

Chapter 2

Related Phenomena

Abstract Brief review of the phenomena closely related to the main subject,—channeling and radiation in PBCr, is given. These include: general features of radiation by relativistic charges, specific types of electromagnetic radiation in external fields (undulator radiation, incoherent and coherent BrS), channeling in straight and bent crystals, channeling radiation.

Prior to analyzing the feasibility of CU and various aspects of the electromagnetic radiation emitted a beam of ultra-relativistic charged particles channeling in a PBCr, below in this chapter we review the phenomena closely related to the main subject. These include: general features of radiation by relativistic charges, specific types of electromagnetic radiation in external fields (undulator radiation, incoherent and coherent BrS), channeling in straight and bent crystals, channeling radiation.

We do not pretend to cover the whole range of problems concerning the mentioned phenomena but present a brief description of the effects. More detailed information on each of the discussed topics one finds in the review chapters and books cited in each section.

2.1 Radiation from Relativistic Charges: Classical, Quantum and Quasiclassical Approaches

An important issue in the study of radiation formed in a CU concerns the choice of the formalism used to describe the phenomenon. This point could have been seen as merely a technical one but it is not so. Contrary to the case of conventional undulators, based on the action of magnetic fields, the physics of CUs is, essentially, a newly arisen research field. Therefore, any theoretical study of the effect, which pretends to go a bit farther than purely academic research, must contain a great part of numerical analysis and numerical data on the basis of which real experimental investigations can be planned. In turn, to obtain the reliable data it is necessary to

choose a theoretical tool which allows one, on the one hand, to treat adequately all principal physical phenomena involved into the problem, and, on the other hand, to effectively carry out the corresponding numerical analysis.

In the CU problem there are three basic phenomena which must be accurately described: (a) the motion of an ultra-relativistic particle in a strong external field (the electrostatic crystalline field), (b) the process of photon emission by the particle, (c) the problem of the radiative recoil, which results in the radiative energy losses of the projectile.

2.1.1 Classical Description

In many cases, the motion of an ultra-relativistic particle, moving in an external field, can be treated, within the framework classical mechanics (see, e.g., [136, 196]). General criterion of the applicability of the classical description is in the condition that the variation of the de Broglie wavelength $\lambda_B = h/p$ of the projectile must be negligible over the distances of the order of λ_B . This condition can be written in the form (see, e.g., [198]) $m\hbar U'_{\max}/p^3 \ll 1$, where m and $p \approx \varepsilon/c$ are projectile's mass and momentum, and $U'_{\max}$ stands for the maximum gradient of the external field (i.e., the maximum force). Taking into account that $U'_{\max} \sim 10^1\text{--}10^2 \text{ GeV/cm}$ for an planar crystalline potential and by approximately an order of magnitude higher for an axial potential (see, e.g., [37]), one demonstrate that the condition is well-fulfilled for projectile positrons and electrons with $\varepsilon \sim 10^2 \text{ MeV}$ and higher (this energy range is of prime interest in the CU problem, as it will be demonstrated below in the book).¹

The process of photon emission can be treated classically provided the photon energy is small compared to that of a projectile: $\hbar\omega/\varepsilon \ll 1$.

Hence, if both of the mentioned conditions are met, one can calculate the spectral-angular distribution the radiated energy E using the following formula of classical electrodynamics (see, e.g., [136]):

$$\frac{d^3E}{d\omega d\Omega} = \frac{e^2}{c} \frac{q^2 \omega^2}{8\pi^2} \int_0^\tau dt_1 \int_0^\tau dt_2 e^{i\omega(\varphi(t_1) - \varphi(t_2))} \left(\frac{\mathbf{v}_1 \cdot \mathbf{v}_2}{c^2} - 1 \right). \quad (2.1)$$

Here $d\Omega$ is the solid angle in the direction $\mathbf{n}$ of the emission, q is the projectile charge in units of the elementary charge e , τ is the time of flight through a spatial domain within which the external field acts on the projectile. The quantities $\mathbf{v}_{1,2} \equiv \mathbf{v}(t_{1,2})$ stand for the projectile velocities at the instants t_1 and t_2 . It is assumed that for an ultra-relativistic particle $v_{1,2} \approx c$. The function $\varphi(t)$ is defined as follows

¹ More accurately, the condition of the applicability of the classical description of the channeling motion is formulated in Sect. 2.3.4.

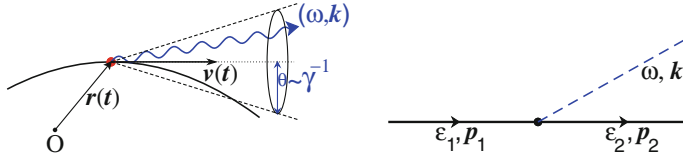


Fig. 2.1 Classical (*left*) and quantum (*right*) approaches to the radiation process. Classical ultra-relativistic charged projectile (red dot on *left panel*), being accelerated (decelerated) by external field, moves along a well-defined trajectory $\mathbf{r} = \mathbf{r}(t)$. The electromagnetic radiation of frequency ω and wave-vector $\mathbf{k}$ is essentially emitted within the cone $\theta \sim \gamma^{-1}$ along the vector of the instant velocity. Within the quantum picture (the *right panel* represents the Feynman diagram) the radiative transition from the initial state of the projectile (initial energy and the asymptotic momentum are ε_1 and $\mathbf{p}_1$) to the final state with ε_2 , $\mathbf{p}_2$ is accompanied by the photon emission (*dashed line*). The circle denotes the vertex of the particle–photon interaction

$$\varphi(t) = t - \frac{\mathbf{n} \cdot \mathbf{r}(t)}{c}. \quad (2.2)$$

The dependence of the position vector on time, $\mathbf{r} = \mathbf{r}(t)$, is found from the classical equations of motions.

The classical description of the radiative process is illustrated by Fig. 2.1 (left), where the solid curve represents the trajectory of the charged particle. The radiation formed in a segment of the trajectory is emitted predominantly within the cone $\theta \sim 1/\gamma$ ($\gamma = \varepsilon/mc^2$ is the Lorentz relativistic factor) along the vector of the instant velocity $\mathbf{v}(t)$.

The classical approach based on (2.1) is commonly used to describe various types of electromagnetic radiation: BrS, synchrotron radiation, undulator radiation and channeling radiation. For more specific and retails information see [8, 14, 36, 37, 46, 53, 54, 136, 142, 193, 196, 269].

The main drawback of the classical framework is that it does not allow a self-consistent description of the decrease of the projectile energy due to the radiation emission. Hence, this scheme implies that the particle moves along the trajectory having the constant value of the total energy, $\varepsilon = \text{const}$.

2.1.2 Quantum Description

The most rigorous approach to the radiation process is based on the formalism of quantum electrodynamics (see, e.g., [61]), where the amplitude of the process is described in terms of a single free-free matrix element of the photon emission taken between the initial and final states of an ultra-relativistic particle in the interplanar field. The Feynman diagram of the process is presented in Fig. 2.1 (right), where the solid line denotes the projectile in the initial (the subscript 1) and the final (the subscript 2) states, the dashed line stands for the emitted photon and the dots marks the vertex of the particle—photon interaction. The energy conservation law implies

$\varepsilon_1 - \varepsilon_2 = \hbar\omega$. The corresponding analytical expression for the amplitude M_{21} is given by

$$M_{21} = qe \int d\mathbf{r} \Psi_{\varepsilon_2 \mathbf{p}_2 \nu_2}^\dagger(\mathbf{r}) (\mathbf{e} \cdot \boldsymbol{\alpha}) \exp(-i\mathbf{k} \cdot \mathbf{r}) \Psi_{\varepsilon_1 \mathbf{p}_1 \nu_1}(\mathbf{r}). \quad (2.3)$$

Here the bi-spinor wavefunction $\Psi_{\varepsilon \mathbf{p} \nu}(\mathbf{r})$ is the solution of the Dirac equation with the external potential U (a so-called Furry approximation, see, e.g., [61]) corresponding to the total energy ε , the asymptotic momentum $\mathbf{p}$. Other quantum numbers which characterize the particle, including its polarization, are incorporated in the subscript ν . The symbol $\dagger$ denotes the hermitian conjugation, $\boldsymbol{\alpha} = \gamma^0 \boldsymbol{\gamma}$ where γ^0 and $\boldsymbol{\gamma}$ are the Dirac matrices. The vectors $\mathbf{k}$ and $\mathbf{e}$ denote the photon wave vector and polarization.

The power of radiation P (the energy per unit time) emitted within the frequency interval $d\omega$ and within the cone $d\Omega$ can be expressed in terms of the differential cross section $d^3\sigma/d\omega d\Omega$ of the process:

$$\frac{d^3 P}{\hbar d\omega d\Omega} = j \omega \frac{d^3 \sigma}{d\omega d\Omega} = \frac{\omega^3 p_2 \varepsilon_2}{(2\pi)^5 c^5} \sum_{\text{pol}} \int_{(4\pi)} d\Omega_{\mathbf{p}_2} |M_{21}|^2. \quad (2.4)$$

Here j stands for the flux of the incoming particles, the summation is carried out over the particle polarizations in the initial and final states as well as over the photon polarizations, the integration is carried out over the scattering angles.

Equations (2.3) and (2.4) are applicable in the whole range of the emitted photon energies, starting from the soft photons, $\hbar\omega \ll \varepsilon_1$ so that $\varepsilon_1 \approx \varepsilon_2$, up to the tip end of the spectrum, when nearly all the initial (kinetic) energy $\varepsilon_1 - mc^2$ is radiated.

In the ultra-relativistic domain the quantum-electrodynamic approach has been used for theoretical and numerical studies of various radiative processes. These include BrS in electron–atom (or/and ion), electron–electron etc collisions (see, e.g., [6, 61, 110, 136] and references therein), coherent BrS (e.g., [7]), synchrotron radiation [136, 147], and channeling radiation (e.g., [14, 194, 237]).

In application to the channeling motion and ChR, as well as to the CUR, the main (technical) limitation of the quantum approach is due to the fact, that in the ultra-relativistic limit, when $\gamma \gg 1$, the number of the energy levels of the transverse motion in the effective interplanar (or, axial) potential increases significantly. Consequently, an accurate numerical calculations of the particle dynamics becomes a formidable task [146]. It is exactly this sort of difficulties which resulted in the absence of any numerical analysis and the data for the emission spectra in [50, 87, 133], where CUR was treated in terms of quantum electrodynamics.

2.1.3 Quasi-Classical Description of Radiation Emission

An adequate approach to the radiation emission by ultra-relativistic projectiles was developed by Baier and Katkov in the late 1960th [34] and was called by the authors

the “operator quasi-classical method”. The details of the formalism, as well as its application to a variety of radiative processes, can be found in [36, 37, 61] and will not be reproduced here.

A remarkable feature of this method is that it allows one to combine the classical description of the particle motion in an external field and the quantum effect of radiative recoil.

The classical description of the motion is valid provided the characteristic energy of the projectile in an external field, $\hbar\tilde{\omega}_0$, is much less than its total energy, $\varepsilon = m\gamma c^2$. The relation $\hbar\tilde{\omega}_0/\varepsilon \propto \gamma^{-1} \ll 1$ is fully applicable in the case of an ultra-relativistic projectile.

The role of radiative recoil, i.e., the change of the projectile energy due to the photon emission, is governed by the ratio $\hbar\omega/\varepsilon$. In the limit $\hbar\omega/\varepsilon \ll 1$ a purely classical description (2.1) of the radiative process can be used. For $\hbar\omega/\varepsilon \leq 1$ quantum corrections must be accounted for.

The quasi-classical approach neglects the terms $\sim \hbar\tilde{\omega}_0/\varepsilon$ but explicitly takes into account the quantum corrections due to the radiative recoil. The method is applicable in the whole range of the emitted photon energies, except for the extreme high-energy tail of the spectrum $(1 - \hbar\omega)/\varepsilon \ll 1$.

Within the framework of quasi-classical approach one derives the following expression for the distribution of the energy radiated in given direction $\mathbf{n}$ by an ultra-relativistic particle (see [37, 61]):

$$\frac{d^3 E}{\hbar d\omega d\Omega} = \alpha \frac{q^2 \omega^2}{4\pi^2} \int_0^\tau dt_1 \int_0^\tau dt_2 e^{i\omega'(\varphi(t_1) - \varphi(t_2))} f(t_1, t_2). \quad (2.5)$$

All notations, except for ω' and $f(t_1, t_2)$, are the same as in the classical formula (2.1). The function $f(t_1, t_2)$ is defined as follows

$$f(t_1, t_2) = \frac{1}{2} \left[\left(1 + (1 + u)^2 \right) \left(\frac{\mathbf{v}_1 \cdot \mathbf{v}_2}{c^2} - 1 \right) + \frac{u^2}{\gamma^2} \right] \quad (2.6)$$

The key point of the quasi-classical method,—the radiative recoil, i.e. the account for the terms $\hbar\omega/\varepsilon$, is contained in the parameters ω' and u , which are defined as follows:

$$u = \frac{\hbar\omega}{\varepsilon - \hbar\omega}, \quad \omega' = (1 + u)\omega. \quad (2.7)$$

In the classical limit $u \approx \hbar\omega/\varepsilon \rightarrow 0$ and $\omega' \rightarrow \omega$, so that (2.5) and (2.6) reproduce (2.1).

Application of the general quasi-classical formula (2.5) to a variety of radiative processes in ultra-relativistic collisions in linear crystals is discussed in the monographs [36, 37]. It was also applied to the problem of synchrotron-type radiation emitted by an ultra-relativistic projectile channeling in a non-periodically bent crystal [21, 22, 255].

2.2 UR from an Ideal Planar Undulator

For the sake of completeness and for further referencing, in this section we present a collection of formulae describing the characteristics of radiation (spectral-angular and spectral distributions, position and width of the peaks of emitted harmonics, etc) by an ultra-relativistic charged particle moving in vacuum with a constant velocity v in the (y, z) plane along the trajectory

$$y(z) = a \sin k_u z, \quad \text{where } k_u = \frac{2\pi z}{\lambda_u}, \quad (2.8)$$

consisting of N segments (periods) each of the length λ_u , which is called an undulator period. We will term a device in which an ultrarelativistic projectile *moves in vacuum along the sinusoidal line* as an “*ideal undulator*”.

Such a motion can be realized in a planar magnetic undulator, in which bending of the particle trajectory is achieved by applying a periodic magnetic field directed perpendicular to the (y, z) plane: $\mathbf{B} = (B_x, 0, 0)$ with $B_x = B_0 \sin(k_u z)$ (see, e.g., [8, 46, 109, 236]).

2.2.1 General Formalism

In an undulator the particle moves quasi-periodically, i.e. during the time interval T it completes a full oscillation along the y direction and simultaneously advances by the interval λ_u along the z direction, which is called the undulator axis, see Fig. 2.2. Hence, the position vector and the velocity of the particle satisfy the conditions

$$\mathbf{r}(t + T) = \mathbf{r}(t) + \langle \mathbf{v}_0 \rangle T, \quad \mathbf{v}(t + T) = \mathbf{v}(t), \quad (2.9)$$

where $\langle \mathbf{v}_0 \rangle = T^{-1} \int_0^T \mathbf{v}(t) dt$ is the mean velocity which is directed along the undulator axis and $\langle v_0 \rangle \approx c$.

Assuming that the Lorentz relativistic factor satisfies a strong inequality $\gamma \gg 1$, one expands the functions $\varphi(t)$ and $f(t_1, t_2)$ in powers of γ^{-1} . Then, retaining the dominant non-vanishing terms, one represents the right-hand side of (2.5) as follows:

$$\frac{d^3 E}{\hbar d\omega d\Omega} = \alpha q^2 \frac{\omega^2 (1+u)(1+w)}{4\pi^2} \left[\frac{w |I_0|^2}{\gamma^2 (1+w)} + |\theta I_0 - \cos \phi I_1|^2 + \sin^2 \phi |I_1|^2 \right] \quad (2.10)$$

Here $w = u^2/2(1+u)$ and (θ, ϕ) are the emission angles with respect to the undulator axis. The notations I_0 and I_1 stand for the integrals

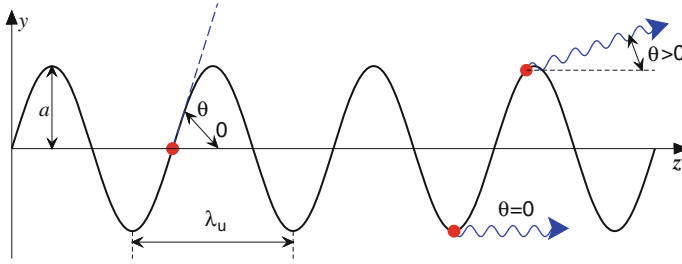


Fig. 2.2 Schematic representation of the ideal planar undulator. An ultra-relativistic charged projectile (filled circle) moves along sinusoidal trajectory (2.8) (thick solid curve). The radiation (wavy lines) is emitted due to the charge acceleration. The maximum turning angle (with respect to the undulator axis z) of the undulating particle is $\theta_0 \sim a/\lambda_u \sim \sqrt{\overline{v_y^2}}/c$, where $\overline{v_y^2}$ is the mean square of the transverse velocity. If $\theta_0 \lesssim \gamma^{-1}$ then all radiation is emitted within the cone $\sim \gamma^{-1}$. This limit corresponds to small values of the undulator parameter, $K < 1$ (see 2.13). In the opposite case, $\theta_0 \gg \gamma^{-1}$ (and, correspondingly, $K^2 \gg 1$), the emission occurs in the cone $\sim \theta_0$. Due to the interference of the waves emitted from spatially different but similar parts of the trajectory, for each θ the intensity of UR is proportional to the square of undulator periods

$$I_0 = \int_{-\infty}^{\infty} dt e^{i\omega' \Phi(t)}, \quad I_1 = \int_{-\infty}^{\infty} dt \frac{v_y(t)}{c} e^{i\omega' \Phi(t)} \quad (2.11)$$

where

$$\Phi(t) = \frac{t}{2\gamma^2} \left(1 + (\gamma\theta)^2 + \frac{K^2}{2} \right) + \frac{1}{2} \int^t dt' \left(\frac{v_y^2(t')}{c^2} - \frac{K^2}{2\gamma^2} \right) - \theta \cos \varphi \frac{y(t)}{c}. \quad (2.12)$$

The quantity K ,—a so-called undulator parameter, is related to the mean-square transverse velocity $\overline{v_\perp^2}$ (i.e., perpendicular to the undulator axis). For a planar undulator this quantity is defined as follows²:

$$K^2 = 2\gamma^2 \frac{\overline{v_y^2}}{c^2} = \frac{2\gamma^2}{c^2} \int_0^T \frac{dt}{T} v_y^2(t) = \left(2\pi\gamma \frac{a}{\lambda_u} \right)^2. \quad (2.13)$$

In other words, the undulator parameter can be defined as the ratio of the maximum turning angle of the undulating ultra-relativistic particle, $\theta_0 \sim \sqrt{\overline{v_y^2}}/c$, to the characteristic cone $\sim \gamma^{-1}$ of the radiation emission from each part of the projectile trajectory: $K \sim \gamma\theta_0$, see Fig. 2.2. The features of the spectral-angular distribution of

² The right-hand side of this equation is written for the sinusoidal trajectory (2.8).

radiation are somewhat different in the two limiting cases: (a) $K < 1$ (or $\theta_0 < \gamma^{-1}$),—a so-called undulator mode, and (b) $K \gg 1$ ($\theta_0 \gg \gamma^{-1}$)—a wiggler regime. The differences will be discussed further in this section.

Formulae (2.10)–(2.13) allow one to analyze, both analytically and numerically, the radiation emitted by an ultra-relativistic projectile moving along arbitrary periodic planar trajectory, $y = S(z)$. In particular, they can be applied to the motion along the sinusoidal line described by (2.8).

The spectral-angular distribution of the energy emitted by an ultra-relativistic particle in an ideal planar undulator can be written in the following form:

$$\frac{d^3 E}{\hbar d\omega d\Omega} = S(\omega, \theta, \phi) D_N(\tilde{\eta}) \quad (2.14)$$

The function $S(\omega, \theta, \phi)$, which does not depend on the undulator length (or, which is equivalent, on the number of periods N), is given by

$$\begin{aligned} S(\omega, \theta, \phi) = & \alpha q^2 \frac{\omega^2}{\gamma^2 \omega_0^2} \frac{(1+u)(1+w)}{4\pi^2} \\ & \times \left(\frac{w}{1+w} |F_0|^2 + |\gamma\theta F_0 - K \cos \phi F_1|^2 + K^2 \sin^2 \phi |F_1|^2 \right). \end{aligned} \quad (2.15)$$

The functions $F_m \equiv F_m(\theta, \phi)$ ($m = 0, 1$) stand for the integrals

$$F_m = \int_0^{2\pi} d\psi \cos^m \psi \exp \left(i \left[\eta \psi + \frac{K^2 \omega'}{8\gamma^2 \omega_0} \sin(2\psi) - \frac{K \omega'}{\gamma \omega_0} \theta \cos \varphi \sin \psi \right] \right). \quad (2.16)$$

The parameter η is given by

$$\eta = \frac{\omega'}{2\gamma^2 \omega_0} \left(1 + \gamma^2 \theta^2 + \frac{K^2}{2} \right). \quad (2.17)$$

The factor $D_N(\tilde{\eta})$ on the right-hand side of (2.14) is defined as follows

$$D_N(\tilde{\eta}) = \left(\frac{\sin \pi \tilde{\eta} N}{\sin \pi \tilde{\eta}} \right)^2, \quad (2.18)$$

where $\tilde{\eta} = \eta - n$ and n is a positive integer such that $n - 1/2 < \eta \leq n + 1/2$.

For $N \gg 1$ the function $D_N(\tilde{\eta})$ has a sharp and powerful maximum in the point $\tilde{\eta} = 0$ (which corresponds to the case $\eta = n = 1, 2, \dots$), where $D_N(0) = N^2$. This behaviour of $D_N(\tilde{\eta})$ results in a peculiar form of the spectral-angular distribution

of UR which clearly distinguishes it from other types of electromagnetic radiation formed by a charge moving in external fields [8, 37, 46, 109, 269]. Namely, for each value of the emission angle the spectral distribution consists of a set of narrow and equally spaced peaks (harmonics). In the soft-photon limit, when the emitted harmonic energy $\hbar\omega_n$ is small compared to the projectile energy ε , the frequencies ω_n are found from the relation

$$\omega_n = \frac{2\gamma^2 \omega_0 n}{1 + \gamma^2 \theta^2 + K^2/2}, \quad n = 1, 2, 3, \dots, \quad (2.19)$$

which coincides with the definition of the harmonic frequencies of UR within the framework of classical electrodynamics [8, 46, 269]. If the terms $\sim \hbar\omega/\varepsilon$ are not neglected, the right-hand side of (2.19) defines the values of ω'_n (see 2.7) [37].

The magnitude of the undulator parameter K determines the number $n_{\max} \sim K^3/2$ of the emitted harmonics [8, 37].

For $K \lesssim 1$ the emission occurs mainly in the first harmonic, which is emitted within the cone $\theta \lesssim \theta_0 \sim \gamma^{-1}$, and the frequency of which, $\omega_1 \approx 2\gamma^2 \omega_0$, does not vary noticeably for $\theta \ll \theta_0$. The peak intensity is proportional to N^2 . This factor reflects the constructive interference of radiation emitted from each of the undulator periods and is typical for any system which contains N coherent emitters.

In the opposite limit, $K^2 \gg 1$, the number of emitted harmonics is large and they are emitted within a wider cone: $\theta \lesssim \theta_0 \sim K/\gamma$. The frequency ω_n of each harmonic is the largest for the emission in the forward direction (i.e., at $\theta = 0$). The emission cone $\Delta\Omega_n$ and the natural bandwidth $\Delta\omega_n/\omega_n$ one derives from (2.19):

$$\Delta\Omega_n = 2\pi \frac{1 + K^2/2}{2nN\gamma^2}, \quad \frac{\Delta\omega_n}{\omega_n} = \frac{1}{nN}. \quad (2.20)$$

Two 3D plots in Fig. 2.3 help one to visualize the spatial behaviour of the angular distribution of UR (at fixed frequency) [156]. The data refer to $K = 3$, $N = 17$ and $\gamma = 10^5$. In these figures the quantity $\log(dE^3/\hbar d\omega d\Omega)$ (measured along the Z -axis) as a function of the azimuthal, ϕ , and the polar, θ , angles of the photon emission is plotted for two harmonics: $n = 7$ (left figure) and $n = 6$ (right figure).

The undulator axis lies along the z direction, and yz is the undulator plane. The positive y direction corresponds to $\phi = 0^\circ$. In these figures the dimensionless variable θ/θ_0 (with $\theta_0 = K/\gamma = 30 \mu\text{rad}$) is used to characterize the distribution in of the radiation with respect to the polar angle.

The graphs illustrate general features intrinsic to the planar UR in the case $K^2 \gg 1$. We first note, that the intensity in the odd harmonic is governed by a powerful maximum in the forward direction, whereas there is nearly no radiation in even harmonics for $\theta = 0^\circ$ (see 2.23 and 2.24 below). The latter reaches its maximal values in the off-axis direction. Apart from the main peak either in the forward (for odd n) or nearly forward (for even n) direction, the radiation in a particular harmonic is emitted in a wide range of the polar angles. In both of the figures there

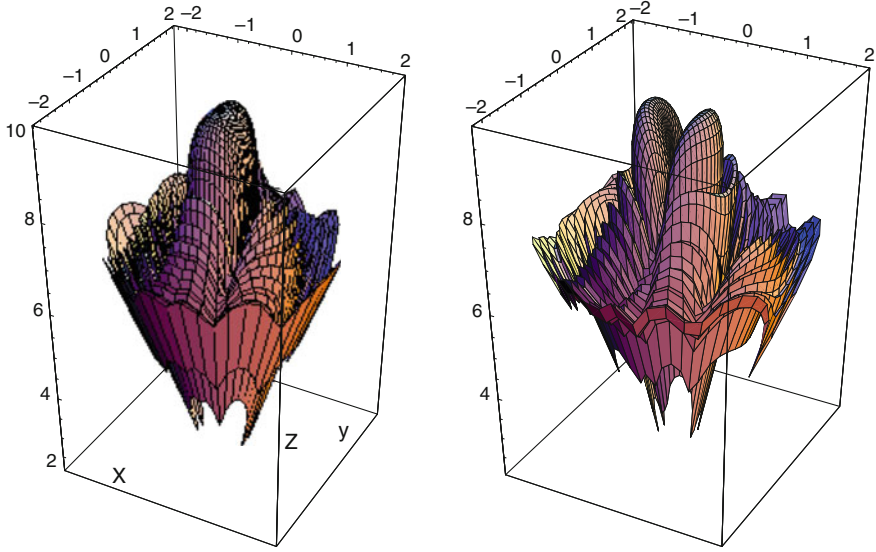


Fig. 2.3 Angular distribution of UR emitted in the odd $n = 7$ (left panel) and even $n = 6$ (right panel) harmonics [156]. The undulator axis lies along the Z -direction. The undulator plane is (yz) . The X and Y axes are scaled with respect to the dimensionless variable θ/θ_0 . Further explanations are given in the text

are several clearly distinct the off-axis peaks in which the radiation intensity reaches the maxima, although the magnitudes of $dE^3/hd\omega d\Omega$ in these secondary maxima at rapidly decrease with the polar angle (recall the log scale along the Z -axis).

Another feature to be mentioned is the absence of the axial symmetry in the shape of angular distribution. More specifically, the radiation emitted within the undulator plane is concentrated in the cone $\theta_{\parallel} \sim \theta_0$, whereas the emission cone in the xz plane is $\theta_{\perp} \sim 1/\gamma$. Thus, the ratio $\theta_{\parallel}/\theta_{\perp} \sim K$ characterizes the asymmetry in the angular distribution with respect to the azimuthal angle ϕ . This peculiarity is more pronounced for odd harmonics.

2.2.2 Spectral Distribution in the Forward Direction

For the sake of reference let us outline the basic formulae which describe the spectrum of radiation emitted in the forward direction (i.e., $\theta = 0$ with respect to the undulator axis). In this case the integrals F_m from (2.16) can be expressed in terms Anger's function $\mathbf{J}_{\nu}(\zeta) = \pi^{-1} \int_0^{\pi} \cos(\nu\phi - \zeta \sin \phi) d\phi$ (see, e.g., [111]):

$$\begin{cases} F_0 = 2\pi e^{i\eta\pi} \cos \frac{\eta\pi}{2} \mathbf{J}_{\eta/2}(\zeta) \\ F_1 = \pi e^{i\eta\pi} \sin \frac{\eta\pi}{2} \left(\mathbf{J}_{\frac{\eta+1}{2}}(\zeta) - \mathbf{J}_{\frac{\eta-1}{2}}(\zeta) \right) \end{cases} \quad \text{where } \zeta = \frac{K^2 \omega'}{8\gamma^2 \omega_0}. \quad (2.21)$$

Using these relations in (2.15), one derives the following explicit formulae for the on-the-axis spectral distribution [166]:

$$\left\{ \begin{array}{l} \frac{d^3 E}{\hbar d\omega d\Omega} \Big|_{\theta=0} = D_N(\tilde{\eta}) S(\omega, 0, \phi) \\ S(\omega, 0, \phi) = \alpha q^2 \frac{\omega^2 (1+u)}{\gamma^2 \omega_0^2} \left(w \cos^2 \frac{\eta\pi}{2} \mathbf{J}_{\frac{\eta}{2}}^2(\zeta) \right. \\ \quad \left. + \frac{K^2 (1+w)}{4} \sin^2 \frac{\eta\pi}{2} \left[\mathbf{J}_{\frac{\eta+1}{2}}(\zeta) - \mathbf{J}_{\frac{\eta-1}{2}}(\zeta) \right]^2 \right) \end{array} \right. \quad (2.22)$$

We remind that $w = u^2/2(1+u)$ with $u = \hbar\omega/(\varepsilon - \hbar\omega)$.

To obtain the on-axis intensity at the frequencies $\omega \approx \omega_n$, one notices that Anger's function of an integer index $\nu = k$ reduces to the Bessel function $J_k(\zeta)$ [111]. Then, taking into account the “selection rules” imposed by the factors $\sin^2 \eta\pi/2$ and $\cos^2 \eta\pi/2$, one derives for integer $\eta = n$:

$$\begin{aligned} \frac{d^3 E}{\hbar d\omega d\Omega} \Big|_{\substack{\theta=0 \\ \omega \approx \omega_n}} &= D_N(\tilde{\eta}) \frac{16\alpha q^2 \gamma^2 n^2}{(1+u)(2+K^2)^2} \\ &\times \left\{ \begin{array}{ll} \frac{K^2(1+w)}{4} \left[J_{\frac{n+1}{2}}(\zeta_n) - J_{\frac{n-1}{2}}(\zeta_n) \right]^2 & n = 1, 3, 5, \dots \\ w J_{\frac{n}{2}}^2(\zeta_n) & n = 2, 4, 6, \dots \end{array} \right. \end{aligned} \quad (2.23)$$

where $\zeta_n = nK^2/(4+2K^2)$. The argument $\tilde{\eta}$ in the factor $D_N(\tilde{\eta})$ is kept to enable one to reproduce the profile of the in the vicinity of the resonance (i.e., for $\omega \approx \omega_n$ where $|\tilde{\eta}| \ll 1$).

At the resonance $D_N(0) = N^2$, so that the peak intensity is proportional to the squared number of the periods. This factor reflects the constructive interference of radiation emitted from each of the undulator periods, and is typical for any system which contains N coherent emitters.

In the soft-photon limit $\hbar\omega/\varepsilon \rightarrow 0$, (2.22) and (2.23) reproduce the formulae obtained means of classical electrodynamics. In particular, setting $u = w = 0$ on the right-hand side of (2.23), one arrives at the following well-known formula for the peak intensity of UR at $\omega = \omega_n$: [8, 142]:

$$\frac{d^3 E^{cl}}{\hbar d\omega d\Omega} \Big|_{\substack{\theta=0 \\ \omega \approx \omega_n}} = D_N(\tilde{\eta}) \frac{4\alpha q^2 \gamma^2 n^2 K^2}{(2+K^2)^2} \times \left\{ \begin{array}{ll} \left[J_{\frac{n+1}{2}}(\zeta_n) - J_{\frac{n-1}{2}}(\zeta_n) \right]^2 & n = 1, 3, \dots \\ 0 & n = 2, 4, \dots \end{array} \right. \quad (2.24)$$

The distinguishing feature of the classical description of UR is the absence of the on-axis emission into even harmonics. Comparing the right-hand side of (2.24) with that of (2.23) one notices that this restriction is lifted when the radiative recoil

is taken into account. However, it can be shown, that if $\hbar\omega \ll \varepsilon$ (this case is of a prime interest for the emission from the undulator) then the intensities of the on-axis emission into even harmonics is much smaller than into odd ones.

Further simplification of the right-hand sides of (2.23) and (2.24) can be achieved in the limit of small undulator parameters, $K^2 \ll 1$. In this limit the argument of the Bessel functions is also small, $\zeta_n = nK^2/(4 + 2K^2) \approx nK^2/4 \ll 1$, therefore, one can write (see, e.g., [1]) $J_\nu(\zeta_n) \approx (\zeta_n/2)^\nu / \Gamma(\nu + 1)$, where $\Gamma(\nu + 1)$ is the Gamma-function. Hence, for the emission in odd harmonics ($\nu = (n \pm 1)/2$) one notes, that the peaks with $n > 1$ are strongly suppressed compared with the fundamental peak $n = 1$. For even harmonics (see the quasiclassical formula 2.23) only the term $n = 2$ can be kept. Therefore, for $K^2 \ll 1$ one derives

$$\left. \frac{d^3 E}{\hbar d\omega d\Omega} \right|_{\substack{\theta=0 \\ \omega \approx \omega_n}} \approx D_N(\tilde{\eta}) \frac{\alpha q^2 \gamma^2 K^2}{(1+u)} \times \begin{cases} 1+w & \omega = \omega_1 \\ 4w & \omega = \omega_2 = 2\omega_1 \end{cases} \quad (2.25)$$

In the classical limit $u = w = 0$ the emission in $\omega = \omega_2$ is nullified, so the factor in front of the curly bracket describes the peak intensity of the fundamental harmonic.

Two graphs in Fig. 2.4 illustrate the mentioned peculiarities of the UR emitted in the forward direction. The spectra are calculated for the two indicated values of the undulator parameter K , and for ε , λ_u and N indicated in the caption. The pronounced peaks correspond to the emission in the odd harmonics: $n = 1, 3, 5$ are visible in the upper graph (although the intensities of the higher harmonics are much smaller than for the fundamental one, $n = 1$), whereas for larger value of K ,—the lower graph, the total number of the emitted (odd) harmonic is ≈ 15 ; only the $n = 1, 3, 5, 7$ harmonics are plotted in the figure. The intensity of the emission into even harmonics is negligible due to the small value of the coefficient w in the first term in the brackets on the right-hand side of the second formula in (2.22). Indeed, for the indicated values of ε and $\hbar\omega_1$ (see the caption) this coefficient, which is determines the even-harmonic peaks (see 2.23), written as $w \approx u^2/2 \approx 0.5 (n\hbar\omega_1/\varepsilon)^2$ does not exceed 10^{-7} for $n = 2$ in the upper graph and for $n = 10$ in the lower graph. Hence, the presented spectra are, in fact, classical since they refer to the limit $\hbar\omega \ll \varepsilon$.

2.2.3 Spectral Distribution Integrated Over the Emission Angles

Integrating (2.14) over the emission angles $\theta \ll 1$ and ϕ , one calculates the spectral distribution of UR

$$\frac{dE}{\hbar d\omega} = \int \frac{d^3 E}{\hbar d\omega d\Omega} d\Omega = \int_0^{\theta_{\max}} \theta d\theta D_N(\tilde{\eta}) \int_0^{2\pi} S(\omega, \theta, \phi) d\phi. \quad (2.26)$$

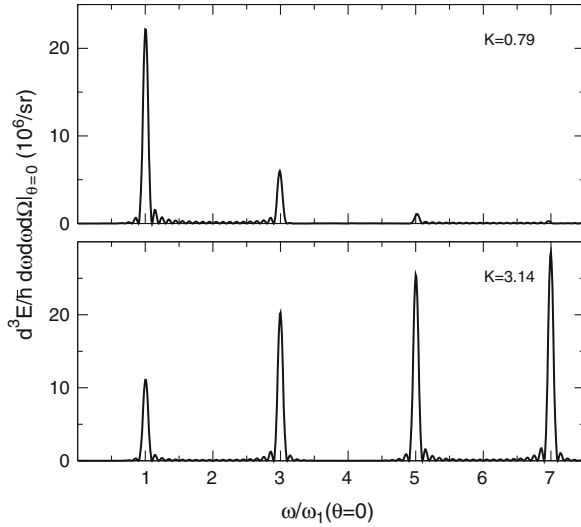


Fig. 2.4 Spectral distribution (2.22) of UR emitted in the forward direction calculated for two values of the undulator parameter K as indicated. Both graphs correspond to the incident energy $\varepsilon = 5$ GeV, the undulator period $\lambda_u = 150 \mu\text{m}$ (it is a CU!) and number of periods $N = 10$. The horizontal axis (identical in both graphs) shows the photon frequency scaled by the frequency of the first harmonic ω_1 in the forward direction (see 2.19 with $\theta = 0$). The value of $\hbar\omega_1$ is 1.2 MeV for the *upper graph*, and 0.265 MeV for the *lower graph*. In both graphs the pronounced peaks correspond to the emission in the odd harmonics, $n = 1, 3, 5, \dots$. See also explanation in the text

As it was mentioned above, the radiation is emitted, effectively, within the cone $\theta_0 \sim \max[K\gamma^{-1}, \gamma^{-1}]$. Therefore, choosing the upper limit of integration $\theta_{\max} \gg \theta_0$, one calculates the total emitted radiation at given frequency. Varying $\theta_{\max}$ within the interval from 0 up to $\sim \theta_0$, one can calculate the spectral distribution for different apertures.

For $\theta \neq 0$ and for an arbitrary value of K , there is no closed analytical expression for the integrals $F_m = F_m(\theta, \phi)$ (2.16) which enter (2.14). The result of analytical evaluation of F_m can be expressed either in series in so-called Anger's and Weber's functions (see [180, 268, 269] for the details of evaluation and [111] for definitions and properties of the special functions) or in series of products of the Bessel functions [46]. In either case, evaluation of the integral over the emission angles involves computational methods. For $K^2 \ll 1$ the integrals F_m are simplified, so that a closed analytical expression can be derived also for the $dE/\hbar d\omega$. The corresponding formula one finds, for example, in [8, 37, 54].

Alternatively, with some care taken on the numerical evaluation of the integrals with rapidly varying functions (for the algorithms, see, for example, [232]), the formulae (2.15) and (2.16), can be easily programmed allowing one to compute the spectrum (2.26) for an arbitrary aperture $\theta_{\max}$.

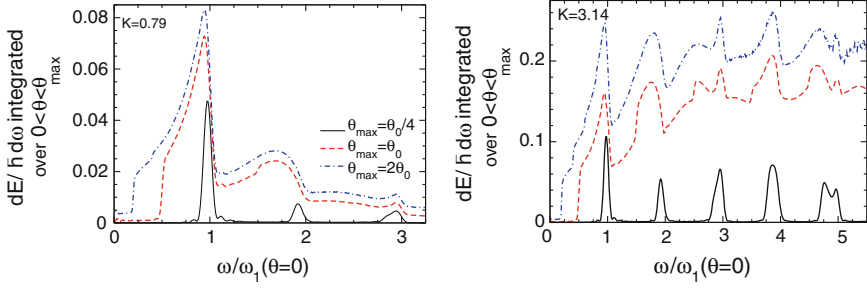


Fig. 2.5 Spectral distribution (2.26) of UR integrated over the emission angles within the cone $0 \leq \theta \leq \theta_{\max}$ for three different values of $\theta_{\max}$, as indicated by the legend in the *left panel*. Two panels refer to the indicated values of the undulator parameter K . Other parameters are as in Fig. 2.4. See also explanation in the text

The dependence of $dE/hd\omega$ on ω/ω_1 , calculated according to (2.26) for three different values of $\theta_{\max}$ (as indicated by the common legend), is presented in Fig. 2.5. The data refer to the same undulators as in Fig. 2.4. For each quoted K value, the cone angle was defined as $\theta_0 = \gamma^{-1} (1 + K^2/2)^{1/2}$, so that the harmonic frequencies as functions of the emission angle can be written in the form (cf. 2.19):

$$\omega_n(\theta) = \frac{\omega_n(\theta = 0)}{1 + \theta^2/\theta_0^2}. \quad (2.27)$$

where $\omega_n(\theta = 0) = 2\omega_0 n/\theta_0^2$ is the harmonic frequency for the on-the-axis emission.

Equation (2.27) explicitly indicates that as for higher emission angles the emission lines become red-shifted. As a result, the integration over θ enhances the yield of the photons lower than $\omega_n(\theta = 0)$, so that the peaks of $dE/hd\omega$ are broadened at the soft-photon side. It is natural to expect the increase of the broadening increases with the aperture $\theta_{\max}$. Both of these features are clearly seen if one compares Figs. 2.4 and 2.5 as well as the curves for different $\theta_{\max}$ in the latter figure. Another peculiarity which appears in the integrated spectrum is the notable emission in the even harmonics.

2.3 Channeling in Straight Crystals

The basic effect of the channeling process in a straight single crystal is in an anomalously large distance which particle can penetrate moving along a crystallographic plane (the planar channeling) or an axis (the axial channeling) and experiencing the collective action of the electrostatic field of the lattice ions [206]. The field is repulsive at small distances for positively charged particles and, therefore,

they are steered into the inter-atomic region, while negatively charged projectiles move in the close vicinity of ion strings or planes.³

Channeling was discovered in the early 1960th by computer simulations of ion motion in crystals of various structure [234, 235]. Large penetration lengths were obtained for low-energy (up to 10 keV) CU ions incident along various crystallographic directions in several crystalline structures of the cubic symmetries. These calculations allowed researches to explain the results of earlier experiments [85, 86] on anomalously long propagation lengths for heavy ions in aluminum. Later, a comprehensive theoretical study by Lindhard [206] demonstrated that propagation of charged particles through crystals strongly depends on the relative orientation of beam and target. The important model of continuum potential for the interaction of energetic projectiles and lattice atoms was formulated. Basing on this model Lindhard demonstrated that under certain conditions the projectile can move through the crystal following a particular crystallographic direction.

There are many reviews on channeling phenomenon in straight crystals in which many of the topics described below are discussed in more details. Let us indicate [37, 54, 68, 81, 106, 206, 228, 259, 287, 289].

2.3.1 Crystallographic Axes and Planes

A specific feature of a crystalline structure (the case of a single crystal is assumed) is that the crystal atom are arranged according to a regular pattern, which can be described in terms of three-dimensional translations of the unit cell, build with three primitive vectors $\mathbf{a}_j$ ($j = 1, 2, 3$) (see, e.g., [144]). The structure of a crystal as well as all its physical properties are invariant with respect to any translation characterized by the vector

$$\mathbf{T} = n_1\mathbf{a}_1 + n_2\mathbf{a}_2 + n_3\mathbf{a}_3, \quad (2.28)$$

where n_j are integers. Three integers define a crystallographic direction (axis) notated as $\langle n_1, n_2, n_3 \rangle$. The plane, perpendicular to this axis is notated as (n_1, n_2, n_3) . Figure 2.6 (left) illustrates the main crystallographic axes for a primitive cubic cell. The dashed area marks the (110) plane which is perpendicular to the $\langle 110 \rangle$ axis.

As a result of such a regular pattern of the crystal structure, the crystal ions can be viewed as been arranged in strings or/and planes. Seen along a crystallographic direction, a crystal can be represented as a set of regularly spaced strings, as it is schematically illustrated by Fig. 2.6 (right), where blue balls represent the atoms in a silicon crystal arranged along the $\langle 020 \rangle$ axis.

³ Channeling effect can be discussed not only for crystals but, in principle, for any structured material which provides “passages”, moving along which a projectile has much lower value of the mean square of the multiple scattering angle than when moving along any random direction. The examples of such materials are nanotubes and fullerites, for which the channeling effects has been also investigated, see, e.g., [20, 56, 77, 88, 108, 112, 297–299].

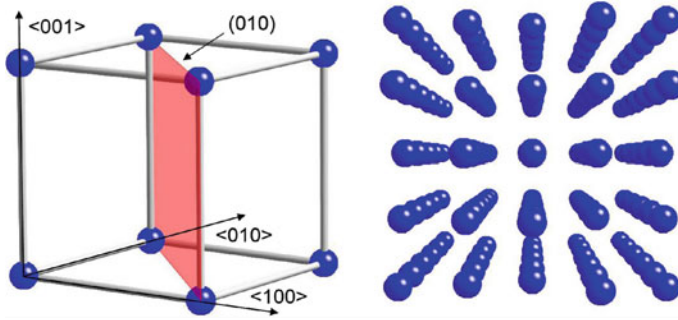


Fig. 2.6 *Left panel* A primitive cubic cell with the main crystallographic directions indicated. The dashed area marks the (110) plane. *Right panel* Silicon single crystal seen along the $\langle 020 \rangle$ axis

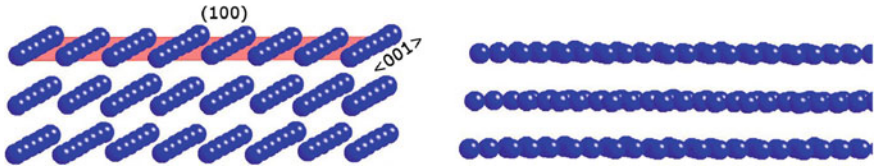


Fig. 2.7 Si(100) planar channels seen from, approximately, $\langle 001 \rangle$ crystallographic direction (*left panel*) and from random (not aligned with any crystallographic axis) direction

Rotating the crystal one can view its atoms aligned along the strings which, in turn, form a set of well-separated crystallographic planes, see Fig. 2.6 (left). The right panel of the figure represents the crystal as a set of planes only. This can be achieved by looking at the crystal along any of its crystallographic planes from a random (i.e., not aligned with any crystallographic axis) direction (Fig. 2.7).

The atoms, arranged either in strings or in planes, create a periodic electrostatic potential which acts on the projectile. Under certain conditions, when the incident angle with respect to the string (or plane) is small enough, the collisions of the projectile with different atoms become strongly correlated, so that it moves further essentially following the direction along the string.

2.3.2 Continuous Potential Model

A charged particle, moving along a string of atoms separated by the distance $d_{\parallel}$, experiences the action of the electrostatic field created by the atoms, see Fig. 2.8.

Provided the incident angle Θ is small enough, the motion of the particle will be governed not by the scattering from the field of individual atoms but rather by the correlated action of many centers of the force. To illustrate this statement one can consider the following arguments. The characteristic time τ of the motion in the

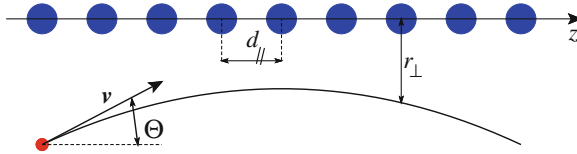


Fig. 2.8 Trajectory and interaction of the charged particles with the string of atoms (The figure adapted from [14])

direction perpendicular to the string can be estimated as $\tau \sim d_{\perp}/v_{\perp}$, where $d_{\perp}$ is the (average) distance between the strings and $v_{\perp}$ is mean velocity in the perpendicular direction. During this interval the particle moves along the string a distance $l_{\parallel} = v_{\parallel} \tau = d_{\perp} v_{\parallel}/v_{\perp} \approx d_{\perp}/\Theta$. Taking into account that in a crystalline structure $d_{\perp} \sim d_{\parallel}$, one derives $l_{\parallel} \sim d_{\parallel}/\Theta \gg d_{\parallel}$ for $\Theta \ll 1$. In other words, within $l_{\parallel}$ the particle experiences a large number correlated collisions. Therefore, to describe the transverse motion of the projectile, one can introduce the continuum approximation [206] within which the potential $V(r_{\perp}, z)$ of each atoms is averaged (“smeared”) along the string, resulting in the following continuous transverse potential $U_s(r_{\perp})$ of the string:

$$U_s(r_{\perp}) = \int_{-\infty}^{\infty} V(r_{\perp}, z) \frac{dz}{d_{\parallel}}, \quad (2.29)$$

where $r_{\perp}$ is the distance from the string.

Similar to the case of atomic string, the motion along a crystallographic plane at small incident angles is described by the continuous transverse potential $U_{\text{pl}}(y)$ of the atomic plane (which is aligned with the (xz) Cartesian plane) defines as follows (see, e.g., [68, 81, 106, 206, 259]):

$$U_{\text{pl}}(y) = n_a d \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} V(r) dz dx, \quad (2.30)$$

where d is the interplanar distance, n_a —the volume density of crystal atoms, y —the coordinate along the perpendicular direction, and $V(r)$ ($r = \sqrt{x^2 + y^2 + z^2}$) is the potential of the projectile—atom interaction.

Depending on the model applied to describe atomic potential $V(r)$, one can obtain different forms for $U_s(r_{\perp})$ and $U_{\text{pl}}(y)$. Many of these can be found in [37, 54, 68, 81, 106, 206, 228, 287, 289].

Total continuous potential acting on the projectile in the crystal is the sum of the potentials created by individual planes (or axes). For the sake of clarity, below we consider the *planar* case.

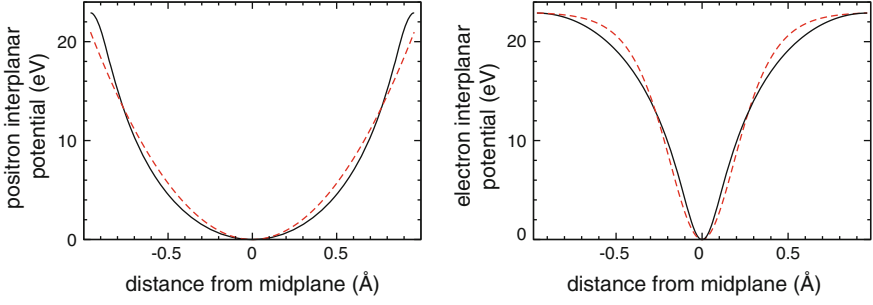


Fig. 2.9 The interplanar potential in Si(110) for positron (*left panel*) and electron (*right panel*). In both panels *solid line* stands for the potential calculated within the Molière approximation (see Appendix D) at crystal temperature 150 K. The *dashed curve* on the *left panel* represents the harmonic approximation $U(\rho) \propto \rho^2$, on the *right panel*—the Pöschl-Teller approximation (see Appendix C)

Taking into account that atomic potential $V(r)$ rapidly decreases at the distances exceeding the (mean) atomic radius,⁴ one can account for two for only two nearest planes when calculating the sum. Then, the continuous *interplanar* potential can be presented in the form [68]:

$$U(\rho) = U_{\text{pl}} \left(\frac{d}{2} - \rho \right) + U_{\text{pl}} \left(\frac{d}{2} + \rho \right) + \text{const.} \quad (2.31)$$

Here $\rho = [-d/2, d/2]$ is the distance measured from the mid-plane. The additive constant can be chosen to ensure $U = 0$ at the midplane (the differences in choosing the mid-plane for positive and negative particles are discussed below).

Figure 2.9 presents interplanar potential for positron (left panel) and electron (right panel) calculated for Si(110) channel using different approximations (see the caption).

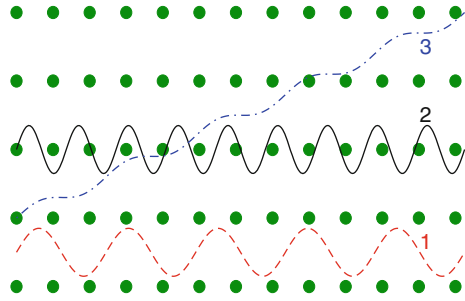
Within the framework of the continuous potential approximation, the transverse motion of an ultra-relativistic projectile of the mass m is described (in terms of classical mechanics) by the Newton equation but with a the “relativistic” mass equal to $m\gamma$ (or, equivalently, to ε/c^2). Written for the planar case the equation of motion reads

$$m\gamma\ddot{y} = -\frac{dU}{dy}. \quad (2.32)$$

Within this picture the energy of the transverse motion, $\varepsilon_{\perp} = p_{\perp}^2/2m\gamma + U(y)$, is the integral of motion. Comparing $\varepsilon_{\perp}$ with the depth U_0 of the interplanar potential (2.31), one can distinguish between two modes of the transverse motion:

⁴ For a quantitative estimate one can use the Thomas-Fermi radius $a_{\text{TF}} = Z^{-1/3}a_0$ as the measure of the mean atomic radius (see, e.g., [198]). Here Z is the atomic number of the crystal atom and a_0 is the Bohr radius.

Fig. 2.10 Schematic representation of the (planar) channeling motion and the motion of the over-barrier particle. Trajectories 1 and 2 stand, respectively, for a positively and a negatively charged particles experiencing planar channeling, trajectory 3 denotes the over-barrier particle (The figure adapted from [13])



1. *Channeling motion* occurs when $\varepsilon_{\perp} < U_0$. In this case the transverse motion is finite, having the form of oscillations in the vicinity of the midplane.
2. *Over-barrier motion* corresponds to $\varepsilon_{\perp} > U_0$ when a particle moves unrestricted, sequentially crossing different crystal planes on its way.

These modes are illustrated by Fig. 2.10.

Taking into account that $p_{\perp} = p\Theta$, one re-writes the channeling condition in terms of the following restriction for the incident angle

$$\Theta < \Theta_L = \sqrt{\frac{2U_0}{pv}} \approx \sqrt{\frac{2U_0}{\varepsilon}}. \quad (2.33)$$

The right-hand side is written for an ultra-relativistic particle for which $pv \approx \varepsilon$.

The critical angle Θ_L was introduced by Lindhard [206]. The values of critical angle in the case of planar channeling are lower by a factor $\sim\sqrt{10}^1$ than those for the axial channeling due to the order of magnitude difference in the depths of the continuous potential well.

2.3.2.1 Dechanneling Effect

Lindhard [206] was also the first who theoretically described the dechanneling process,—the phenomenon of a gradual increase of the transverse energy of a channeled particle due to collisions with nuclei and electrons of the crystal.

Within the continuous potential model the transverse energy $\varepsilon_{\perp}$ is conserved. However, the real potential, equal to the sum of potentials due to the point charges, differs from $U_s(r_{\perp})$ or/and $U_{pl}(y)$. Therefore, the projectile motion determined within the framework of the continuous potential model is perturbed by the action of the additional potential. This action can be considered in terms of uncorrelated scattering from crystal constituents. In each act of scattering the magnitude of $\varepsilon_{\perp}$ changes by some small value (except for, comparatively, rare events of close collisions with nuclei which may result in large-angle scattering and, thus, to a noticeable change in $\varepsilon_{\perp}$). As a consequence, initially channeled particle during its passage through the

crystal gains the transverse energy higher than the continuous potential barrier. At this point the particle leaves the channel and is, basically, lost for the channeling process.

The scale which defines the (average) interval for a particle to penetrate into a crystal until its dechannels is called a *dechanneling length*, L_d . This quantity depends on the parameters of a crystal (these include the charge of nuclei, the inter-atomic spacing, the mean atomic radius, the amplitude of thermal vibrations) and the parameters of a channeled particle,—the energy and the charge. It is important to note that for negative and for positive projectiles the dechanneling occurs in different regimes (see also the Sect. 2.3.3). As mentioned above, the interplanar (or axial) potential is repulsive at small distances for positively charged particles and is attractive for negatively ones. Therefore, negatively charged particles tend to channel in the regions around the nuclei whereas positive particles are pushed away. Consequently, the number of collisions with the crystal constituents is much larger for negatively charge particles and they dechannel faster.⁵ Typically, the dechanneling lengths of positive charges exceed those for negative ones (of the same energy and charge modulus) by the order of magnitude or more [80, 192, 193]. This statement is valid, both for axial and planar channeling, and for various pairs of positive/negative particles of the same charge modulus: for e^+/e^- [283], π^+/π^- [12], and $p/\bar{p}$ [285].

2.3.3 Positron Versus Electron Channeling

At small distances from the atom, the field is repulsive for a positively charged particle and but attractive for a negative one. The same feature characterizes the continuous potentials of planes and strings. Therefore, positively charged projectiles they are steered into the interatomic region, while negatively charged projectiles move in the close vicinity of strings or planes (see, e.g., [259]). To be specific, we compare the motion of positrons and electrons, although the main features discuss below are applicable to any type of positively and negatively charged projectiles.

For the planar case, the differences in the channeling regime for positrons and electrons are illustrated by Fig. 2.11 where the left column of graphs refers to the positron and the right one to the electron channeling.

The Si(110) interplanar potential $U(y)$, calculated within the Molière approximation (see Appendix C), versus the distance from the midplane is presented by the two upper graphs. The electron potential equals to the inverted positron one but shifted upward by the constant value of approximately $U_0 = 22.9\text{ eV}$ (the magnitude of the Si(110) interplanar potential well) to match the minimum value (which is set to zero for both particles). By definition, the midplanes are associated with the minima of $U(y)$. In the case of positron channeling the midplanes are located half-way

⁵ More detailed quantitative description of the dechanneling process is presented below in the book in Sect. 4.3.1 (positively charged ultra-relativistic projectiles) and in Sect. 6.1.1 (electrons).

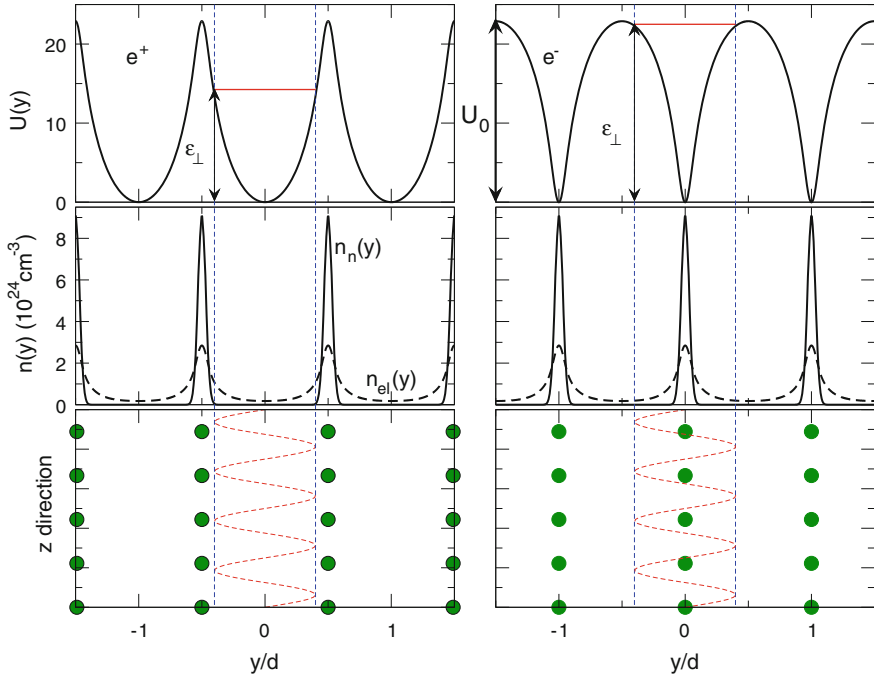


Fig. 2.11 Positron (left column) vs. electron (right column) planar channeling. Two upper graphs represent the interplanar potential (2.31) in Si(110) channel calculated within the Molière approximation at the crystal temperature $T = 150$ K. The middle graphs present the distribution of nuclei density (solid curves) and electron density (dashed curves). The lower graphs illustrate schematically the channeling motion of the projectiles: while advancing along the crystallographic planes (filled circles) the particle oscillates in the transverse direction, where its motion is restricted by the interplanar potential. The channeling oscillations occur with a fixed value of the transverse energy $\varepsilon_{\perp}$ which is indicated in the upper graphs. The figure stresses the main difference between electron and positron channeling: channeled electrons oscillate in the interplanar space whereas electrons channel in the vicinity of the plane

between two neighbouring crystal planes, whereas for an electron they coincide with the crystal planes.

As a result, a positron channels in between two planes,—see left bottom graph, where the wavy line represents the positron trajectory and filled circles denote the crystal atoms. This domain is characterized by low content of crystal electrons and by, essentially, absence of the nuclei (see the middle graphs which represents the volume densities of crystal nuclei n_n and electrons n_{el} as functions of the distance from midplane). In contrast, an electron (as well as any other negatively charged projectile) channels the vicinity of the plane, i.e., in the domain with values of n_n and n_{el} . Hence, the probability to experience the uncorrelated collision is much higher for electrons rather than for positrons. Therefore, during the planar channeling the electrons dechannel faster than positrons [192, 193].

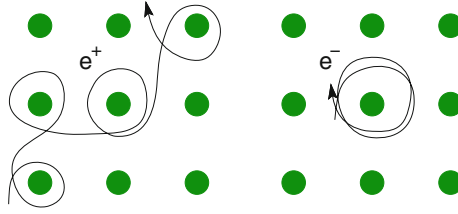


Fig. 2.12 Schematic representation of the axial channeling for a positron (*left*) and an electron (*right*). Attractive character of the atomic string potential for electron keeps the negatively charged projectile in the vicinity of a single string (axial channel). For positrons, the repulsive core at the string allows the projectile to freely move from channel to channel

The difference in channeling motion for positively and negatively charged projectiles is even more pronounced in the case of axial channeling, Fig. 2.12. The potential of atomic string (2.30), being repulsive for a positively charged projectile, pushes a positron away, so that the positron freely moves from axis to axis. In contrast, the trajectory of the channeling electron is mainly concentrated in the vicinity of a single string (see, e.g., [259]).

2.3.4 Classical Versus Quantum Description

In the case of a channeling particle the notion of trajectory becomes meaningful provided the particle coordinate ρ and the transverse momentum $p_{\perp}$ can be measured with accuracies (uncertainties) $\Delta\rho$ and Δp_y , which are much smaller than the channel width d and typical value $p_{\perp, \text{ch}}$ of the transverse momentum of in the channeling regime. Thus, using Heisenberg's uncertainty principle, one writes

$$p_{\perp, \text{ch}} d \gg \Delta p_{\perp} \Delta \rho > \hbar. \quad (2.34)$$

Kinetic energy, associated with the transverse motion of the channeling particle, cannot exceed the depth U_0 of the interplanar potential well. Therefore, to estimate $p_{\perp, \text{ch}}$ one can use the relation

$$\frac{p_{\perp, \text{ch}}^2}{2\varepsilon/c^2} < U_0. \quad (2.35)$$

Combining (2.34) and (2.35) one finds the condition which validates the use of the classical description of the projectile motion:

$$\varepsilon \gg \frac{(\hbar c)^2}{2d^2 U_0}. \quad (2.36)$$

Applying typical values $d \sim 2 \text{ \AA}$ and $U_0 \sim 10 \text{ eV}$ (the case of planar channels is considered) one obtains $E \gg 0.1 \text{ MeV}$.

However, the sign $\gg$ in (2.36) must be understood as “larger by at least three orders of magnitude” [176].

To demonstrate this, let us consider the harmonic approximation for the interplanar potential (which is adequate at least for positively charged projectiles, see e.g. [68] and Fig. 2.9 (left)): $U(\rho) = 4\rho^2 U_0/d^2$. Then, the frequency of channeling oscillations is given by $\Omega_{\text{ch}} = c\sqrt{U''(0)/\varepsilon} = c\sqrt{8U_0/d^2\varepsilon}$. The number of quantum levels in the potential well is $N_{\varepsilon_{\perp n}} = U_0/\hbar\Omega_{\text{ch}} = \sqrt{d^2 U_0 \varepsilon / 8 \hbar^2 c^2}$. Hence

$$\varepsilon \approx 16N_{\varepsilon_{\perp n}}^2 \frac{(\hbar c)^2}{2d^2 U_0}. \quad (2.37)$$

The right-hand sides of (2.36) and (2.37) coincide up to the factor of $16N_{\varepsilon_{\perp n}}^2$. The classical description of the channeling oscillations is valid provided the number of levels is sufficiently large. Assuming $N_{\varepsilon_{\perp n}} = 10$ one obtains $16N_{\varepsilon_{\perp n}}^2 = 1,600$, thus specifying the sign in (2.36) [176].

Hence, the classical model can be always applied to ultra-relativistic heavy projectiles. It is also applicable to light positively charged projectiles (positrons) if their energy is in the hundred MeV range or higher.

The applicability conditions for an electron is somewhat stricter than for a positron, because the interplanar potential well is narrower for negative projectiles, cf. two panels in Fig. 2.9. Numerical results presented in Appendix C.3 indicate that the number $N_{\varepsilon_{\perp n}}$ of levels for a channeling electron is approximately two times smaller than for the positron of the same energy. Figure C.2 suggests that for both projectiles $N_{\varepsilon_{\perp n}} \propto \varepsilon^{1/2}$. This functional dependence is well-known for the planar channeling. For the axial case $N_{\varepsilon_{\perp n}} \propto \varepsilon$ for both types of projectile [259, 286].

Therefore, the motion of positrons and electrons in the GeV range and higher can be described in classical terms for both planar and axial channeling.

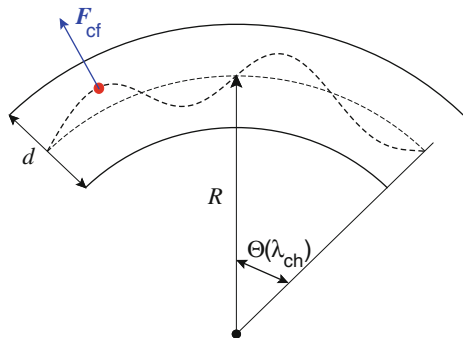
2.4 Channeling in Bent Crystals

Let us consider a crystallographic channel which is bent with a constant curvature with the radius R , see Fig. 2.13. An ultra-relativistic particle with the energy ε moving in the bent channel experiences the action of the centrifugal force

$$F_{\text{cf}} = \frac{pv}{R} \approx \frac{\varepsilon}{R}. \quad (2.38)$$

Channeling motion in the bent crystal is possible if F_{cf} is less than the maximum transverse force U'_{max} acting on the particle in the interplanar (or, axial) potential

Fig. 2.13 In a bent channel, in addition to the interplanar force (not indicated in the figure), the particle experiences the action of the centrifugal force F_{cf} . The curvature radius R and the interplanar spacing d satisfy the condition $R \gg d$



[281]. It is convenient to quantify this statement in terms of the dimensionless parameter C defined as follows⁶

$$C = \frac{F_{cf}}{U'_{\max}} \approx \frac{\varepsilon}{RU'_{\max}} \leq 1. \quad (2.39)$$

The value $C = 0$ corresponds to a straight channel. The critical radius $R_c = \varepsilon/U'_{\max}$ (frequently called as Tsyganov's radius) corresponds to $C = 1$.

Let us mention that the *channeling condition* (2.39) can be formulated in somewhat different terms [255], which are especially convenient in connection with axial channeling in bent crystals. Namely, one can require the bending angle $\Theta(\lambda_{ch})$ over one period λ_{ch} of the channeling oscillations to be smaller than the Lindhard critical angle Θ_L , (2.33).

To construct the effective potential in a bent channel one adds the centrifugal term to the continuous interplanar (or axial) potential. In terms of the bending parameter C the centrifugal potential (within a single channel) can be written as $-CU'_{\max} \rho$ with ρ being the distance from the midplane. Then, the effective potential $U(C; \rho)$ is written as

$$U(C; \rho) = U(\rho) - CU'_{\max} \rho. \quad (2.40)$$

The centrifugal term modifies the potential in which the projectile moves in the transverse direction. The potential becomes (a) asymmetric with respect to the midplane $\rho = 0$, and (b) shallower, i.e., the depth of the potential well ΔU_C decreases as C increases. Both of these features are illustrated by Fig. 2.14 where the effective potential in Si(110) planar channel is presented for several values of C , as indicated. The interplanar potential $U(\rho)$ was calculated within the Molière approximation at $T = 150$ K (see Appendix D).

In accordance with (2.39), the depth potential well disappears at $C = 1$ making channeling effect essentially impossible. For $C < 1$ the value ΔU_C equals to the

⁶ Below in the book, this quantity will be frequently called *bending parameter*.

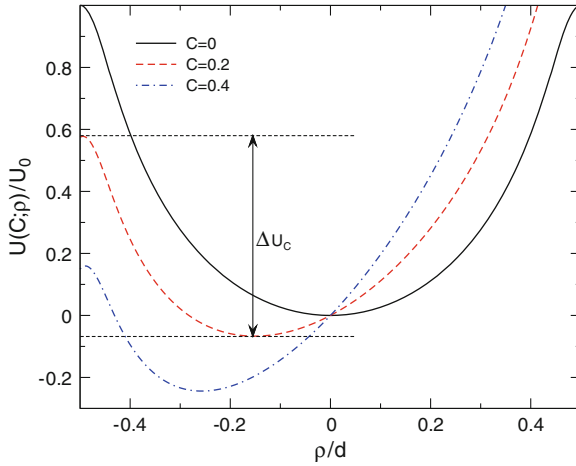


Fig. 2.14 The effective potential $U(C; \rho)$ versus distance ρ from the midplane calculated for bent Si(110) planar channel. The curve $C = 0$ stands for the interplanar potential $U(\rho)$ within the Molière approximation at the crystal temperature 150 K. The values of $U(C; \rho)$ are scaled by $U_0 = U(d/2)$, the ρ values—by $d = 1.92 \text{ \AA}$, which is the interplanar distance. The notation ΔU_C stands for the depth of the effective potential well

maximum transverse energy of the channeling motion which, in turn, defines the critical angle $\Theta_c(C)$ in bent channel

$$\Theta_c(C) = \sqrt{\frac{2 \Delta U_C}{\varepsilon}}. \quad (2.41)$$

In straight channel $C = 0$, and $\Theta_c(0)$ coincides with Lindhard's critical angle Θ_L .

The necessary condition for a projectile to be trapped into the channeling mode of motion at the crystal entrance reads $\Theta < \Theta_c(C)$ where Θ is the angle between the particle velocity and the channel centerline [281].

Theoretical analysis of various phenomena related to the channeling in bent crystals (classical treatment), including the dechanneling effect, can be found in [94, 95, 97, 189, 274]. The review of theoretical models and computational methods can be found in [68, 271].

Since its first theoretical prediction [281] and experimental support [38, 93], the idea to deflect high-energy beams of charged particles by means of a planar channeling in bent crystals had attracted a lot of attention worldwide and still is of a great interest due to its growing practical application. Indeed, the crystals with bent crystallographic planes are used to steer high-energy charged particle beams replacing huge dipole magnets. We refer to the review chapters which describe in detail initial stages of the development of the idea as well as the experiments carried out up to 2,000 [52, 68, 81, 222, 271].

Bent crystal have been routinely used for beam extraction in the Institute for High Energy Physics, Russia [4]. A series of experiments on the bent crystal deflection and collimation of proton and heavy ion beams were performed at different accelerators [18, 82, 101, 239, 240, 244, 258] throughout the world. The bent crystal method has been proposed to extract particles from the beam halo at CERN's Large Hadron Collider [286]. The possibility of deflecting positrons [58, 242], electrons [242, 257, 258], and π^- mesons [121, 241, 243, 245] has been studied as well.

Another possible application of bent crystal concerns its possible use to focus beams of ultra-relativistic heavy-ions [246] or protons [89]. To this end a crystal is needed in which the crystal axes are no longer parallel, but are slanted more and more the farther away they are from the axis of the beam. Then the bending angle of the particles far away from the beam axis would be largest and a general focusing effect will result. Such a crystal can in principle be produced by varying Ni to Cu (or Sb to Bi) ratio in a mixed crystal [246]. Similar idea was suggested also for producing periodically bent crystals, see Sect. 3.2 for more detail.

Recently, it was suggested to use bent nanotubes to steer beams of charged particle [112, 299].

2.5 Radiative Processes in Crystals

2.5.1 *Bremsstrahlung*

2.5.1.1 The Elementary Process of Bremsstrahlung

In the *elementary process* of BrS a charged projectile emits a photon being accelerated by the static field of a target (nucleus, ion, atom, etc). The basic description of BrS, both classical and quantum, can be found in textbooks (see, e.g., [61, 123, 136, 196]). The importance and the fundamental character of the BrS process was recognized long ago (for a review of the historical background see, e.g., [231]). Since then it has been intensively studied theoretically, numerically and experimentally in a wide range of the projectile and the emitted photon energies, different geometries of the emission, and a variety of atomic and ionic targets (see the reviews [122, 231] and references therein).

Diagrammatic representation of the BrS amplitude is given by Fig. 2.1 (left). In the chapters cited above (see also [249]), various approaches are described concerning the choice of the initial and final state wavefunctions of the projectile.

For relativistic projectiles, the Bethe-Heitler (BH) approximation [64] is the simplest and the most widely used one, since all the related cross sections for radiation in a point-charge Coulomb field can be expressed in closed form in terms of the elementary functions. The mentioned cross sections include the spectral distribution $d\sigma \equiv d\sigma/d\omega$ which one obtains from the double cross section $d^2\sigma \equiv d^2\sigma/d\omega d\Omega$ by integrating over the emission angles $\Omega = (\theta, \phi)$; in turn, $d^2\sigma$ one calculates

by integrating the triply differential cross section $d^3\sigma \equiv d^3\sigma/d\omega d\Omega d\Omega_{\mathbf{p}_2}$ over the scattering angles $\Omega_{\mathbf{p}_2} = (\theta_{\mathbf{p}_2}, \phi_{\mathbf{p}_2})$. The BH formula results from the application of the Born approximation for the integration of the scattering relativistic electron with the potential, retaining only the first non-zero terms (for the details, see, e.g., [6]). To take into account the screening of the atomic nucleus by the electrons one multiplies the cross section $d^3\sigma$, calculated in the Coulomb field Ze/r of the nucleus, by the atomic form factor [64] $F(q) = 1 - Z^{-1} \int \rho(\mathbf{r}) \exp(i\mathbf{q} \cdot \mathbf{r}) d\mathbf{r}$, where $\rho(\mathbf{r})$ is the electron charge density in the atom (or ion), and $\mathbf{q} = \mathbf{p}_1 - \mathbf{p}_2 - \hbar\mathbf{k}$ is the momentum transferred to the nucleus during the collision (see Fig. 2.1 (left) for the notations).

Formally, the applicability of the BH formula is linked to that of the Born approximation, which implies that the initial (v_1) and final (v_2) projectile velocities satisfy the relation $v_{1,2}/c \gg \alpha Z$ (α is the fine structure constant). As a result, for heavy elements, $\alpha Z \sim 1$, the data on $d^2\sigma$ and $d^3\sigma$ within the BH approximation strongly deviate from the results of more accurate calculations (see [249] and references therein). However, the discrepancies become much less pronounced for the cross section $d\sigma$ integrated over the emission and scattering angles [231].

The BH cross section $d\sigma$ for the case of an ultra-relativistic electron (positron) scattering in the field of a neutral atom can be written as follows [6]:

$$\frac{d\sigma_{\text{BH}}}{d\omega} \approx \frac{16}{3} \frac{Z^2 \alpha r_0^2}{\omega} \left(1 - x + \frac{3}{4}x^2\right) \ln(183Z^{-1/3}) \quad (2.42)$$

where $x = \hbar\omega/\varepsilon_1$ and r_0 is the classical electron radius.

Equation (2.42) accounts for the photon emission in the process of “elastic” BrS, i.e. when the atom does not change its state in the collision. In addition to this, the processes of “inelastic” BrS, in which the photon emission is accompanied by excitation or ionization of the target, also contribute to the spectrum. To include this part, one recalls that for an ultra-relativistic particle the BrS cross section of a projectile electron does not depend on the mass of the target particle but is proportional the square of its charge. Therefore, ignoring the differences in logarithmic factors, the cross section of elastic BrS in the field of a screened nucleus exceed that of inelastic BrS in collision with an atomic electron by a factor Z^2 . Then, to account for the inelastic BrS in collision with an atom with Z electrons, one carries out the following substitution on the right-hand side of (2.42): $Z^2 \rightarrow Z(Z+1)$ (see, e.g., [61]).

On passing through a medium, an electron loses its energy emitting photons. The efficiency of this process can be characterized by the *radiation length* L_r , i.e., the mean distance over which a projectile loses all but $1/e$ of its energy via BrS. To calculate L_r , one multiplies (2.42) by the photon energy $\hbar\omega$ and integrates over ω from 0 up to $(\varepsilon_1 - mc^2)/\hbar$. Then, multiplying the result by the factor n_a/ε_1 (n_a is the volume density of the medium atoms) and accounting for the substitution mentioned above, one derives:

$$L_r = \left[4Z(Z+1)\alpha r_0^2 n_a \ln(183Z^{-1/3})\right]^{-1}. \quad (2.43)$$

The radiation length is derived on the basis of (2.42), which defines the BrS spectrum in a single electron-atom collision. The applicability of this approach is justified if L_r greatly exceeds the characteristic length, L_c , termed the *coherence length* [275], over which the radiation is formed in a single collision. Omitting the details of evaluation (see, e.g., [61, 275]), we present the final result

$$L_c = \frac{\hbar}{q_{\min}} \approx \frac{2\varepsilon_1\varepsilon_2}{\omega m^2 c^3}, \quad (2.44)$$

where $q_{\min} = p_1 - p_2 - \hbar\omega/c \approx \hbar\omega mc^3/2\varepsilon_1\varepsilon_2$ (for $\varepsilon_{1,2}/mc^2 \gg 1$) is the minimum transferred momentum $|\mathbf{q}|$ in the radiative collision.

The formulae obtained for BrS at an isolated atom can be valid also for passage through a medium only if over the distance L_c there is no secondary photon emission or/and electron scattering. The absence of the former is ensured by the condition $L_c \ll L_r$. However, the condition of no electron scattering is violated much sooner, so that at the distances $\sim L_r$ a projectile experiences multiple scattering with the medium atoms. As a result, the distances, over which (2.42) is applicable for evaluation of the BrS spectrum in medium, become much smaller and result in the condition $L_c \ll \alpha L_r$. At larger distances the yield of BrS (as well of other electrodynamic processes, such as pair creation) is suppressed due to the multiple scattering of the projectile from the medium atoms, a so-called Landau-Pomeranchuk-Migdal effect [200, 217].

2.5.1.2 Coherent Bremsstrahlung

In an amorphous medium, an ultra-relativistic projectile experiences uncorrelated collisions with the constituent atoms. As a result, the total intensity of BrS will be proportional to the number of atoms n_a (per unit volume), i.e., it is equal to the incoherent sum of the intensities formed in collisions with individual atoms (a so-called *incoherent* BrS).

Passing through a crystalline media along a line close to any crystallographic direction, a projectile “feels” the periodicity of the potential. The periodicity will strongly influence the emission process provided the coherence length (2.44) noticeably exceeds the spatial period l of the potential [230, 275, 282]. In this case, the amplitudes of the waves, emitted in collisions of the projectile with each of these $N_c \sim L_c/l$, may add coherently, so that the resulting emitted intensity will be enhanced relative to incoherent BrS by a factor $\sim N_c$. For each emission angle θ the coherence effect will be mostly pronounce if θ and ω (which enters via the coherence length) satisfy the condition $l/(L_c\theta) = 2\pi n$, with n being an integer [230, 275, 282].

This process, commonly termed as the *coherent* BrS, is associated with an over-barrier particle (see trajectory No. 3 in Fig. 2.10) which moves along quasi-periodic trajectory, traversing the crystallographic planes (or axes) under the angle greater than Lindhard’s critical angle, $\Theta > \Theta_L$.

Within the framework of the Born approximation it can be shown (see, e.g., [275]), that the triply differential cross section of BrS in a medium can be related to the BH cross section $d^3\sigma_{\text{BH}} \equiv d^3\sigma_{\text{BH}}/d\omega d\Omega d\Omega_{\mathbf{p}_2}$ as follows

$$\begin{aligned} d^3\sigma &= d^3\sigma_{\text{BH}} \left| \sum_a e^{i\mathbf{q}\cdot\mathbf{r}_a/\hbar} \right|^2 = d^3\sigma_{\text{BH}} \left(N_a + \sum_{a \neq a'} e^{i\mathbf{q}\cdot\Delta_{aa'}/\hbar} \right) \\ &= d^3\sigma_{\text{nc}} + d^3\sigma_{\text{c}}, \end{aligned} \quad (2.45)$$

where $\mathbf{q} = \mathbf{p}_1 - \mathbf{p}_2 - \hbar\mathbf{k}$ is the transferred momentum, $\Delta_{aa'} = \mathbf{r}_a - \mathbf{r}_{a'}$, and the sums are carried out over the lattice atoms of total number N_a .

The term $d^3\sigma_{\text{nc}} \equiv N_a d^3\sigma_{\text{BH}}$ stands for the contribution of the incoherent BrS, while the second one, $d^3\sigma_{\text{c}} = N_a d^3\sigma_{\text{BH}} \sum_{a \neq a'} e^{i\mathbf{q}\cdot\Delta_{aa'}/\hbar}$ represents the coherent part of the BrS cross section.

In an amorphous medium, where the atomic position vectors $\mathbf{r}_a$ and $\mathbf{r}_{a'}$ are not correlated, the sum $\sum_{a \neq a'} (\dots) = 0$, so that the coherent term vanishes.⁷ Then, integrating $d^3\sigma_{\text{nc}}$ over the emission and scattering angles, one finds that the spectral distribution $d\sigma_{\text{nc}}/d\omega$ of incoherent BrS is defined by formula (2.42) multiplied by N_a . The BH spectrum is a smooth function of the photon energy, therefore, $d\sigma_{\text{nc}}/d\omega$ represents a smooth background of the total BrS spectrum formed in a crystal.

The coherent term $d^3\sigma_{\text{c}}$ becomes dominant in a crystalline medium. Not pretending to overview all important theoretical results related to the phenomenon of coherent BrS (this one can find, for example, in [7, 11, 14, 37, 53, 250, 259, 275, 276, 287, 289]), we just mention several qualitative features of $d^3\sigma_{\text{c}}$ which originate from the factor $\Sigma(\mathbf{q}) \equiv \sum_{a \neq a'} e^{i\mathbf{q}\cdot\Delta_{aa'}/\hbar}$. The detailed evaluation of this factor for various types of cubic lattices is presented in [230]. As a function of the transferred momentum $\Sigma(\mathbf{q})$ is strongly peaked in the vicinity of $\mathbf{q} = \mathbf{g}$, where $\mathbf{g}$ is a reciprocal lattice vector. Therefore, for particular geometries of radiation and scattered electron, $d^3\sigma_{\text{c}}$ has very pronounced maxima. In the maximum $d^3\sigma_{\text{c}} \propto N_a N_c$ with N_c standing for the number of atoms within the coherence length L_c related to the transferred momentum as $L_c \sim \hbar/q_{\parallel}$, where $q_{\parallel} = (\mathbf{p}_1 \cdot \mathbf{q})/p_1$ is the component of $\mathbf{q}$ along the incident momentum. These peaks will lead to the non-monotonous structure of the cross section $d\sigma_{\text{c}} = d\sigma_{\text{c}}/d\omega$ integrated over the emission and scattering angles. This feature is illustrated by Fig. 2.15 (left) which presents the experimental results on the enhancement factor, i.e. on the ratio $d\sigma_{\text{c}}/d\sigma_{\text{nc}}$ for a 150 GeV electron traversing Si crystal as indicated in the capture [216].

Figure 2.15 (right) stresses another feature of coherent BrS, namely its sensitivity to the sign of a projectile. It is seen, that under the same conditions (incident energy, crystal and the direction of traversing) the positron and electron are much alike (as well as the BH spectrum, which is just identical). This is in contrast with spectral dependences of ChR, see Fig. 2.16 in the subsequent section.

⁷ This term is also negligibly small when a projectile moves in a random direction through the crystal.

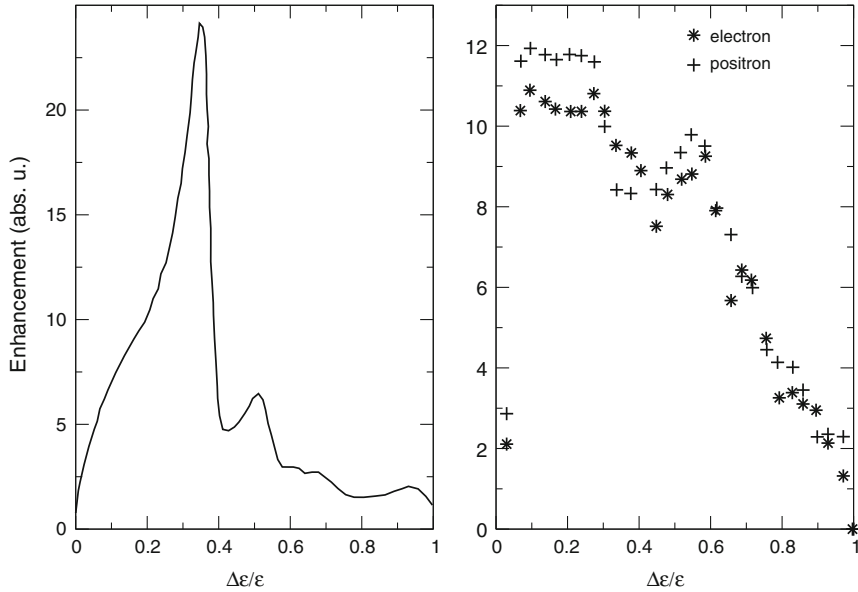


Fig. 2.15 *Left graph* radiation from 150 GeV electrons incident at the angle 0.08 mrad to the (110) plane in 0.6 mm Si. The polar angle to the (110) axis is 8.2 mrad. *Right graph* the radiation from electrons and positrons traversing 0.5 mm Si at 0.52 mrad from the axis and 27 μ rad from the (100) plane. In both graphs the enhancement is with respect to an amorphous target of the same material and thickness. The dependences were obtained by digitalizing Figs. 3b and 2a in [216]

To conclude this section, we mention that coherent BrS from high energy electrons and positrons in a bent crystal was recently considered in [73], where analytical expressions for the spectrum of emitted radiation were derived. The process of coherent BrS by relativistic electrons in PBCr was considered in [251, 252]. The authors predicted sharp jumps in the radiation spectrum in the high-frequency range of emitted photons.

2.5.2 ChR in Straight and Bent Crystals

Channeling of charged particles in crystals is accompanied by the channeling radiation [191].⁸ This specific type of electromagnetic radiation arises due to the transverse motion of the particle inside the channel under the action of the interplanar or axial field (the channeling oscillations, see the illustrative Fig. 2.10). The phenomenon of ChR of a charged projectile in a linear crystal, see e.g. [7, 13, 14, 16, 37, 39, 40, 48, 53, 54, 62, 81, 193, 194, 237, 271, 283, 284, 287, 289], as well as in a “simple” (i.e. non-periodic, one-arc) bent channel [21, 22, 51, 137, 255, 272, 273], are known,

⁸ See [53, 54] for a detailed comparative survey of several early chapters of the mid-1970th devoted to theoretical description of the phenomenon.

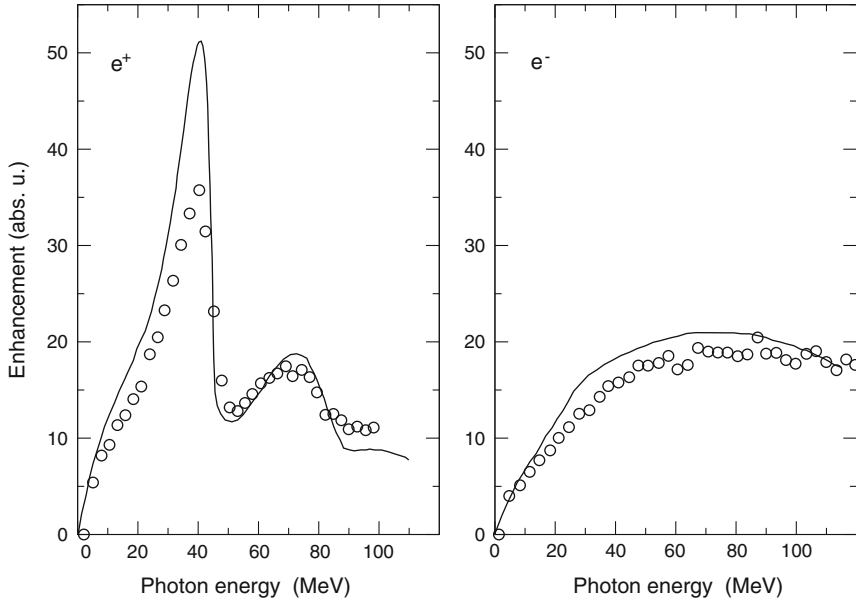


Fig. 2.16 Enhancement spectra for ChR by 6.7 GeV positrons (*left*) and electrons (*right*) channeling through a 105 μm Si crystal along the (110) plane. *Open circles* show the experimental data, *solid lines* stand for theoretical expectations. All dependences were obtained by digitalizing the graphs in Fig. 1 from [284]

although in the latter case the theoretical and experimental data are scarce. The study of the ChR, initially proposed for particles moving in crystals, was later extended to the case of nanotubes [107, 148, 149].

For our purposes it is important to mention several well-established features of ChR.

First, as well as in any other radiative process of a charge moving in an external potential, the intensity of ChR is inversely proportional to the squared mass of projectile. Hence, a channeled electron/positron emits $(m_p/m_e)^2 \sim 10^6$ times more intensively than a proton with the same value of the relativistic factor γ .

Second, for both electrons and positrons the intensity of radiation in the channeling mode greatly exceeds (by more than an order of magnitude) that by the same projectile in an amorphous medium (see, e.g., [39, 40, 283]). This is a direct consequence of the motion of a channeling particle which, being quasi-periodic, bears close resemblance with the undulator motion [191]. As a result, constructive interference of the waves emitted from different but similar parts of the channeling trajectory increases the intensity. Similar to the UR, for each value of the emission angle θ the coherence effect will be mostly pronounced for the radiation into harmonics. In the case of planar channeling, the harmonic frequencies can be calculated from (cf. (2.18))

$$\omega_{\text{ch},n}(\varepsilon_{\perp}) = \frac{2\gamma^2 \Omega_{\text{ch}}(\varepsilon_{\perp})}{1 + \gamma^2 \theta^2 + K_{\text{ch}}^2(\varepsilon_{\perp})/2} n, \quad n = 1, 2, 3, \dots \quad (2.46)$$

Here, $\Omega_{\text{ch}} = 2\pi/T_{\text{ch}}$ is the frequency of channeling oscillations and $K_{\text{ch}} = \sqrt{2\gamma^2 \langle v_{\perp}^2 \rangle}$ is the undulator parameter related to them (see Appendix B). Within the framework of continuous potential approximation, these quantities are dependent on the magnitude of the transverse energy $\varepsilon_{\perp}$ which, in turn, determines the amplitude of oscillations. The only exception is the case of harmonic continuous interplanar potential, $U \propto \rho^2$ (with ρ being the distance from the midplane), for $\Omega_{\text{ch}} = \sqrt{U''(0)}/m\gamma$ is independent on the amplitude.

It was mentioned above (see also Fig. 2.9), that harmonic approximation is adequate for the positron channeling whereas for the electron the interplanar potential is strongly anharmonic. Hence, the variation of $\Omega_{\text{ch}}(\varepsilon_{\perp})$ as well as of $\omega_{\text{ch},n}(\varepsilon_{\perp})$ with $\varepsilon_{\perp}$ will be more pronounced for an electron than for a positron. As a result, the spectrum of ChR, which one calculates by averaging over channeling trajectories with various $\varepsilon_{\perp}$ from 0 up to U_0 , will be more undulator-like for a positron but much less peaked for an electron.

This qualitative statement is illustrated by Fig. 2.16, which was plotted by digitalizing the data from Fig. 1 in [284]. Two graphs in the figure represent the dependences of the spectral intensity of ChR scaled by that of incoherent bremsstrahlung (within the Bethe-Heitler approximation) on the emitted photon energy for 6.7 GeV positron (left graph) and electron (right graph) channeling in the planar channel Si(110). Open circles stand for the experimental data, the curves represent the results of calculations. More details on the experimental setup and on the used theoretical framework can be found in [39].

It is clearly seen that in the positron case the emission spectrum resembles that of the undulator radiation with $K^2 < 1$ (see Fig. 2.5 (left)).⁹ The first peak in Fig. 2.16 (left) corresponds to the emission in the harmonic with $n = 1$, and the second, less accented one, is due to the emission into the second harmonic.

In the case of electron channeling the undulator effect is smeared completely due to strong anharmonicity of the interplanar potential.

Hence, in contrast to coherent BrS, where electron and positron radiative spectra are very close, the difference in the interplanar potentials for the two types of light projectiles results in noticeable difference in the spectra of ChR.

However, despite close similarity of the positron ChR and UR spectra integrated over the emission angles, there are discrepancies in the spectra-angular distributions. Namely, for any value of the emission angle θ where will be sizable difference in the bandwidth $\Delta\omega_n/\omega_n$ of the peaks. For the ideal undulator $\Delta\omega_n/\omega_n = 1/nN$ (see 2.20) is the natural width determined by the finite size of the undulator ($\Delta\omega_n \rightarrow 0$ as $N \rightarrow \infty$). Although this mechanism adds to broadening the lines of ChR, it is not dominant. Main contribution to the line width of ChR is due to the deviation of the positron interplanar potential from the harmonic shape [96].

To estimate $\Delta\omega_{\text{ch},n}$ due to the anharmonic effects, in Fig. 2.17 we present the dependences of the first harmonic of ChR $\omega_{\text{ch},1}(\varepsilon_{\perp})$ in the forward direction (see 2.46 with $n = 1$ and $\theta = 0$) scaled by its value at $\varepsilon_{\perp} \rightarrow 0$, $\omega_{\text{ch},1}(0)$, i.e., in vicinity of the potential minimum where the parabolic approximation is fully applicable. The

⁹ Using (B.5) one estimates the (average) value of the squared undulator parameter as $\langle K_{\text{ch}}^2 \rangle \approx 0.4$.

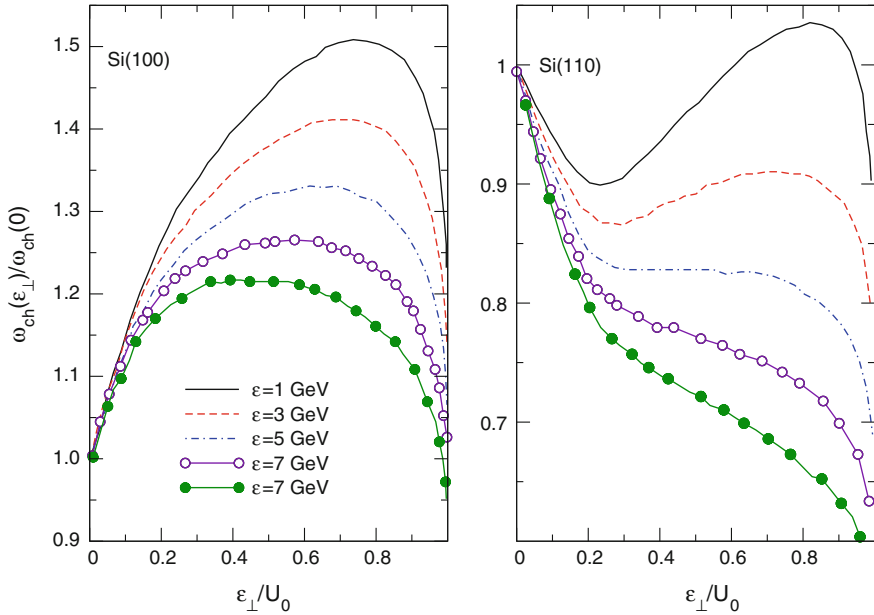


Fig. 2.17 Frequency $\omega_{\text{ch},1}(\varepsilon_{\perp})$ of the first harmonic of ChR in the forward direction (see 2.46) as a function of the energy $\varepsilon_{\perp}$ of the transverse motion of a positron channeling in Si(100) (left panel) and Si(110) (right panel) planar channels. The vertical axis $\omega_{\text{ch},1}(\varepsilon_{\perp})$ scaled by its value at $\varepsilon_{\perp} \rightarrow 0$; the horizontal axis $\varepsilon_{\perp}$ scaled by the depth U_0 of the interplanar potential well. Different curves correspond to the indicated values of the projectile energy ε . All dependences were obtained by digitalizing graphs (a) and (b) in Fig. 14 from [39]

figure was plotted by digitalizing the data from Fig. 14a and b in [39]. The curves, calculated for different energies of projectile positron (as indicated on the left graph), correspond to the channeling in Si(100) (left graph) and Si(110) (right graph). The interplanar potential was treated in the Doyle-Turner approximation [91] (in [39] similar curves were plotted using the Molière potential).

It is seen, that the ratio $\omega_{\text{ch},1}(\varepsilon_{\perp})/\omega_{\text{ch},1}(0)$ varies from 10 to 50 % for $\varepsilon_{\perp}$ within the interval of allowed values, from 0 up to U_0 . This variation is the leading mechanism of broadening the lines of ChR.

To conclude this section we mention that various aspects of radiation formed in crystals bent with the constant curvature radius R were discussed in [21, 22, 51, 137, 255, 272, 273] although with various degree of detail.

The motion of a channeling particle in a bent crystal contains two components: the channeling motion itself and translation along the centerline of the bent channel, see Fig. 2.13. The latter motion gives rise to the synchrotron-type radiation, i.e. the one, which is emitted by a charge moving along a circle (or, its part) under the action of the magnetic field. Therefore, the total spectrum of radiation formed by an ultra-relativistic projectile in bent crystal contains the features of ChR and those of synchrotron radiation. It was shown in [51], that in the case of planar channeling the crystal bending noticeably affects the spectrum only if the condition $\Theta_c \gamma \leq 1$ is met

(Θ_c is the critical angle in the bent channel). In this case, the total spectrum preserves the features of ChR if $L_c \gg \lambda$ (here, $L_c = R/\gamma$ is the formation (coherence) length of the radiation emitted from an arc of the circle of the radius R), but becomes of the quasi-synchrotron type in the opposite limit [51, 271]. The peculiarities which appear in spectral and spectra-angular distributions of the emitted radiation due to the interference of the two mechanisms of radiation were analyzed in [21, 22, 255, 272, 273] by analytical and numerical means.

Channeling and Radiation in Periodically Bent Crystals

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