quad (2.39) \\
 T &= T_{\infty} \quad \text{at} \quad x \rightarrow \pm\infty, z \rightarrow \pm\infty
 \end{aligned}$$

The quasi-steady state solution for the temperature distribution on the moving coordinate axis in this case is obtained to be

$$T(x, z) = T_{\infty} + \frac{2\alpha\dot{q}''}{\pi kV} \left\{ \begin{aligned} &\int_{X-B}^{X+B} \exp(-\lambda) K_0(\sqrt{Z^2 + \lambda^2}) d\lambda \\ &- \pi H \exp(HZ) \int_0^{\infty} \tau \exp(H^2\tau^2) \operatorname{erfc}\left(\frac{Z}{2\tau} + H\tau\right) \times \left[\begin{aligned} &\operatorname{erf}\left(\frac{X+B}{2\tau} + \tau\right) \\ &-\operatorname{erf}\left(\frac{X-B}{2\tau} + \tau\right) \end{aligned} \right] d\tau \end{aligned} \right\} \quad (2.40)$$

where $X = \frac{Vx}{2\alpha}$ $Z = \frac{Vz}{2\alpha}$ $B = \frac{Vb}{2\alpha}$ $H = \frac{2\alpha h}{kV}$

When the convection heat transfer coefficient is small, i.e. $h \rightarrow 0$ the solution becomes

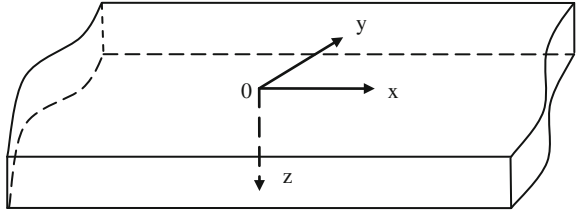
$$T(x, z) = T_i + \frac{\dot{q}''\alpha}{kV} \frac{2}{\pi} \int_{X-B}^{X+B} \exp(\lambda) K_0(\sqrt{Z^2 + \lambda^2}) d\lambda \quad (2.41)$$

as expected.

2.4 Slab (Plate) of Thickness L

When the medium has a finite thickness L rather than being semi-infinite the boundary conditions on both sides of the domain affect the solution (Fig. 2.26). For the case of continuous uniform heat flux \dot{q}'' (W/m²) on one surface for $t > 0$ and constant temperature on the other surface the initial and boundary conditions are given as

$$T = T_i \quad \text{at} \quad t = 0$$

Fig. 2.26 Schematic view of plate geometry

$$-k \frac{\partial T}{\partial z} = q'' \quad z = 0 \quad (2.42)$$

$$T = T_i \quad z = L$$

The solution for the transient temperature distribution in the plate satisfying the above conditions can be obtained as

$$T(z, t) = T_i + \frac{\dot{q}''}{kL} \left[1 - \frac{z}{L} + \frac{8}{\pi} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^2} \sin\left(\frac{(2n+1)\pi(L-z)}{2L}\right) \exp\left(-\frac{(2n+1)^2 \pi^2 \alpha t}{4L^2}\right) \right] \quad (2.43)$$

As time goes to infinity the steady state solution is reached. The above solution yields the steady state solution as

$$T(z) = T_i + \frac{\dot{q}''}{kL} \left[1 - \frac{z}{L} \right] \quad (2.44)$$

On the other hand the temperature variation on the surface $z = 0$ which is subjected to uniform and continuous heat flux is obtained to be

$$T(0, t) = T_i + \frac{\dot{q}''}{kL} \left[1 + \frac{8}{\pi} \sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} \exp\left(-\frac{(2n+1)^2 \pi^2 \alpha t}{4L^2}\right) \right] \quad (2.45)$$

2.4.1 Moving Point Source \dot{q} (W) on the Surface of an Insulated Infinite Plate

Now consider an infinite plate of thickness L that is insulated on both sides is subjected to a moving point source \dot{q} (W) in direction of x-axis with a velocity of V on one side of the medium. The quasi-steady state solution for the temperature on the moving coordinate axis is given by

$$T(x, y, z) = T_i + \frac{\dot{q}}{kL} \left\{ \frac{1}{2\pi} K_0 \left(\frac{Vr}{2\alpha} \right) + \frac{1}{\pi} \sum_{n=1}^{\infty} K_0 \left[\frac{Vr}{2\alpha} \sqrt{1 + \left(\frac{2\alpha n\pi}{VL} \right)^2} \right] \times \cos \left(\frac{n\pi z}{L} \right) \exp \left(\frac{Vx}{2\alpha} \right) \right\} \quad (2.46)$$

where $r = \sqrt{x^2 + y^2}$.

2.4.2 Moving Line Source on the Surface of an Insulated Infinite Plate

In the case of the infinite y-axis line source moving along the x-axis with a velocity of V the quasi-steady state solution for the temperature distribution on the moving coordinate system is

$$T(x, z) = T_i + \frac{\dot{q}'\alpha}{VkL} \left\{ 1 + 2 \sum_{n=1}^{\infty} \left[\frac{1}{\sqrt{1 + \left(\frac{2\alpha n\pi}{VL} \right)^2}} - 1 \right] \times \cos \left(\frac{n\pi z}{L} \right) \exp \left(\frac{Vx}{2\alpha} \right) \right\} \quad (2.47)$$

The heat loss through convection from the surfaces of the plate is neglected in the above solution.

2.5 Thin Slab (Sheet)

When the thickness of the plate is negligible the variation of the temperature across the thickness of the plate is neglected. In this case thin slab assumption is used.

2.5.1 Spot Welding

Spot welding can be characterized as the instantaneous line heat release q across the thin sheets of total thickness δ to be welded between the two electrodes. The transient temperature distribution in the sheets is given as

$$T(r, t) = T_i + \frac{(q/\delta)}{4\pi kt} \exp \left(-\frac{r^2}{4\alpha t} \right) \quad (2.48)$$

where $r = \sqrt{x^2 + y^2}$.

Fig. 2.27 Temperature profiles in the 2 mm steel sheet after a spot welding at various times

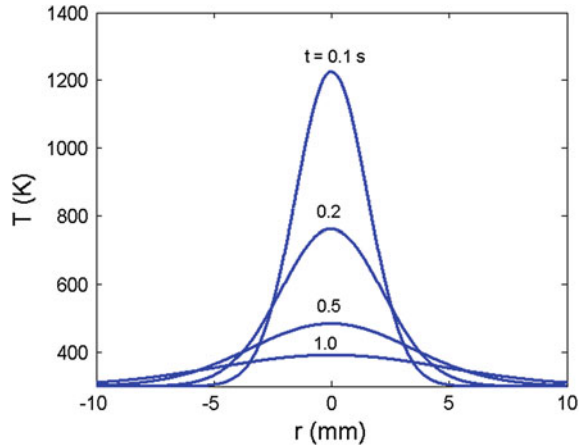


Figure 2.27 shows the temperature distribution in a 2 mm thick steel sheet after a spot welding operation in which 100 J heat release across the thickness of the sheet at the origin. The peak temperature at the weld spot decreases sharply as a result of heat diffusion that takes place in the radial direction in the sheet.

Example 12 In the spot welding operation on a 2 mm steel sheet an instantaneous heat release of 1 kJ occurs. Determine the maximum temperature at a location of 5 mm away from the spot and at what time this maximum temperature is reached.

Solution:

The time at which a maximum temperature is reached at a given r location in the sheet material can be obtained by differentiating Eq. (2.48) with respect to t and setting the result equal to zero, i.e.

$$T(r, t) = T_i + \frac{(q/\delta)}{4\pi kt} \exp\left(-\frac{r^2}{4\alpha t}\right)$$

$$\frac{\partial T(r, t)}{\partial t} = \frac{\partial T}{\partial t} \left[T_i + \frac{q/\delta}{4\pi\alpha t} \exp\left(-\frac{r^2}{4\alpha t}\right) \right] = 0$$

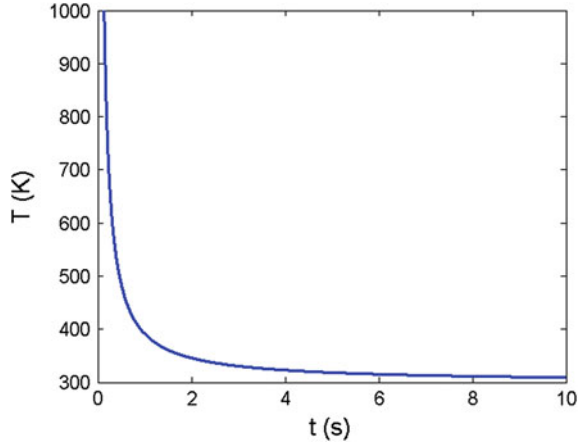
Carrying out the differentiating the time for the maximum temperature is obtained as

$$t = \frac{r^2}{4\alpha}.$$

Substituting the values we get

$$t = \frac{0.005^2}{4 \times 1.172 \times 10^{-5}} = 0.53 \text{ s}$$

Fig. 2.28 Temperature versus time at the spot of a steel sheet of 2 mm thickness after a heat release of 100 W at the spot



Therefore the maximum temperature 5 mm away from the spot becomes

$$T(0.005, 0.53) = 300 + \frac{1000/0.002}{4\pi \times 43 \times 0.53} \exp\left(-\frac{0.005^2}{4 \times 1.172 \times 10^{-5} \times 0.53}\right) = 938 \text{ K}$$

Temperature at the spot varies with time as

$$T(0, t) = T_i + \frac{(q/\delta)}{4\pi kt}. \quad (2.49)$$

The temperature decreases inversely proportional with time as shown in Fig. 2.28.

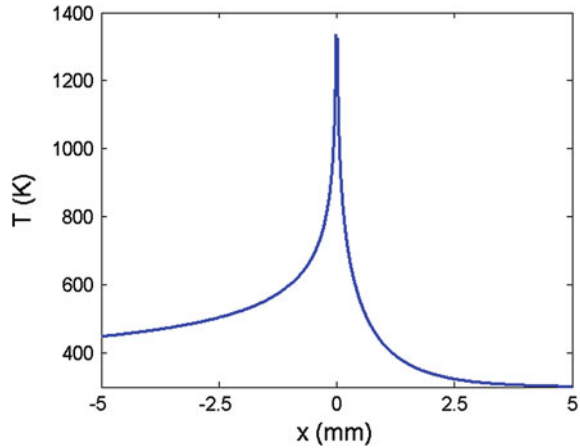
2.5.2 Moving Line Heat Release q Across the Thin Sheets of Total Thickness δ to Be Welded Between the Two Electrodes Along x -Direction

In the case of the line heat release across the thin sheets is continuous and moves along x -axis with a velocity V the quasi-steady state solution on moving coordinates is given as

$$T(x, y) = T_i + \frac{(q/\delta)}{2\pi k} \exp\left(-\frac{Vx}{2\alpha}\right) K_0 \left[\sqrt{x^2 + y^2} \sqrt{\left(\frac{V}{2\alpha}\right)^2 + \frac{h_1 + h_2}{k\delta}} \right] \quad (2.50)$$

where h_1 and h_2 are the convection heat transfer coefficients on both each of the sides of the sheets. Figure 2.29 shows the temperature distribution in a 2 mm steel sheet. The line heat release is considered to be 100 W/m and the velocity of the

Fig. 2.29 Temperature profile in the 2 mm steel sheet subjected to a line heat release of 100 W/m and a speed of 10 mm/s in the direction of the x-axis



line heat release is 10 mm/s in the direction of the x-axis. The convection heat transfer coefficients h_1 and h_2 are both assumed to be $100 \text{ W/m}^2 \text{ K}$.

Example 13 In a seam welding operation of steel sheets that can be characterized by a moving line heat source of 1 kW with a speed of 10 mm/s in the direction of x-axis the total thickness of the sheets to be welded is 4 mm. Determine the temperature 2 mm behind and 2 mm in front of the heat source location during this welding process. Assume that the convection heat transfer coefficients on both the top and bottom surfaces are $100 \text{ W/m}^2 \text{ K}$.

Solution:

Equation (2.50) can be written along the direction of x-axis (i.e. for $y = 0$) as

$$T(x) = T_i + \frac{(q/\delta)}{2\pi k} \exp\left(-\frac{Vx}{2\alpha}\right) K_0 \left[x \sqrt{\left(\frac{V}{2\alpha}\right)^2 + \frac{h_1 + h_2}{k\delta}} \right]$$

So, the temperature at the location 2 mm behind the heat source is

$$\begin{aligned} T(-0.002) &= 300 + \frac{(1000/0.004)}{2\pi \times 43} \exp\left(-\frac{0.01 \times (-0.002)}{2 \times 1.172 \times 10^{-5}}\right) \\ &\quad \times K_0 \left[(-0.002) \sqrt{\left(\frac{0.01}{2 \times 1.172 \times 10^{-5}}\right)^2 + \frac{100 + 100}{43 \times 0.004}} \right] \\ &= 1428.5 \text{ K} \end{aligned}$$

Similarly, the temperature 2 mm in front of the heat source becomes

$$\begin{aligned}
 T(0.002) &= 300 + \frac{(1000/0.004)}{2\pi \times 43} \exp\left(-\frac{0.01 \times (0.002)}{2 \times 1.172 \times 10^{-5}}\right) \\
 &\quad \times K_0 \left[(0.002) \sqrt{\left(\frac{0.01}{2 \times 1.172 \times 10^{-5}}\right)^2 + \frac{100 + 100}{43 \times 0.004}} \right] \\
 &= 504.8 \text{ K}
 \end{aligned}$$

2.6 Solid Rod

2.6.1 Friction Welding of Long Rods

For the case of moving plane source $\dot{q}(W)$ (over the cross sectional area of the rod) in the axial direction (x) of an infinite rod the quasi-steady state solution for the temperature distribution on the moving coordinate system is given as

$$T(x) = T_{\infty} + \frac{\dot{q}}{kA} \frac{\alpha}{U} \exp\left(-\frac{Vx - |x|U}{2\alpha}\right) \text{ where } U = \sqrt{V^2 + 4\alpha^2 \frac{hp}{kA}} \dot{q}(W) \quad (2.51)$$

T_{∞} is the surrounding temperature. Figure 2.30 shows the temperature variation in the axial direction of a rod subjected to the moving plane source of 1 kW with a speed of 1 mm/s in the direction of the x-axis. The temperature in the back side of the moving front decreases slightly in the negative direction of the x-axis as a result of the convection cooling. The temperature on the right side of the moving front shows a steep temperature gradient since the heat diffusion is yet to reach this part as the moving front approaches.

For a special case when the velocity is zero ($V = 0$) the temperature variation is obtained as

$$T(x) = T_{\infty} + \frac{\dot{q}}{2\sqrt{hp}kA} \exp\left(-\sqrt{\frac{hp}{kA}}|x|\right) \quad (2.52)$$

This solution is useful when radius of the rod is small and the radial temperature variation in the rod is negligible. Figure 2.31 shows the temperature variation in a rod of 2 cm radius with a plane source of 1 kW at the origin perpendicular to the axial direction. The temperature variation is symmetric around the origin. It decreases exponentially in both directions as a result of diffusion and convection. The convection heat transfer coefficient is considered to be $1000 \text{ W/m}^2 \text{ K}$.

Fig. 2.30 Temperature distribution in the steel rod of 2 mm radius subjected to 1 kW moving plane source with a velocity of 1 mm/s

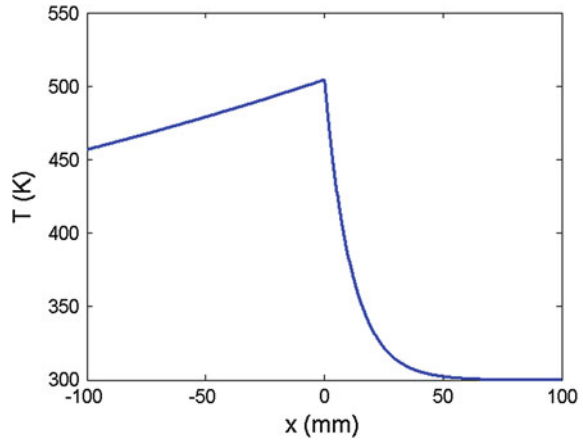
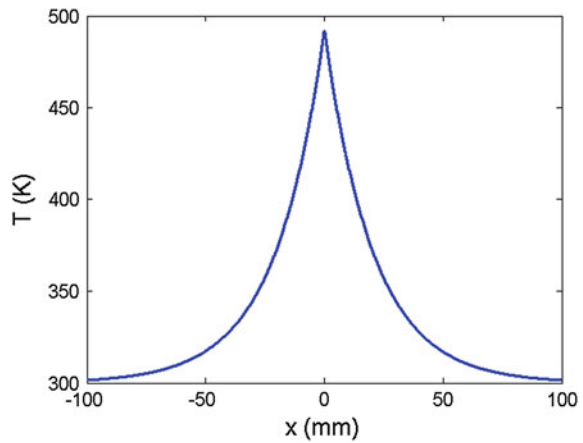


Fig. 2.31 The temperature variation in a rod of radius 2 cm and subjected to stationary plane source of 1 kW



Example 14 In a friction welding process of cylindrical steel rods of diameter 4 cm a temperature of 1000 K is needed at the interface. The convection heat transfer coefficient is estimated to be $10 \text{ W/m}^2 \text{ K}$. Determine the rate of heat needed for this process at the interface of the rods. If the equivalent relative radial speeds of rods at the interface is 10 m/s and the friction coefficient is assumed to be 0.2, determine the compression force needed at the interface to achieve the friction welding.

Solution:

At the interface ($x = 0$) Eq. (2.50) becomes

$$T(0) = T_{\infty} + \frac{\dot{q}}{2\sqrt{hpkA}}$$

Therefore the rate of heat needed is

$$\begin{aligned}\dot{q} &= 2\sqrt{hpkA}[T(0) - T_\infty] \\ &= 2 \times \sqrt{100 \times \pi \times 0.04 \times 43 \times \pi \times 0.02^2} \times [1000 - 300] = 3648 \text{ W}.\end{aligned}$$

From Eq. (2.1) the compression force needed at the interface is

$$F = \frac{\dot{q}}{\mu V} = \frac{3648}{0.3 \times 10} = 1216 \text{ N}.$$

2.6.2 Time Variable Heat Source in Rod

In the actual friction welding process the source of heat is time-varying in the exponential form. When stationary coordinate system is used the formulation of the problem in this case can be written as

$$\frac{1}{\alpha} \frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} - \frac{hp}{kA}(T - T_\infty) + \frac{\dot{q}_o'''}{k} \exp(at) \delta(x) \quad (2.53)$$

where $\delta(x)$ is Dirac delta function and \dot{q}_o''' (W/m³).

The initial and boundary conditions are

$$T(x, 0) = T_\infty \quad \frac{\partial T}{\partial x}(\pm\infty, t) = 0 \quad (2.54)$$

The last term on the right hand side considers the variation of the strength of the heat source in the exponential form. The solution for the transient temperature variation in the rod is given by

$$T(x, t) = T_\infty + \frac{\dot{q}_o \sqrt{\alpha}}{2kA} \frac{\exp(at)}{2\sqrt{m^2\alpha + a}} \left\{ \begin{aligned} &\exp\left[-\sqrt{m^2 + \frac{a}{\alpha}}x\right] \times \operatorname{erfc}\left[\frac{x}{2\sqrt{at}} - \sqrt{(m^2\alpha + a)t}\right] \\ &- \exp\left[\sqrt{m^2 + \frac{a}{\alpha}}x\right] \times \operatorname{erfc}\left[\frac{x}{2\sqrt{at}} + \sqrt{(m^2\alpha + a)t}\right] \end{aligned} \right\} \quad (2.55)$$

where $m = \sqrt{\frac{hp}{kA}}$ and \dot{q}_o (W).

For the steady heat release at the interface for $t > 0$ the exponent $a = 0$ and the solution for the transient temperature distribution becomes

$$T(x, t) = T_\infty + \frac{\dot{q}_o}{4\sqrt{hpkA}} \left\{ \exp[-mx] \times \operatorname{erfc}\left[\frac{x}{2\sqrt{at}} - m\sqrt{at}\right] - \exp[mx] \times \operatorname{erfc}\left[\frac{x}{2\sqrt{at}} + m\sqrt{at}\right] \right\} \quad (2.56)$$

The temperature profiles in a transient heating of 10 cm diameter steel rod subjected to a steady plane heat source of 10 kW is shown in Fig. 2.32. The profiles correspond to times 10, 20, 30, 40, and 50 s.

Fig. 2.32 Temperature profiles during the transient friction welding operation of steel rods of 10 cm diameter subjected to 10 kW of plane heat source at the origin at different times from 10 to 50 s

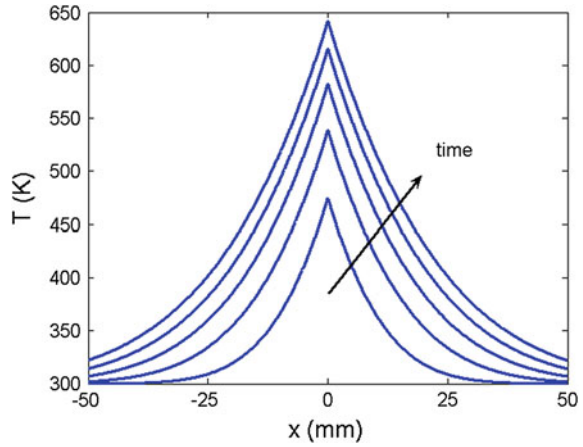
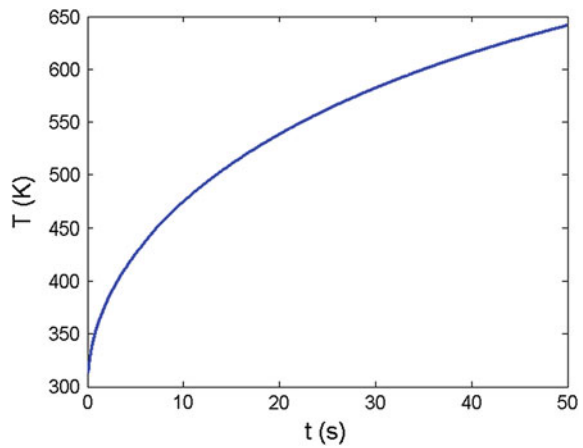


Fig. 2.33 The temperature rise at the origin in a 10 cm diameter rod during the transient heating due to steady plane heat source at the interface $x = 0$



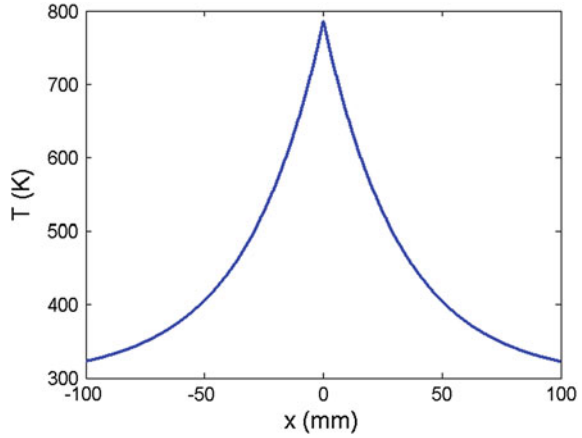
The temperature at the interface $x = 0$ is obtained to be

$$T(0, t) = T_{\infty} + \frac{\dot{q}_o}{2\sqrt{h\rho k A}} \operatorname{erf}(m\sqrt{\alpha t}) \approx T_{\infty} + \frac{\dot{q}_o'''}{A} \frac{\sqrt{t}}{\sqrt{\pi\rho C_p k}} \exp(-m^2 \alpha t) \quad (2.57)$$

in which the approximation $\operatorname{erf}(x) \approx \frac{2}{\sqrt{\pi}} x \exp(-x^2)$ is used. The rise of temperature at the interface of the rod for the above case is shown in Fig. 2.33.

Example 15 Determine the time needed for the interface temperature to reach 1000 K in the friction welding process given in example 14.

Fig. 2.34 The steady temperature variation in a steel rod of 10 cm diameter subjected to a steady plane heat source of 10 kW at the interface $x = 0$ with a heat transfer coefficient $1 \text{ kW/m}^2 \text{ K}$



Solution:

Equation (2.57) is

$$T(0, t) = T_{\infty} + \frac{\dot{q}_o}{2\sqrt{h\rho k A}} \operatorname{erf}(m\sqrt{\alpha t}) = T_{\infty} + \frac{\dot{q}_o}{2\pi R \sqrt{2hkR}} \operatorname{erf}\left(\sqrt{\frac{2h}{\rho C_p R}} t\right)$$

$$1000 = 300 + \frac{8860}{2\pi \times 0.02 \times \sqrt{2 \times 1000 \times 43 \times 0.02}} \operatorname{erf}\left(\sqrt{\frac{2 \times 1000}{7800 \times 473 \times 0.02}} t\right)$$

Solving for t using trial and error method the time needed for the friction welding is obtained to be

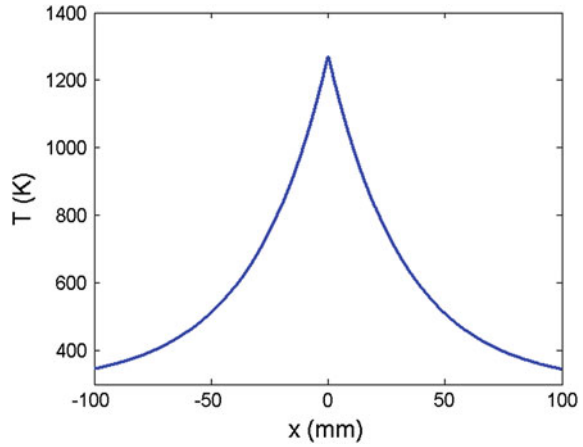
$$t = 29 \text{ s.}$$

On the other hand the steady state temperature distribution is obtained as time goes to infinity as

$$T(x) = T_{\infty} + \frac{\dot{q}_o}{2\sqrt{h\rho k A}} \exp(-mx). \quad (2.58)$$

Figure 2.34 shows the steady temperature variation in the steel rod of 10 cm diameter subjected to 10 kW plane heat source and heat transfer coefficient of $1 \text{ kW/m}^2 \text{ K}$. Because of the convection heat transfer the temperature reaches a steady profile. The peak temperature occurs at the interface and it decreases sharply in both sides reaching almost the ambient temperature 10 cm away from the interface.

Fig. 2.35 Steady temperature distribution in a finite rod of 10 cm length in both sides and 10 cm diameter subjected to 10 kW of plane heat source at the interface $x = 0$



2.6.3 Finite Rod

The analysis of finite rod of length L is done by considering the boundary conditions at both ends of the rod. Consider the case where a continuous heat source of \dot{q}_o (W) presents at one end and the other end of the rod is insulated. In this case the transient temperature distribution in the rod is given by

$$T(x, t) = T_{\infty} + \frac{\dot{q}_o}{\sqrt{hpkA}} \left\{ \frac{\cosh[m(L-x)]}{\sinh(mL)} + \frac{\exp(-m^2 \alpha t)}{mL} + \frac{2mL}{\pi} \sum_{n=1}^{\infty} \frac{1}{n^2 + m^2 L^2 / \pi^2} \cos\left(\frac{n\pi x}{L}\right) \exp\left[-\frac{(\pi^2 n^2 + m^2 L^2) \alpha t}{L^2}\right] \right\} \quad (2.59)$$

The steady state temperature distribution becomes

$$T(x) = T_{\infty} + \frac{\dot{q}_o}{\sqrt{hpkA}} \frac{\cosh[m(L-x)]}{\sinh(mL)} \quad (2.60)$$

Figure 2.35 shows the steady temperature distribution along the axial direction of a finite steel rod of 10 cm length in both sides and 10 cm diameter. The steady plane heat source is considered to be 10 kW and the convection heat transfer coefficient is assumed to be $1 \text{ kW/m}^2 \text{ K}$.

Thus the steady temperatures at both ends are found to be

$$T(0) = T_{\infty} + \frac{\dot{q}_o}{\sqrt{hpkA}} \frac{1}{\tanh(mL)} \quad (2.61)$$

and

$$T(L) = T_{\infty} + \frac{\dot{q}_o}{\sqrt{hpkA}} \frac{1}{\sinh(mL)} \quad (2.62)$$

respectively.

The analytical solutions discussed above provide a general insight to various welding processes. Thermal analysis of most of the practical welding processes can be approximately simulated by these analytical solutions. In cases where the welding process involves more parameters such as temperature dependent material properties, utilization of assisting gas and complex geometry of materials to be welded, the analytical solutions may not be available. The thermal analysis in such cases must be carried out numerically. In the next section a numerical solution is presented for a typical friction welding of dissimilar rods during which an assisting inert gas is used.

2.7 Numerical Analysis of Friction Welding

Consider the two cylindrical rods with radius R to be welded as shown in Fig. 2.36. The rods are assumed to be semi-infinite in z direction. Rod I rotates with an angular velocity ω while rod II is stationary. At the surface of the intersection of the two rods, the coefficient of friction μ and the pressure P are assumed to be constant and uniform throughout the surface. The thermal diffusivities are given by α_1 and α_2 for rod I and rod II, respectively. Likewise, the thermal conductivities and the heat transfer coefficients are given by k_1 , h_1 and k_2 , h_2 respectively. The jet velocity of inert gas around the interface is V_j and ambient temperature is T_{∞} .

With the foregoing assumptions and using the dimensionless parameters; $\theta = \frac{T-T_{\infty}}{\mu P \omega R^2 / k_1}$, $\tau = \frac{\alpha_1 t}{R^2}$, $\rho = \frac{r}{R}$, $\zeta = \frac{Z}{R}$, $r_{\alpha} = \frac{\alpha_1}{\alpha_2}$, $r_k = \frac{k_2}{k_1}$, $B_1 = \frac{h_1 R}{k_1}$, and $B_2 = \frac{h_2 R}{k_2}$ the formulation of the problem together with the initial and boundary conditions becomes:

$$\frac{\partial \theta_1}{\partial \tau} = \left[\frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial \theta_1}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2 \theta_1}{\partial \phi^2} + \frac{\partial^2 \theta_1}{\partial \zeta^2} \right] \quad (2.63)$$

$$\frac{\partial \theta_2}{\partial \tau} = r_{\alpha} \left[\frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial \theta_2}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2 \theta_2}{\partial \phi^2} + \frac{\partial^2 \theta_2}{\partial \zeta^2} \right] \quad (2.64)$$

$$\theta_1(\rho, \phi, \zeta, 0) = 0; \quad \theta_2(\rho, \phi, \zeta, 0) = 0 \quad (2.65)$$

$$\frac{\partial \theta_1}{\partial \zeta}(\rho, \phi, \infty, \tau) = 0; \quad \frac{\partial \theta_2}{\partial \zeta}(\rho, \phi, -\infty, \tau) = 0 \quad (2.66)$$

Fig. 2.36 Schematic view of friction welding of cylindrical rods of dissimilar materials

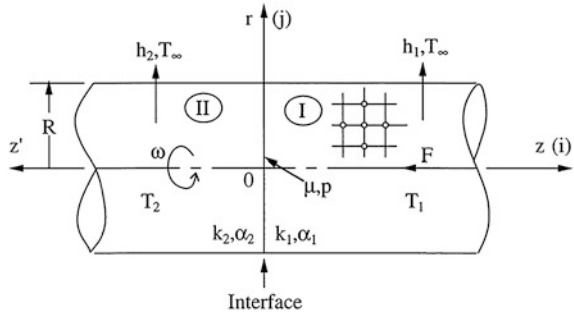
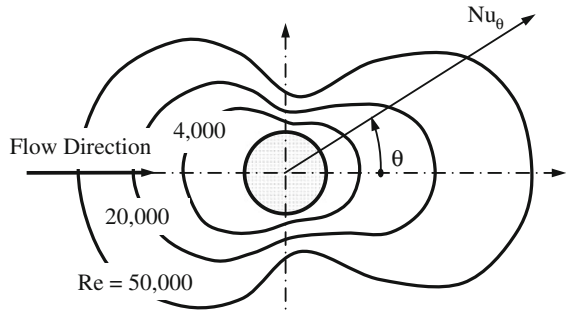


Fig. 2.37 Circumferential variation of the local heat transfer coefficient for a circular cylinder in air cross flow



$$\frac{\partial \theta_1}{\partial \rho}(0, \phi, \zeta, \tau) = 0; \quad \frac{\partial \theta_2}{\partial \rho}(0, \phi, \zeta, \tau) = 0 \quad (2.67)$$

$$\begin{aligned} \frac{\partial \theta_1}{\partial \rho}(1, \phi, \zeta, \tau) - B_1(\phi)\theta_1(1, \phi, \zeta, \tau) &= 0; \\ \frac{\partial \theta_2}{\partial \rho}(1, \phi, \zeta, \tau) - B_2(\phi)\theta_2(1, \phi, \zeta, \tau) &= 0 \end{aligned} \quad (2.68)$$

$$\theta_1(\rho, 0, \zeta, \tau) = \theta_1(\rho, 2\pi, \zeta, \tau); \quad \theta_2(\rho, 0, \zeta, \tau) = \theta_2(\rho, 2\pi, \zeta, \tau) \quad (2.69)$$

$$\frac{\partial \theta_1}{\partial \phi}(\rho, 0, \zeta, \tau) = \frac{\partial \theta_1}{\partial \phi}(\rho, 2\pi, \zeta, \tau); \quad \frac{\partial \theta_2}{\partial \phi}(\rho, 0, \zeta, \tau) = \frac{\partial \theta_2}{\partial \phi}(\rho, 2\pi, \zeta, \tau) \quad (2.70)$$

$$\theta_1(\rho, \phi, 0, \tau) = \theta_2(\rho, \phi, 0, \tau) \quad (2.71)$$

$$\frac{\partial \theta_1}{\partial \zeta}(\rho, \phi, 0, \tau) = 1 + r_k \frac{\partial \theta_2}{\partial \zeta}(\rho, \phi, 0, \tau) \quad (2.72)$$

The local heat transfer coefficient $Nu(\phi) = \frac{hD}{k_f}$ for a non-rotating circular cylinder in air stream is given by Kreith and Bohn [12] and shown in Fig. 2.37. This is used in obtaining the Biot number variation around the stationary rod. According to Chiou and Lee [13], a more uniform heat transfer rate can be achieved in the case of rotating cylinder in the cross-flow; therefore the heat transfer coefficient (Biot number) around the rotating rod is taken to be circumferentially uniform.

Fig. 2.38 Radial temperature profiles

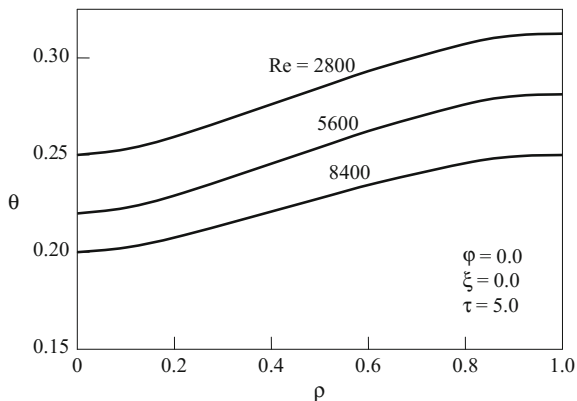
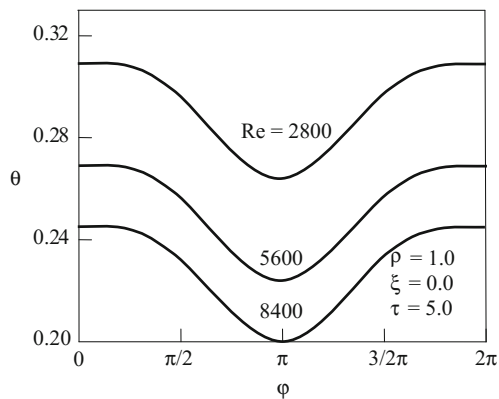


Fig. 2.39 Temperature profiles along the circumferential direction in the rod



An explicit central difference scheme in space is used by selecting appropriate mesh sizes and number of nodes in each direction to ensure accuracy of the solution. The time step size for the stability of the numerical backward difference scheme is also determined and used in the solution. The effect of the jet velocity on the temperature rise around the vicinity of the weld interface is also considered. For simplicity, the thermal conductivity is assumed to be constant and the heat radiation is neglected in the present solution, although these effects can easily be implemented in the numerical scheme.

Let us consider welding of only similar materials, where $r_x = 1$ and $r_k = 1$. The quality of weld depends primarily on the uniformity of temperature obtained on the interface. Therefore the focus of discussion is on the rise and distribution of temperature around the welding interface, $\zeta = 0$.

Figure 2.38 shows the temperature profiles in the radial direction for different Reynolds numbers ($Re = 2V_j R/\nu$) at the back stagnation point ($\phi = 0$) on the interface where the convective cooling is expected to be less than that at the front stagnation point. The maximum temperature occurs at the surface of the cylinder due to variation of frictional heat generation along the radial direction. However, the location of maximum temperature moves towards the center while the

Fig. 2.40 Temperature profiles along the axial direction in the rod

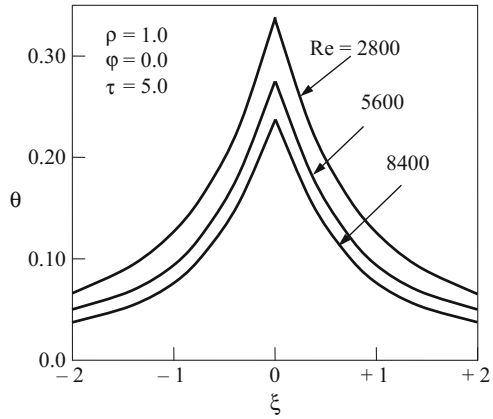
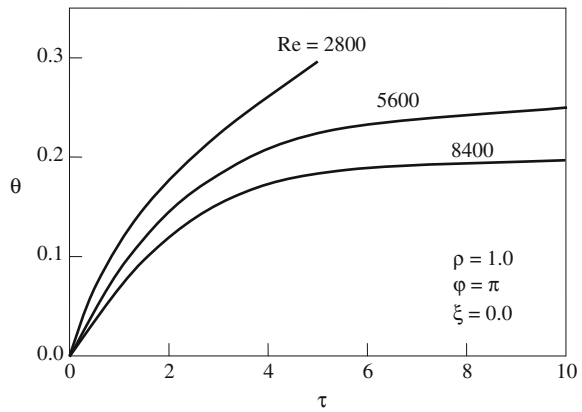


Fig. 2.41 Temperature rise at the front stagnation point at the interface



Reynolds number is increased. A considerable temperature difference is observed along the radial direction.

Figure 2.39 shows the temperature profiles along the circumferential direction on the surface at the interface for the same set of Reynolds numbers as given in Fig. 2.37. The minimum temperature is observed at the front stagnation point ($\phi = \pi$), and the variation of temperature is significant especially for high Reynolds numbers.

The temperature profiles along the axial direction at the back stagnation point along the surface are given in Fig. 2.40 for the same Reynolds numbers as mentioned above. It should be noted that, even though similar materials are used, the temperature profiles are not symmetric around the interface, because of the different surface boundary conditions prevailing on the two domains. The rotating rod which is on the left side in Fig. 2.36 experiences lower and circumferentially more uniform temperature distribution than that for the stationary rod. This suggests that, in welding of different materials, the rod with lower thermal conductivity should be the rotating one in order to achieve more uniform temperature distribution.

The temperature rise at the front stagnation point ($\phi = \pi$) on the interface is given in Fig. 2.41. The temperature rises very sharply in the early stages of the process and approaches quickly the steady state temperature especially for high Reynolds numbers. So the plastic range temperature which is needed for welding process may never be reached when the Reynolds number is high. On the other, hand melting temperature may be reached at certain locations due to the large variation of temperature in the radial and circumferential direction.

The above thermal analysis suggests that the temperature uniformity cannot be achieved during a welding process especially when the thermal conductivity of the parts to be welded is low. Low Reynolds number yields sharp rise of interface temperatures and decreases the time required for welding process, however it causes large radial temperature variation. High Reynolds number decreases the radial temperature variation for the rotating rod, but increases the circumferential temperature variation especially in the stationary rod.

The plastic range temperature needed for welding process may never be reached for high Reynolds numbers at locations where heat transfer coefficient is high. The rod with lower thermal conductivity should be the rotating one in order to achieve more uniform temperature distribution in welding of different materials.

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