Chapter 2
Research on the Combination of IGS Analysis-Center Solution for Station Coordinates and ERPs

Min Li and Tian-he Xu

Abstract This paper mainly focus on the issues of combining the station coordinates and ERPs based on the SINEX file, and discusses the constraints and normal equation's reconstruction in the SINEX file in details. The combination model and computational steps are given. A weighted-combination method based on the polynomial fitting residuals is proposed for the pole motion parameters. Computations and comparisons are performed using the proposed methods. The results show that the SINEX combination solution have the consistent accuracy with those provided by IGS. The accuracy of station coordinates in x and y direction is about 3, and 4 mm in direction z. The accuracy of pole motion parameters and their rate are 0.02 mas and 0.05 mas/d respectively. The accuracy of ERP solution based on the SINEX file is higher than that of the weighted-combination method.

Keywords Station coordinates · ERPs · SINEX · Transformation parameters · Polynomial fitting

M. Li (✉)
Chang'an University, Xi'an, Shanxi, China
e-mail: liminbmw760@163.com

T. Xu
State Key Laboratory of Geo-information Engineering, Xi'an, Shanxi, China

T. Xu
State Key Laboratory of Geodesy and Earth's Dynamics, Wuhan, Hubei, China

T. Xu
Xian Research Institute of Surveying and Mapping, Xi'an, Shanxi, China

T. Xu
State Key Laboratory of Astronautic and Dynamics, Xi'an, Shanxi, China

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2.1 Introduction

Since 1999, IGS (International GNSS Services) have released many high-precision products including the global GNSS tracking station coordinates, velocity fields and the ERPs (Earth Rotation Parameters) etc. through a combination of at least seven AC (analysis-center) products [1]. IGS AC product combination is an important step for the IERS (International Earth Reference Services) to realize the ITRF (International Terrestrial Reference Frame) by fusing multi-source spatial geodetic data [1, 2]. For the IGS, the estimation of the station coordinates is the core task of realizing and maintaining the ITRF. Another important task is the determination of ERPs since they are necessary physical parameters in the conversion of Celestial coordinate system and Earth-Centered coordinate system, and they are also the basic data in satellite precise orbit determination, high accuracy positioning and navigation [3–5]. In order to facilitate and combine the AC products, IGS proposed the SINEX (Software INdependent EXchange) format file, from which the normal equation systems can be recovered. In this paper, we will focus on the issues of combination of the station coordinates and ERPs based on the SINEX file.

In this paper, we mainly focus on the theory and method of fusing the SINEX solutions. A weighted-combination method based on the polynomial fitting residuals is proposed for the pole motion parameters, computations and comparisons are performed using the proposed methods and some useful conclusions are obtained.

2.2 Strategy to Fuse the SINEX Solutions

During the combination, all the SINEX files from the ACs need to be preprocessed by rejecting the gross errors, eliminating the apriori constraints, apriori transformation of normal equations, unifying parameters apriori values, reconstructing normal equations and so on.

2.2.1 The Combination Model

The combination model is similar to that of ITRF, and here we don't consider the station velocity parameters. The model can be expressed as [6]:

$$X_s^i = X_c^i + X_k + D_k X_c^i + R_k X_c^i \tag{2.1}$$

where X_s^i is the solution for station i, X_c^i is the combined solution. T_k, D_k, R_k are the translation, scale and rotation parameters for the kth AC. For simplification, X_s^i is replaced by X, seven transformation parameters is expressed as T_k, so the model written in the way of normal equations can be expressed as:

$$\begin{pmatrix} A1_s^T \\ A2_s^T \end{pmatrix} P_s \begin{pmatrix} A1_s & A2_s \end{pmatrix} \begin{pmatrix} X \\ T_k \end{pmatrix} = \begin{pmatrix} A1_s^T P_s B_s \\ A2_s^T P_s B_s \end{pmatrix} \tag{2.2}$$

where $A1_s$, $A2_s$ is the designed matrix defined by each station, which can be expressed as:

$$A1_s^i = \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix}, \quad A2_s^i = \begin{pmatrix} A_s^i & 0 \\ 0 & A_s^i \end{pmatrix} \tag{2.3}$$

where P_s is the weight matrix which is the inverse of each solution's variance-covariance matrix, B_s is the difference value of observation and calculation, A_s^i is the approximate coordinate values, i changes from 1 to n, n is the number of stations, the A_s^i is expressed as:

$$A_s^i = \begin{pmatrix} . & . & . & . & . & . & . \\ 1 & 0 & 0 & x_0^i & 0 & z_0^i & -y_0^i \\ 0 & 1 & 0 & y_0^i & -z_0^i & 0 & x_0^i \\ 0 & 0 & 1 & z_0^i & y_0^i & -x_0^i & 0 \\ . & . & . & . & . & . & . \end{pmatrix} \tag{2.4}$$

Equation (2.1) should be extended by adding the following equation if the ERP parameters are considered:

$$\begin{cases} x_s^p = x_c^p + R2_k \\ y_s^p = y_c^p + R1_K \\ \dot{x}_s^p = \dot{x}_c^p \\ \dot{y}_s^p = \dot{y}_c^p \\ LOD_s = LOD_c \\ UT_s = UT_c \end{cases} \tag{2.5}$$

Eq. (2.5) contains the pole motion vectors x_s^p and y_s^p and their transformation parameters $R1_k$ and $R2_k$, the pole motion velocity vectors $\dot{x}_s^p$ and $\dot{y}_s^p$, length of day LOD_s, universal time, UT_s. As the transformation parameters of LOD and UT vector is not obvious in the pure GPS intra-technique combination, so they are ignored.

2.2.2 The Combination Steps

2.2.2.1 Normal Equations Restoration

The process to restore the normal equations with the variance factor of unit weight $\hat{\sigma}_0^2$, estimation value $\hat{X}$, variance-covariance matrix $D_{\hat{X}}$, apriori value X^0, apriori variance-covariance D_{X^0} is described in detail in literature [5, 7]. Please notice that the normal equations restored here need to be revised during the next step, because when the parameters are pre-eliminated, the variance factor is no more accurate. It need to determinate the variance factor again through by iteration using Helmert variance component estimation [8]. The apriori standard deviation value in SOLUTION/ESTIMATE model is better to be used as apriori variance factor, otherwise the minus variance factor may occur during iteration.

2.2.2.2 Constraints Handling

One difficulty in the post-processing for GPS precise positioning and orbit determination with SINEX file is how to deal with the constraints well, the constraints added to each AC is not always appropriate and consistent, so it needs to be eliminated in advance.

We can identify the constraints though the first line in the SINEX file, it's shown with a mark: 0 stands for the fixed/tight constraint, 1 stands for the significant constraint, 2 stands for no constraint. Generally speaking, the constraints in the SINEX file can be classified into three parts [9]:

Case 0: the solutions are thought to be obtained by co-adjustment with the data unchanged during the process. Such kind of solutions is rare in the SINEX file now.

Case 1: the solutions are thought to be obtained by weighted parameters estimation, the apriori value and apriori variance-covariance matrix are given.

Case 2: the solutions need to be identified. Sometimes it's the loose constraints and a large variance such as 100 m is given. Sometimes it's the minimum constraints, seven or less than seven parameters and variance values are given in the apriori batch. Nowadays the latter is usually used in the IERS data combination and IGS weekly solution combination.

No fixed constraints are added to the station coordinates in all the SINEX files used in this paper, only the UT parameters are added with fixed constraints by several ACs. The process of eliminating the significant constraints is as follows:

$$N_f = D_{\hat{X}}^{-1} - D_{X^0}^{-1} \tag{2.6}$$

$$W_f = D_{\hat{X}}^{-1}\hat{X} - D_{X^0}^{-1}X^0 \tag{2.7}$$

Then we may get the unconstrained normal equations with N_f and W_f:

$$N_f X = W_f \tag{2.8}$$

As to the IGS SINEX files, the unconstrained matrix is almost singular after eliminating the significant constraints, because the matrix may be rank defect if it is related to the seven transformation parameters and their rates. Even if it's not rank defect, the normal equation maybe singular and the solution won't be stable. As to $N\hat{x} = b$, when it's singular, the module of N tends to be very small, the solution will change obviously if b changes a little. In this paper, we use the same procedures of ITRF to deal with the issues of rank defect and normal equation's singularity.

The combination work and realization of ITRF is mainly done by IGN in France. They add the minimum constraints before stacking the normal equations [5]. The principle of minimum constraints applied here is expressed as Similar Transformation constraints. It can be expressed by seven parameters transformation model as:

$$X_2 - X_1 = A\theta \tag{2.9}$$

where X_2, X_1 are station coordinates in two coordinate systems; θ is the transformation parameters expressed as in Eq. (2.1); A is the same as that of Eq. (2.4).

If we know several common points, θ can be solved by the principle of least-squares adjustment as:

$$\theta = \left(A^T P_x A\right)^{-1} A^T P_x (X_2 - X_1) \tag{2.10}$$

where P_x is the weight matrix of station coordinates, Let $B = \left(A^T P_x A\right)^{-1} A^T P_x$, we can get:

$$\theta = B(X_2 - X_1) \tag{2.11}$$

In order to eliminate the rank defection of Eq. (2.8), we introduce the condition equation expressed as:

$$B\left(\hat{X} - X_0\right) = 0, \left(\sum\nolimits_\theta\right) \tag{2.12}$$

The condition number equals to the number of rank defection, $\sum_\theta$ is the diagonal matrix, the value on the diagonal is the variance corresponding to the transformation parameters which is quite small. The corresponding normal equation of Eq. (2.12) is expressed as:

$$(B^T \Sigma_\theta^{-1} B)\hat{X} = (B^T \Sigma_\theta^{-1} B)X^0 \tag{2.13}$$

We may get the normal equation with minimum constraints if we combine Eq. (2.13) with Eq. (2.8):

$$\left(N_f + B^T \Sigma_\theta^{-1} B\right)\hat{X} = W_f + \left(B^T \Sigma_\theta^{-1} B\right)X_0 \tag{2.14}$$

From the equation above, we can find that the realization of Similar Transformation constraints is to transform the station coordinate of control net to the known coordinate system. As the added condition number equals to the number of rank defection and no extra constraints are introduced, so the benchmark information of the control net itself is not affected.

What need to be emphasized here is that the solutions to the normal equations with minimum constraints are still unstable, because the condition number is still very large and the normal equations are singular. So another organization in Germany called DGFI dealing with the realization of ITRF suggests to multiply a factor k^2 to the minimum constraints matrix, thus the condition number of the coefficient matrix in Eq. (2.14) is minimal. The minimum constraints is added after stacking the normal equations, thus the negative effect to the results caused by over parameterization is avoid.

The principle of Tykhonov-Phillips Regularization can also deal with the singular normal equations well, and the regularization parameters are solved by the Optimal Regularization method [5]. The results from this method are consistent but not the same as those of DGFI. As it's not studied in this paper, more details will not be shown here.

2.2.2.3 Normal Equation Systems Pre-processing

Apriori Helmert transformation to the normal equation systems without constraints is mainly to unify the reference benchmark and epoch, check the duplication of station names besides detecting and rejecting the gross error. The gross error can affect the following procedures a lot, for example the Helmert variance component estimation and the accuracy of the final combination and it should be detected by comparing the coordinates transformed by seven parameters with the ITRF solution at the same epoch.

2.2.2.4 Normal Equation Systems Reconstruction

Normal equation systems reconstruction means to classify, eliminate and merge the parameters, unify apriori value, stack the normal equations, introduce the transformation parameters and etc. The theory of parameter transformation is applied for all of them, which is the basic and core algorithm for estimating various parameters.

Suppose that two kinds of parameters as $\tilde{X}$ and $\hat{X}$, the transformation equation can be expressed as:

$$\hat{X} = C\tilde{X} + dx \tag{2.15}$$

where C is the coefficient matrix, dx is usually the constant matrix. Then the normal equation:

$$N\hat{X} = W \tag{2.16}$$

It can be rewritten as follows:

$$C^T NC\tilde{X} = C^T(W - Ndx) \tag{2.17}$$

Let, $\tilde{W} = C^T(W - Ndx)$ $\tilde{N} = C^T NC$ and $\tilde{W} = C^T(W - Ndx)$, we can get the following normal equation as:

$$\tilde{N}\tilde{X} = \tilde{W} \tag{2.18}$$

Equation (2.18) can be applied in a lot of aspects, such as apriori value unification and parameter pre-elimination. Apriori values of the unknown parameters in normal equations need to be unified before stacking, otherwise the normal equation should be transformed. For example, the parameter UT in ERPs supplied by GFZ is TAI-UT1 [10], while in most ACs it's the UT1 corrected with pole motion value, so they need to be unified. Parameter pre-elimination. Only the station coordinates and ERPs are considered in Eq. (2.1), so the parameters like apparent geocenter and satellite antenna phase bias are pre-eliminated.

It's not easy to solve the nine transformation parameters (seven for station coordinates and two for pole motion). The number of stations needs to be suitable for both the demand of benchmark and the robustness of normal equations. The rotation parameters for ERPs have better be solved every week.

2.2.2.5 Determination of Relative Weight Factor

The relative weight factor is determined for station coordinates and ERPs computation. Suppose the number of AC is N_i, P_k is the post-processing weight for solution k, σ_0^2 is the post-processing variance factor and then the average post-processing variance for AC i can be expressed as:

$$\sigma_i^2 = \sum_{k=1}^{N_i} \sigma_0^2 p_k^{-1} / N_i \tag{2.19}$$

Then the relative weight factor for each AC is [11] :

$$w_i = \frac{1/\sigma_i^2}{\sum_{i=1}^{I} 1/\sigma_i^2 / I} \tag{2.20}$$

2.3 Strategy to a Weighted-Combination Method Based on the Polynomial Fitting Residuals

The weighted-combination method developed in this paper mainly aims at the combination of the pole motion parameters x^p, y^p. As the main period of pole motion is Chandler wobble and yearly wobble, the data for one year from all the ACs is used for fitting, the weight for each AC is determined by residuals, then we may carry out the combination. The fitting model is as follows [12]:

$$f(t) = a + bt + A_c \sin(2f_c t + \varphi_c)\pi + A_a \sin(2f_a t + \varphi_a)\pi \tag{2.21}$$

where a is constant terms for linear trend, b is the quotient term, A_c, A_a is the amplitude of Chandler wobble and yearly wobble, f_c, f_a is the corresponding frequency, φ_c, φ_a is the corresponding phase. For the convenient of computation, Suppose that $a_k = A_k \cos(\varphi_k)$, $b_k = A_k \sin(\varphi_k)$, then Eq. (2.21) can be written as:

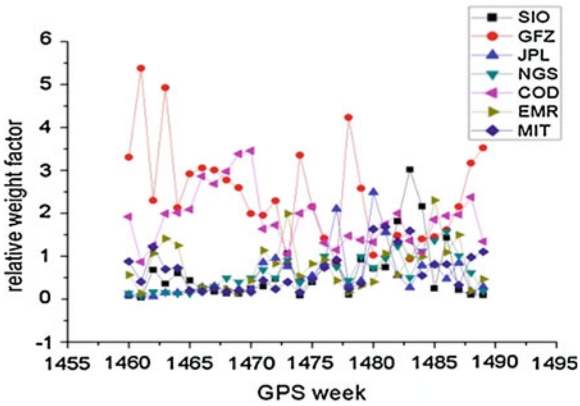
$$f(t) = a + bt + \sum_{1}^{2} (a_k \sin(2\pi f_k t) + b_k \cos(2\pi f_k t)) \tag{2.22}$$

All the parameters in Eq. (2.22) can be solved by the principles of least-squares:

$$\hat{\beta} = \left(A^T A\right)^{-1} A^T l \tag{2.23}$$

where $\hat{\beta} = \begin{bmatrix} a & b & a1 & b1 & a2 & b2 \end{bmatrix}^T$, A is the design matrix, l is the observation vector of pole motion data. The pole motion parameters can be fitted after solving Eq. (2.23) by least-squares without considering the observation accuracy of EOPs.

Fig. 2.1 Relative weight factors of 7 ACs



2.4 Calculation and Analysis

The SINEX files from 1460 to 1489 of GPS week in year 2008 are used with seven ACs, that is COD, EMR, GFZ, JPL, MIT, NGS and SIO. The relative weight factors determined by this paper are shown in Fig. 2.1. From it, we can find that GFZ and COD take a larger weight especially for the weeks before 1480. From weeks of 1480 to 1489, the parameters are added and new parameters are introduced in some ACs. For example, 120 and 170 parameters are added to the solutions of SIO and NGS separately. GFZ also introduces the satellite antenna phase bias, smaller differences of all the weight factors can be found in Fig. 2.1.

2.4.1 The Combination of Station Coordinates

The combination of station coordinates is based on ITRF05 reference frame with the file ITRF_IGS05.SNX. The reference epoch is the average epoch of all the weekly SINEX files. Three rotation parameters are not obvious while carrying out T-test to the transformation parameters, so they are rejected while computation. The standard deviation of the residuals between the combined solution and the each of the AC & ITRF in X, Y, Z direction are shown in Figs. 2.2, 2.3 and 2.4 respectively. The statistics information between the combined solution and that of ITRF is shown in Table 2.1.

From Figs. 2.2, 2.3 and 2.4, we can conclude that the standard deviation between the combined solution and those of ITRF in direction X and Y is from 2 to 4 mm, direction Z is from 2 to 4.5 mm. The standard deviation between the combined solution and that of each AC in direction X, Y and Z is almost below 7 mm and it is quite lower for that of COD and GFZ. Our solutions is almost equal to the IGS solutions shown in literature 1 whose standard deviation of each AC is under 3.5 mm in direction X, Y and 10 mm in direction Z.

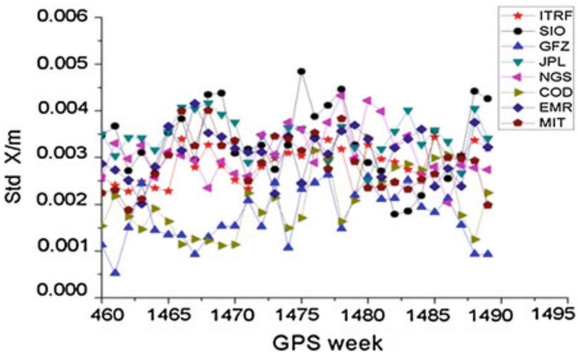


Fig. 2.2 Standard deviation in X direction between the combined solution and 7 ACs & ITRF

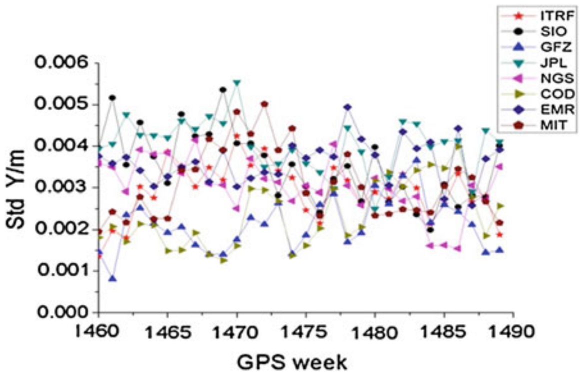


Fig. 2.3 Standard deviation in Y direction between the combined solution and 7 ACs & ITRF

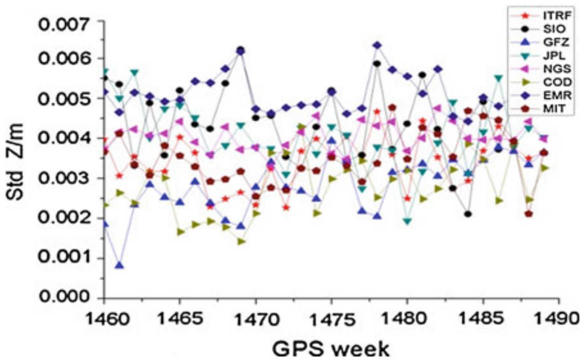


Fig. 2.4 Standard deviation in Z direction between the combined solution and 7 ACs & ITRF

According to Table 2.1, the maximum difference between the combined solutions and those of ITRF in all directions is less than 2 cm and the difference of standard deviation is between 3 and 4 mm. So, our results and IGS final results are at the same accuracy level.

Table 2.1 Statistics result of difference between the combined solution and ITRF in X, Y, Z direction

Difference	X	Y	Z
Maximum	0.0106	0.0093	0.0195
Minimum	−0.0087	−0.0091	−0.0201
Average	0.0007	0.0004	−0.0011
Standard deviation	0.0029	0.0028	0.0038

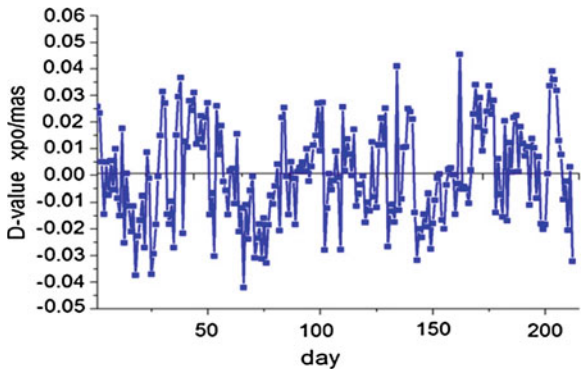


Fig. 2.5 Difference of xpo between the combined solutions and IGS

2.4.2 The Combination of ERPs

2.4.2.1 Combination of SINEX Solutions

The differences in ERPs between our solution and IGS solution are shown in Figs. 2.5, 2.6, 2.7, 2.8, 2.9 and 2.10, the statistical results are shown in Table 2.2.

From Figs. 2.5, 2.6, 2.7, 2.8, 2.9, 2.10 and Table 2.2, we can conclude that the maximum difference of pole motion vector between our combination results and IGS is less than 0.05 mas, the pole motion rate vector is less than 0.3 mas/d, the LOD is less than 0.015 ms and the UT is less than 0.07 ms. The maximum standard deviation of them is less than 0.02 mas, 0.06 mas/d, 0.002 and 0.013 ms separately. The accuracy of our results is comparative to those of literature [1].

2.4.2.2 The Weighted-Combination

The linear formula of Eqs. (2.22) and (2.23) are used to compute the standard deviation with the data of ERPs in 2008 of seven ACs, and then the weighted combination is performed. The results are shown in Figs. 2.11, 2.12 and Table 2.3.

From Figs. 2.11, 2.12 and Table 2.3, we can see that the standard deviation compared to the IGS solutions is about 0.05 mas while the corresponding RMS (Root-Mean-Square) of SINEX solutions is about 0.02 mas which obtained by only 30 weeks of data. So we can conclude that the accuracy of the combination

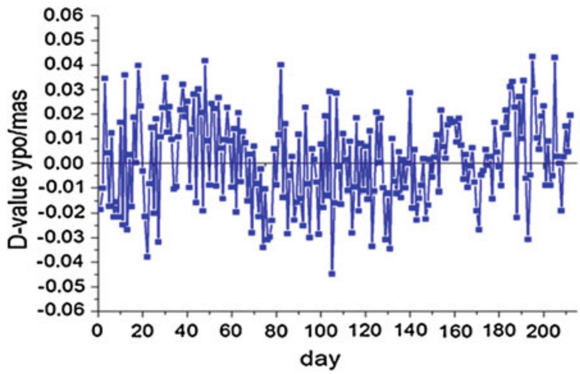


Fig. 2.6 Difference of ypo between the combined solutions and IGS

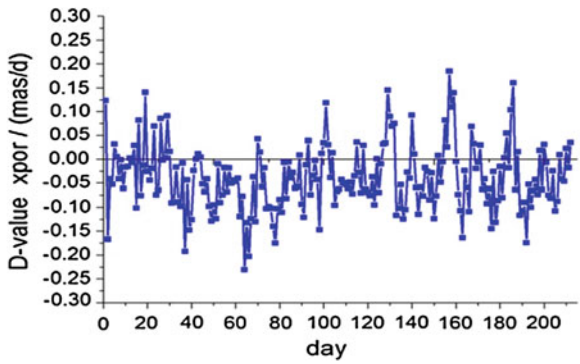


Fig. 2.7 Difference of xpor between the combined solutions and IGS

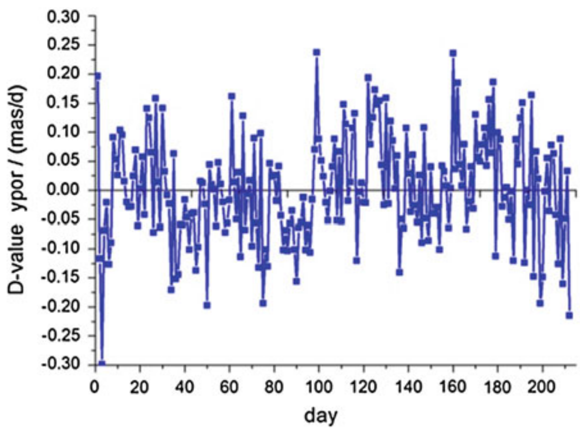


Fig. 2.8 Difference of ypor between the combined solutions and IGS

accuracy of SINEX is higher than that of the weighted-combination. However, the former needs to deal with a large amounts of computation relative to the normal equations, so the weighted-combination method is easier and more efficient which

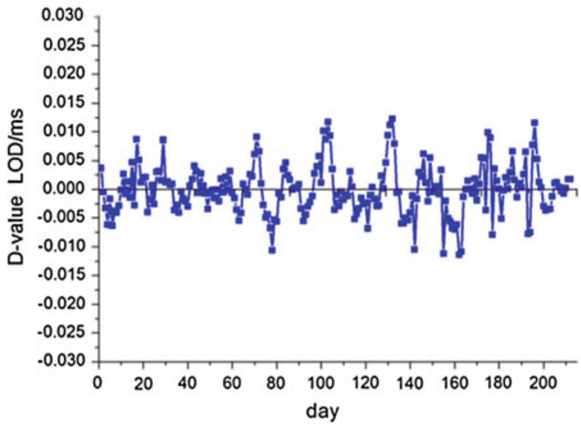


Fig. 2.9 Difference of LOD between the combined solutions and IGS

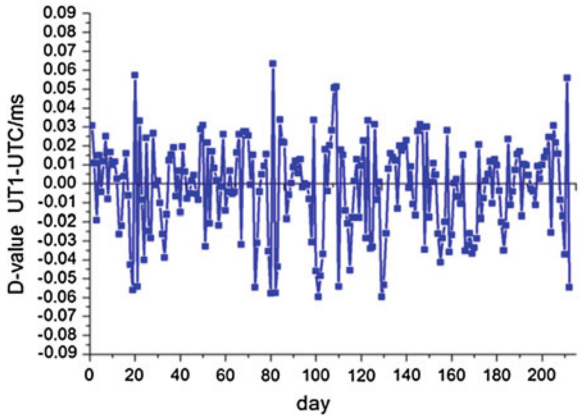


Fig. 2.10 Difference of UT between the combined solutions and IGS

Table 2.2 Statistics results of ERPs between the combined solutions and IGS

Difference	Xpo/mas	Ypo/mas	Xpor/mas/d	Ypor/mas/d	LOD/ms	UT/ms
Maximum	0.046	0.044	0.192	0.248	0.012	0.065
Minimum	−0.042	−0.047	−0.234	−0.300	−0.011	0.060
Average	−0.009	−0.004	−0.059	0.014	0.0013	0.005
Standard deviation	0.017	0.015	0.049	0.055	0.0021	0.013

can be used for fast combination of ERP. It should be noticed that the average difference of the weighted-combination is smaller than that of SINEX, which means that no obvious system error exists in the weighted-combination solutions.

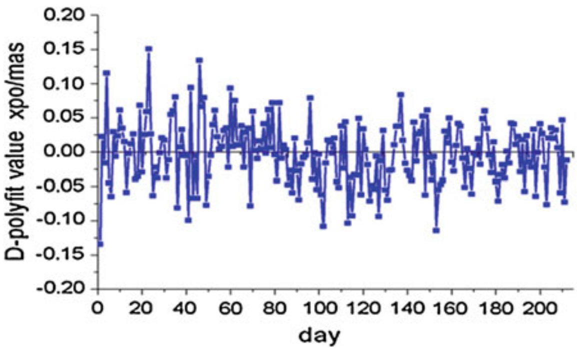


Fig. 2.11 Difference of xpo between the weighted-combination solutions and IGS

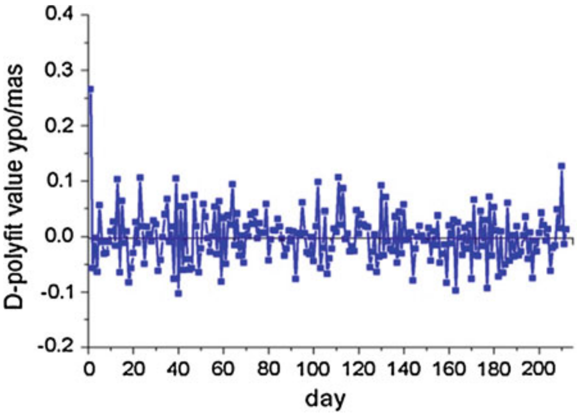


Fig. 2.12 Difference of ypo between the weighted-combination solutions and IGS

Table 2.3 Statistics results of pole parameters between the weighted-combination solutions and IGS

Difference	Xpo/mas	Ypo/mas
Maximum	0.151	0.266
Minimum	−0.134	−0.103
Average	−0.002	0.0007
Standard deviation	0.047	0.047

2.5 Conclusions

This paper mainly deals with the combination of station coordinates and ERPs based on weekly SINEX file provided by IGS. For the SINEX file of IGS, it's essential to deal with the constraints and properties of the normal equations well. The computation shows that the results from the combination of SINEX files have higher accuracy and are more reliable than those from the weighted-combination. The accuracy of former is consistent with IGS and higher efficiency for ERP combination solution can be obtained by by the latter. It can be concluded that a higher accuracy of ERP will be gotten if longer period of data is used.

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