

2. Effects of Uncertainty on the Policy Design

2.1. From Uncertainty to Uncertainty

The previous chapter has elaborated different accounts on the discrimination of risk and probability on the one hand and uncertainty on the other. It has been shown that various types and definitions of what is loosely called ‘uncertainty’ exist. Especially the notions of Keynes, Knight, Davidson, Shackle, and Ellsberg have been taken into account. Despite several linguistic differences all authors share a very similar tenor, at least regarding their definition of uncertainty. The major part of literature defines uncertainty with respect to the notion and definition of Knight (1921). Accordingly, situations where any kind of probability relation can be formed and (all) the future events can be entitled are situations of risk. In these situations *a priori* as well as *empirical* statements can be given. These situations can be encompassed by tabbing instances further and further into more or less homogeneous groups (Knight 1921).¹

Yet, in most decision problems neither of these situations occur. Hence, there is no chance of achieving a probability judgment. For such situations only an *estimate* can be given. This might be due to several reasons. To Knight (1921) the two main reasons why probability relations can not be given are due to measurement problems and due to the fact that some events are so unique that they are not even held to be contingent before they actually occur. This definition of uncertainty, whereby it is not possible to give a well defined probability distribution, is supported by various other authors, especially Keynes (1921).²

Despite the findings of Knight (1921) and other to cope with uncertainty, modern macroeconomics utilize well defined probability distributions, which are characterized by a known mean as well as a known variance. The assumption of agents who behave rational and live in a world of calculable risk “is by now a bedrock assumption in the social sciences”

¹ If, however, only the possible future states are known and the probability relation related to any specific state is only theoretically given a situation of *ambiguity* is prevailing rather than risk (Ellsberg 1961).

² Albeit the reasoning might differ to some extent as shown before.

(Nelson and Katzenstein 2011:10). Any modern treatment which incorporates uncertainty in fact only claims to deal with uncertainty, but *de facto* uncertainty is replaced merely by a risk analysis and uncertainty has been more or less relegated to the margin of the economic discipline.

An explanation for this development is hard to tell, although, a key role plays the decision theory of the late forties and early fifties, which is strongly connected with the works of von Neumann and Morgenstern (1944) and Savage (1954). While the former have introduced the concept of expected utility with the help of lottery bets, the latter has refined expected utility theory into a theory of *subjective* expected utility, which presumes that people always act ‘as if’ they have a well defined probability distribution in mind. This led to a subsequent warping of uncertainty in the notion of Knight (1921) and Keynes (1921) by most economists (Camerer and Weber 1992, Nelson and Katzenstein 2011).

On the other hand, Post-Keynesian literature accuses Tobin (1958) and the following rational expectation literature for the issue of a marginalization of uncertainty in (economic) thinking (Weisman 1984, Rosser 2001). Tobin (1958) is said to have “hijacked” (Rosser 2001:545) the concept of uncertainty developed by Keynes (1921), and has turned it into a concept of risk by equalizing risk and uncertainty. Thereby, Tobin has paved the way for the rational expectation literature, whereby people act as if their subjective probability distribution equals the objective distribution. This simplification, however, actually rules out uncertainty in the notion of Knight, vividly depicted in Table 1.1 of the previous chapter (Lucas 1977). In fact, what is not covered by rational expectation models is uncertainty arising out of the non-stationarity of the world, which includes situations or events which are not held to be contingent – like the occurrence of the *black swan* of Taleb (2008) (Davidson 2010). Knight’s and Keynes’s 1921 concept of uncertainty permits this non-stationarity (Meltzer 1982). These situations, the *unknowables*, however, are different from the *unknown* – which are both said to be uncertainty situations, in common literature.

No matter who to blame for the intermixture of risk and uncertainty, from a practical point of view uncertainty is simply ruled out and replaced by probability statements, because it is impossible to work with situations one could even not imagine. This practical problem is not objected by Keynes, it is rather confirmed by acknowledging that “[n]evertheless, the necessity for action and for decision compels us as practical men to do our best to overlook this awkward fact and to behave exactly as we should if we had behind us a good Benthamite calculation [...] multiplied by its appropriate probability, waiting to be summed” (Keynes 1937:214). Also, on the other side, the rational expectation theory indeed acknowledges the uncertainty concept of Knight (1921), but mentions

that “[i]n cases of uncertainty, economic reasoning will be of no value”(Lucas 1977:15). Hence, both protagonists of thought appreciate the difference between probability and uncertainty, but also acknowledge that in daily business the concept of uncertainty in the spirit of Knight (1921) and Keynes (1921) can hardly be applied.

In what follows, I give a brief overview of possible sources, forms, and consequences of uncertainty. Accordingly, some aspects are examined exemplary to show the effects caused by uncertainty. For the mathematical part of this work I follow the established linguistic usage and speak of uncertainty, although in fact risk is assumed.

The variety of possible sources and forms of uncertainty is large. The most trivial case would be just an exogenous shock which hits the economic system, and thus, causes variables to deviate from their desired values. Those shocks are unforeseen and may arise due to different reasons which make them even harder to predict. Additionally, it is possible for them to unfold only during the current period, but have originated long before. This again complicates an identification and thus a proper handling.

Apart from this, economists have to predict the future development of key macro variables. Essential for a good forecast is the proper evaluation of the current state. Yet, this could cause another problem. Values and data of macro variables, for example, GDP, inflation, unemployment, or capacity utilization, are in most cases only available with a considerable time lag and are revised several times later on. During this lag, policy must rely on imprecise estimates of those variables. Moreover, some variables such as potential output are not observable at all, likewise trend output or the real interest rate. These synthetic variables need to be estimated or inferred by variables which are themselves subject of noise. Against this background, policy rules such as the famous Taylor rule are particularly vulnerable due to the fact that they may contain uncertain variables like the output gap as arguments. Both components of the output gap, the actual output and the potential output, are to a high degree uncertain, especially at the moment they are needed the most. As a result, policy settings which have been appropriate at one point in time can be inappropriate given the new, revised information set.

Yet, even if there are no exogenous disturbances and the state of the economy would be known perfectly, there would be problems inferring from one point in time to the other, due to the fact of an only partly known transmission channel. Generally speaking, the effect of one parameter on to the other is uncertain and may vary over time. Reasons for this can be found in the fact that different economic situations produce different transmission behaviors. For example, the process of financial intermediation has changed the transmission of policy signals significantly. This again bears the possibility of a formerly optimal response to be no longer appropriate due to changes in the functional chain. Closely related to the issue of a changing transmission channel is that one can not be

sure about the reaction of private agents or the ‘financial market’ in response to particular policy actions. It might be that some action undertaken by the central bank could itself change relations, which have been assumed to be constant over time. Especially the formation and change of expectations are at the core of this kind of uncertainty.

Finally, if all the information about the impact or size of the transmission parameters would be known, the right data basis is employed, and it is accounted for unforeseen, exogenous shocks, one could even not be sure about the correctness of the assumed underlying model structure. Different schools of thought produce different models and approaches, some of them competing against each other. Hence, without shocks, valid data, and the correct estimation of parameters, there would still be no guarantee of whether the functional form of the workhorse model is correct or not.

This walk through a set of possible sources of uncertainty has revealed the most common classification of uncertainty. The first being *additive* uncertainty, relying on exogenous shocks that hit the system. The second is called *data* uncertainty, capturing measurement as well as estimation errors. *Multiplicative* or *parametric* uncertainty is prevailing if the intensity of the transmission process, that is, the effect of one variable onto the other can not be said with certainty. Thereby, it is often distinguished between a policy which is uncertain about the structure she is implemented into or about the effect she has on the structure, the latter being also called *strategic* uncertainty. This could be the case, for example, if reactions of private agents are incorporated into the decision of whether tightening or easing a respective policy stance. Extending parametric uncertainty to uncertainty covering not only the values of parameters, but also the functional form of models leads to *model* uncertainty. While model uncertainty can be simply understood as another type of uncertainty, it could also be used as a superior type, covering all other forms of uncertainty as sub-forms.

Although this classification seems to be common, see, e.g., Blinder (1998), Dow (2004) or Spahn (2009), a more comprehensive classification distinguishes only between *data* and *model* uncertainty, the former covering ‘classical’ concerns about the availability of data and the problem of data revisions, the latter covering general model, as well as additive or shock, and multiplicative uncertainty concerning the transmission process (see, e.g., Bundesbank 2004, González-Páramo 2006). Another categorization could be between uncertainty about the state, the structure of the economy, and strategic uncertainty. Thereby, uncertainty about the ‘state’ is defined as covering data and additive uncertainty. Uncertainty about the structure of the economy covers parameter and model uncertainty. Strategic uncertainty deals with interactions or expectations of market participants (Issing 2002). Apart from these different arrangements, in general one can identify four main

categories of uncertainty, namely additive, data, parameter, and model uncertainty. This is the categorization used in this work.³

Yet, knowing about the existence of uncertainty does not mandatorily tell the respective policy maker how to react correctly. Although it is accepted that under a linear framework with a quadratic loss function *certainty equivalence* prevails, thus the policy maker should act ‘as if’ everything would be known with certainty, the advice might not be that clear for parametric uncertainty. If uncertainty about the transmission of monetary signals prevails it could be that only a relatively small reaction is needed in order to change, for example, expectations about the future path of inflation. The more caution stance has found its expression in the literature as the *Brainard principle*, with respect to the seminal work of Brainard (1967). It suggests that under parametric uncertainty the monetary authority should act less aggressive than she would under certainty. However, this concept has been criticized, for example, by Söderström (2002) who singles out that in some constellations it could be superior to react more aggressive than being gradual. According to Söderström (2002) it is crucial whether there is uncertainty about the impact or persistence parameters of the model.

In addition, to induce the intended reaction it could be necessary to give a rather strong signal to the market by cutting rates more drastic than under certainty (Spahn 2009). Acting too cautious could lead to a situation where the central bank runs into the risk of what is called ‘falling behind the curve’ (Goodhart 1999). In this case, if, e.g., the economy is picking up, an only gradual policy stance could boost price developments, calling for an even stronger policy reaction in the future with all its consequences on investment and employment (González-Páramo 2006).

Orphanides (2003a) shows how the presence of noise may lead into exaggerations when setting the interest rate in accordance with mismeasured data. Thus, when faced with data uncertainty it could be useful to assign less weight on uncertain variables – if they can be identified at all. This implies if the central bank follows a Taylor rule with output-gap and inflation-gap as arguments, and output data is noisy, she should assign less weight on the output-gap and respectively more on the inflation-gap in order to reduce excess volatility. In extreme, she should not account for measured output deviations at all (Orphanides 2003a).

With regard to model uncertainty there has been a development onto two approaches. Hansen and Sargent (see, e.g. 2001, 2003b, 2008) have transferred the problem of model uncertainty into the robust control literature. Given a true but unknown model the pur-

³ This categorization excludes strategic uncertainty as a single source and rather covers this issue under the aspect of model or parameter uncertainty. This seems to be reasonable, as changes in the behavior or expectations of agents in fact change the behavior of the transmission channel, which is covered by the analysis of the already mentioned sources of uncertainty.

pose is to find a strategy which performs best given the worst conceivable consequences of different models surrounding the reference model of the central bank. In this setting they construct a game theoretic context with an evil agent playing against the central bank. The advantage of this type of modeling lies in the possibility to account for unquantifiable Knightian uncertainty, although there must be some agreement about the boundaries of the different models (Dow 2004, Dennis 2005).

Alternatively one could choose between a range of models, not so much focusing on one specific model. If so, one is able to calculate the ‘cost’ of a strategy which is made on the basis of a specific model, whilst the prevailing correct model is actually a different one. To work with both alternatives it is necessary to constrain the model somehow. In the first case, it is the budget of the malevolent player, in the second case, its the range of models being compared, which limits the analysis (Dow 2004).

A second approach to model uncertainty follows Bayesian principles, and targets a policy rule that works well over a range of models with different set-ups. This model averaging approach was mainly brought forward due to the impossibility to come to a compromise regarding the best strategy in dealing with model uncertainty. Within this category is also the idea of implementing possible switches between a set of models. The monetary authority must then calculate her optimal response with respect to a possible regime switch. However, both approaches have some strong limitations. Most notably, in either case the set of alternative models must be fully specified and probabilities need to be assigned, which precludes Knightian uncertainty (Issing 2002, Dennis 2005, Svensson and Williams 2005, Wagner 2007, Blake and Zampolli 2011).

Table 2.1 is taken from Dow (2004). It lists four different kinds of uncertainty, summarizing the above mentioned classification. Actually, only three *sources* are listed, because the latter two are both attached to model uncertainty, depending on whether Knightian uncertainty is precluded or not.

U	Type of uncertainty	Measure
U_1	additive	σ_ϵ^2 of equation
U_2	multiplicative	σ_ϵ^2 of parameter
U_{3a}	model	S_w
U_{3b}	model	unmeasurable

Table 2.1.: Classification of uncertainty

Uncertainty enters the model set-up either by the variance of the error term or the variance of the parameter values, whether additive or multiplicative uncertainty is prevailing. U_{3a} covers model uncertainty, where, although the model specification is unknown, the error terms distribution can be specified by S_w . For these three types, uncertainty can

be more or less measured and expressed formally.⁴ It is important to distinguish what is known from what is unknown. In Table 2.1 uncertainty can be identified and assigned to a specific source. If this is not the case, i.e., if there is no way what so ever to grasp what is known and what is not U_{3b} is prevailing. In this case uncertainty predominates in a Knightian sense. In reference to the previous chapters, in this situation it does not matter, whether this impossibility to assign probabilities is due to the real world or the inability of understanding, i.e., not matter if following Knight's aleatory or Keynes' epistemic notion of probability and uncertainty (Dow 2004).

The following sections take a closer look on the briefly mentioned forms of uncertainty and provide examples to underpin the argumentation.

2.2. Additive Uncertainty and Certainty Equivalence

The previous section has highlighted the seminal work of Brainard (1967). Together with Poole (1970) these two publications have notably shaped the discussion on monetary policy under uncertainty during the last century. Brainard (1967) demonstrates how *parametric* uncertainty changes the behavior of the monetary authority, and Poole (1970) analyzes, which instrument may serve best and delivers thus a smaller loss if *additive* uncertainty prevails, i.e., the possibility of an exogenous shock hitting the system.

Additive uncertainty summarizes all kinds of exogenous shocks, which could hit the economic world and affect either demand or supply factors or both. These stochastic shocks, which can not be anticipated, occur from time to time and prevent the system from reaching its desired level of the target variables. Thereby, supply side shocks – opposed to demand driven shocks – are of special concern due to the trade-off situation they create. Reducing the deviation of one target variable, say inflation, can only be done at the expense of missing the target of the output (Goodhart 1999, Spahn 2009).

In a taxonomy of potential sources of uncertainty additive uncertainty would be the most trivial form, especially when it comes to modeling. However, it offers some first valuable insights concerning properties and the general assessment of conducting policy under uncertainty, most notably the concept of *certainty equivalence*. Given the model set up additive uncertainty is introduced by adding a stochastic term with zero mean and positive but limited variance to the demand and/or supply equation. The variance of the added error term affects the dependent variables, hence, adding several stochastic terms,

⁴ I would suggest to add data uncertainty by defining it as the variance of a specific variable. Yet, following Issing (2002) or Swanson (2000), data uncertainty would be already covered by additive uncertainty. This treatment, however, is only in part true, as I will show in Section 2.4.

all with a zero mean, equals ‘adding variance’ to the actual dependent variable which is object of a minimization procedure (Dow 2004).

The model in the subsequent paragraphs relies on the work of Poole (1970). He studied the different consequences of the choice of the instrument – in his case money supply or interest rate policy – in response to an additive goods or money market shock. For a better understanding, first of all the concept of *certainty equivalence* is briefly introduced. Afterwards the classical model of Poole (1970) is examined graphically as well as analytically.

2.2.1. The Concept of Certainty Equivalence

If only additive shock uncertainty prevails, the central bank is told to ignore these uncertainties, that is, the decision rule should be equivalent to the certainty case. The central bank behaves ‘as if’ she knows everything with certainty, although she is actually confronted with uncertainty. However, this rule is only applicable to a very limited amount of models and forms of uncertainty. To see why this proposition holds suppose an environment which can be described by a linear relationship of the form

$$y = a_1 r + u. \quad (2.2.1)$$

In Equation (2.2.1) the target variable is given by y , which could be, for example, a desired level of output. The policy instrument is given by r , and $a_1 < 0$ determines the response of the target variable to a policy action. For the additive shock vector u it holds that the expected value equals zero and the variance is limited and positive, $u \sim N(0, \sigma_u^2)$. Yet, any value of $u > 0$ affects the value of the dependent variable with the distribution now given as $y \sim N(y_f, \sigma_u^2)$. The deterministic character of the target variable has changed into a stochastic, due to the stochastic noise term. Hence, no guarantee can be given that $y = y_f$, that is, output equals its target or optimal level y_f .

The variance of the target variable is determined by the variance of the additive shock term. No matter what action is made by the monetary authority it can not alter the distribution of y , but only shift it, so that $y \sim N(a_1 r, \sigma_u^2)$. The uncertainty about the target variable is in the system and it can not be affected by any central bank action. Indeed, the additive shock vector has added some variance to the dependent variable, however, it has not altered the mean of it. This explains why the best advice one could give is to behave ‘as if’ one would act under certainty.⁵

⁵ The picture changes, though, if no longer additive uncertainty is prevailing, but rather uncertainty about the transmission parameter a_1 . In this case the monetary authority would not only shift, but also alter the distribution of the target variable.

It is further supposed that the central bank follows a quadratic loss function, which penalizes output deviations from its target of the form

$$L = (y - y_f)^2. \quad (2.2.2)$$

Since the policy parameter has to be set before the shock term reveals, the central bank has to make an estimate about the target variable in order to achieve a minimal loss.⁶ The expected loss is given by

$$L = E \left[(y - y_f)^2 \right] \quad (2.2.3)$$

$$\begin{aligned} &= E \left[(a_1 r + u - y_f)^2 \right] \\ &= a_1^2 r^2 + E \left[u^2 \right] + y_f^2 + 2a_1 r E[u] - 2a_1 r y_f - 2y_f E[u] \\ &= a_1^2 r^2 + \sigma_u^2 + y_f^2 - 2a_1 r y_f. \end{aligned} \quad (2.2.4)$$

Equation (2.2.4) reveals that the loss of the monetary authority is determined *inter alia* by the variance of the shock vector, σ_u^2 .

Due to the fact that the expectation operator only applies to the shock term, but the parameter and the variables a_1 , r , and y_f are supposed to be known, the expression $E[L(r, y)]$ equals $L(r, E[y])$. In order to achieve the minimal loss defined in Equation (2.2.2), the derivative of (2.2.4) with respect to the policy instrument delivers the optimal setting of the policy parameter r^* according to

$$\begin{aligned} \frac{\partial L}{\partial r} &= 2a_1^2 r - 2a_1 y_f \stackrel{!}{=} 0 \\ r^* &= \frac{y_f}{a_1}. \end{aligned} \quad (2.2.5)$$

Equation (2.2.5) shows that the decision rule is independent of what so ever the variance of the disturbance vector might be, as long as its zero mean remains, the constraint is linear, and the loss function is quadratic. This is known as the *certainty equivalence* principle, a concept ascribed to the work of Tinbergen (1952), Simon (1956), and Theil (1957). It states that, although the objective function is altered in its value by the additive disturbance vector, or rather its (co-) variance, the policy maker can ignore this uncertainty and go on in setting his policy ‘as if’ everything is known with certainty. Hence, to proceed the uncertain additive variable is merely replaced by its expected value (Simon 1956, Ljungqvist and Sargent 2004).⁷

⁶ With a time subscript the above system must be written as $y_{t+1} = a_1 r_t + u_{t+1}$ to highlight the chronological order.

⁷ Taking expected instead of true values indicates already a related issue which is discussed later in this work but should be mentioned already at this point. The concept of certainty equivalence also involves

2.2.2. Additive Uncertainty: The Poole-Model

The concept of certainty equivalence has revealed that additive uncertainty does not affect the policy reaction function of the monetary authority, no matter how large the variance of the shock might be. The conclusion of this finding is that a central bank does not have to care about additive uncertainty, due to the fact that she can not actively reduce the impact of uncertainty on the target variable. However, Poole (1970) shows that if the variance of the target variable is included into the loss function policy action must be set very well under consideration of the noise term, even if it is additive. The underlying question of Poole (1970) is whether the monetary authority should act through interest rate changes or money supply changes. Depending on the situation one or the other instrument might be superior.

Interestingly enough, from today's point of view and through the experiences gathered throughout the recent financial turmoil, Poole (1970) picks the interest rate as a representative for overall "money market conditions" (Poole 1970:199) and mentions that the conditions also cover "the level of free reserves in the banking system, the rate of growth of bank credit [...], or the overall "tone" of the money market" (Poole 1970:199). However, the interest rate is selected as being the best single variable to represent those conditions in an analytical analysis. This is due to the fact that Poole was not so much interested in the issue of uncertainty *per se*, but rather in the question of controlling quantities vs. controlling rates in conducting monetary policy. His primary goal is not to describe how central bank behavior changes when confronted with uncertainty – as it is the case, e.g., in Brainard (1967) – but rather to find reasons in favor or against different strategies in monetary policy. Nevertheless, his 1970 work is referred to as one of the major works in the field of monetary policy under uncertainty. Moreover, some authors, see, e.g., Collard and Dellas (2005), have translated the original model of Poole into a New Keynesian framework to show that the basic conclusions continue to hold under certain assumptions (Schellekens 2000).

To show why a consideration of the policy instrument matters the previous equations are extended and complemented by a money market equation. In the nomenclature of

the so-called *separation principle*. If the true value of any variable is hidden or the measurement is noisy, accordingly, the estimation problem is separated from the optimization problem. Hence, in a first step the central bank forms the best possible estimate of the unobserved variable and in a second step she optimizes with this estimate as if the values are certain. Forecasting and optimization need to be done separately and are thus independent from each other. This leads to the situation that the response of the optimization problem is independent of the degree of completeness of information, i.e., the variance as long as the best estimate is utilized. If this procedure is adopted to the problem stated here, the best estimate one could make for the value of the additive shock is zero. Hence, any optimal policy should go on by treating the exogenous shock as not existent when optimizing (Svensson and Woodford 2003b).

Poole (1970) the goods and money market equation, as well as the loss function are given respectively by

$$y = a_0 + a_1 r + u, \quad (2.2.6)$$

$$m = b_0 + b_1 y + b_2 r + v, \quad (2.2.7)$$

$$L = E (y - y_f)^2. \quad (2.2.8)$$

It holds that $a_1 < 0$, $b_1 > 0$, $b_2 < 0$. Hence output, y , depends negative from the interest rate, r . The money stock level, m , depends positive on the output and negative on the interest rate. The goods and money shock terms, u and v , exhibit zero mean and a positive variance. In accordance with the previous equations the target output is denoted by the subscript f , whereas the instrument of choice is denoted by the asterisk superscript.

Non-Stochastic Case

The aim of Poole (1970) is to show why identifying the shock properly matters for the right choice of the policy instrument, and thus, for the size of the realized loss. Yet, before the situation of uncertainty is considered, the non-stochastic case is taken into account. To minimize her loss function, which constitutes itself as deviations from the target output level, the central bank is equipped with two different instruments, namely the interest rate and the money supply.

(1) First, it is assumed that the interest rate is picked as the instrument of choice, i.e., $r = r^*$. Output is thus determined via the goods market equation as

$$y_{r^*} = a_0 + a_1 r^*. \quad (2.2.9)$$

The money stock which is demanded to finance this level of output, given r^* , is determined by the money market equation as

$$m = b_0 + b_1 y + b_2 r^* \quad (2.2.10)$$

$$\begin{aligned} &= b_0 + b_1 (a_0 + a_1 r^*) + b_2 r^* \\ &= b_0 + b_1 a_0 + (b_1 a_1 + b_2) r^*. \end{aligned} \quad (2.2.11)$$

For this deterministic setting one would simply move along the goods market curve by varying the interest rate until the desired level of output, y_f , is met. Money stock and output evolve endogenously in order to fulfill the optimal interest rate level, which is the only exogenous variable in this setup. Hence, the money market curve becomes horizontal.

(2) On the other hand, the instrument of choice could be the money supply or money stock change, $m = m^*$. The interest rate is determined by m^* according to

$$m^* = b_0 + b_1 y + b_2 r \quad (2.2.12)$$

$$\begin{aligned} &= b_0 + b_1 a_0 + (b_1 a_1 + b_2) r \\ r &= (a_1 b_1 + b_2)^{-1} (m^* - b_0 - a_0 b_1). \end{aligned} \quad (2.2.13)$$

The corresponding output level can be found by combining Equation (2.2.6) with the resulting interest rate of Equation (2.2.13) to get

$$\begin{aligned} y_{m*} &= a_0 + a_1 r \\ &= (a_1 b_1 + b_2)^{-1} [a_0 b_2 + a_1 (m^* - b_0)]. \end{aligned} \quad (2.2.14)$$

If the instrument of choice is the money supply, one would move the money market curve, which is no longer a horizontal, until y_f is met. Money supply is exogenous, whereas output and interest rate evolve endogenously.

For convenience the relevant equations for an interest rate and money supply policy are respectively

$$y_{r*} = a_0 + a_1 r^*, \quad (2.2.15)$$

$$m = b_0 + b_1 a_0 + (b_1 a_1 + b_2) r^*, \quad (2.2.16)$$

$$y_{m*} = (a_1 b_1 + b_2)^{-1} [a_0 b_2 + a_1 (m^* - b_0)], \quad (2.2.17)$$

$$r = (a_1 b_1 + b_2)^{-1} (m^* - b_0 - a_0 b_1). \quad (2.2.18)$$

The optimal values of the interest rate and the money stock for a given level of output can be found by the reduced Equations (2.2.15), (2.2.17), respectively as

$$r^* = a_1^{-1} (y_f - a_0), \quad (2.2.19)$$

$$m^* = a_1^{-1} [y_f (a_1 b_1 + b_2) - a_0 b_2 - a_1 b_0]. \quad (2.2.20)$$

In a deterministic world where all additive shocks equal zero, it would not matter which instrument is chosen to achieve the desired level of output. If the interest rate is the instrument of choice it is easy to see that one would just move along the goods market curve in order to meet the target output. The money supply would evolve endogenously in order to fulfill the interest rate target. Alternatively, if money supply is the chosen instrument, one just ‘moves’ the money market curve along the goods market curve, until

the desired level of output is achieved. Given the goods market curve, the interest rate compatible with the target output level, y_f , would merely be a residual.

Because r^* is the result if money supply is the instrument of choice and money demand will be m^* if the interest rate is the chosen instrument, in this deterministic setting the choice of instrument would be “a matter of convenience, preference, or prejudice, but not of substance” (Poole 1970:204).

Due to the fact that the target output y_f is achieved anyway, the loss function would in both cases deliver a value of zero. Hence, even under loss aspects, without uncertainty the chosen policy – whether the interest rate or the money supply is the leading instrument – is equivalent (Poole 1970, Froyen and Guender 2007).

Stochastic Case

The assumption of certainty is relaxed by permitting stochastic shocks to hit the system. Because of certainty equivalence, the minimization problem should have the same solution under certainty, as well as under additive uncertainty. If it holds that $E(y) = y_f = y$, minimizing the loss function is equivalent to minimizing the variance of the output, $Var(y)$. By permitting additive uncertainty to enter the equation setup the dependent variable output becomes a random variable as well, due to the newly shock component. However, once the instrument has been chosen the situation should equal one of certainty and is thus certainty equivalent.

(1) Again, first the interest rate is chosen to be the instrument of choice. The loss function

$$L_{r*} = Var(y), \quad (2.2.21)$$

is equipped with Equation (2.2.15), but now complemented with the random disturbance vector u ,

$$y_{r*} = a_0 + a_1 r^* + u. \quad (2.2.22)$$

The variance of output is given by

$$Var(y) = E[(y - E[y])(y - E[y])] \quad (2.2.23)$$

$$= E[(a_0 + a_1 r^* + u - a_0 + a_1 r^*)^2] \quad (2.2.24)$$

$$L_{r*} = Var(y) = \sigma_u^2 \quad (2.2.25)$$

taking into account that $E[u] = E[v] = 0$, $E[u^2] = \sigma_u^2$, and $E[v^2] = \sigma_v^2$.

Equation (2.2.25) shows if the interest rate is set by the monetary authority and money evolves endogenously only goods market shocks enter the loss function. For a money market shock the value of the loss function remains zero.

(2) If the money stock is the instrument of choice, the corresponding loss function L_{m*} is derived by taking the reduced form Equation (2.2.17), which displays the optimal output level with money stock as the instrument, together with Equation (2.2.20), which equals the reaction function in this setting and therefore is needed in order to find the appropriate money supply to achieve y_f .

$$y_m = \underbrace{(a_1 b_1 + b_2)^{-1} [a_o b_2 + a_1 (m^* - b_0)]}_{\text{brace}} + b_2 u - a_1 v \quad (2.2.26)$$

$$y_m = y_f + (a_1 b_1 + b_2)^{-1} (b_2 u - a_1 v) \quad (2.2.27)$$

The first part of Equation (2.2.26), covered by the brace, equals Equation (2.2.17), which delivers the optimal output level if money supply is the instrument. If the money supply is the chosen instrument, the variance of the output is given by

$$Var(y) = E[(y - E[y])(y - E[y])] \quad (2.2.28)$$

$$= E[(y_f + (a_1 b_1 + b_2)^{-1} (b_2 u - a_1 v) - y_f)^2]$$

$$= E[(a_1 b_1 + b_2)^{-1} (b_2 u - a_1 v)^2]$$

$$L_{m*} = Var(y) = (a_1 b_1 + b_2)^{-2} (a_1^2 \sigma_v^2 + b_2^2 \sigma_u^2). \quad (2.2.29)$$

Opposed to the previous finding a clear conclusion concerning the influence of uncertainty on the loss can not be drawn. The difference between L_{r*} and L_{m*} results from the fact that reaching the target level of output, y_f , with the interest rate as instrument only involves the goods market. Walking along the goods market curve is sufficient for an equilibrium. On the other hand, using the money stock as instrument would involve both sides of the model, the goods market as well as the money market (Poole 1970, Froyen and Guender 2007)

Comparing Loss

To find an answer to the question whether or not it is useful to steer the economy by money supply or via the interest rate two kinds of shocks (goods market and money market) are allowed to hit the system. They are compared in terms of losses they create.

For convenience, the respective loss functions under an ‘interest rate regime’ or ‘money supply regime’ are repeated here

$$L_{r*} = \sigma_u^2, \quad (2.2.30)$$

$$L_{m*} = (a_1 b_1 + b_2)^{-2} (a_1^2 \sigma_v^2 + b_2^2 \sigma_u^2). \quad (2.2.31)$$

For a goods market shock ($\sigma_u^2 > 0$) it holds that $L_{r*} > L_{m*}$, hence, the appropriate instrument should be the money supply, due to the smaller loss. On the other hand, for a money market shock ($\sigma_v^2 > 0, \sigma_u^2 = 0$) it holds that $L_{r*} < L_{m*}$, thus, in this case the interest rate as instrument is superior to the money supply.⁸ These findings can also be traced in Figure 2.1. On the left-hand side, a money market shock is displayed which shifts the money market curve to the left or right, depending whether this is a positive or negative shock. Setting the interest rate and letting the money supply evolve endogenously makes the money market curve horizontally and the loss equals zero. Instead, using the money supply as instrument creates a loss greater than zero. On the right-hand side a possible goods market shock is demonstrated, which shifts the goods market curve to the left or right depending on the sign of the shock. As can be seen clearly, a horizontal money market curve, caused by an endogenous money supply, creates a larger loss compared to the case using the money supply as an instrument and letting the interest rate fluctuate as it likes.

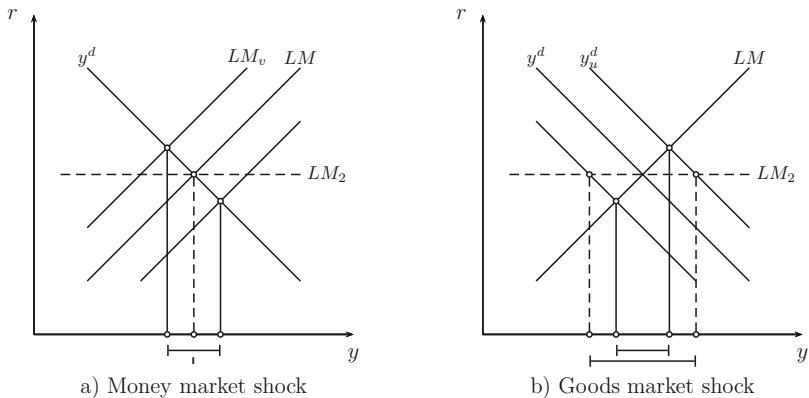


Figure 2.1.: Additive uncertainty in the model of Poole (1970)

⁸ The pronunciation of the relative loss difference depends of course also on the parameter values. The closer b_2 is to zero, the smaller is the difference of Equation (2.2.30) and (2.2.31).

The result is not surprising. Equation (2.2.30) shows that a positive money market shock does not even enter the loss function, in case the interest rate is the chosen instrument. This is due to the fact that an increase in money demand is completely accommodated by the central bank, so that output is not affected at all. The interest rate remains at its previous level. The money market curve is horizontal and the output is purely defined by the goods market. If, however, the money supply is being fixed, a rise in money demand would lift the interest rate, causing a decline in output and thus, causing a higher value of the loss function.

If on the other hand, a positive goods market shock occurs, output rises above its target level so that $y > y_f$. The rise in money demand will, due to its money supply constraint, force the interest rate to rise, causing a dampening effect on demand. For an endogenous money supply, that is, the interest rate is pinned down on a fixed level, this effect will be absent with the consequence of an even larger deviation of output from its desired level. The rise of the interest rates thus provides a stabilizing effect on the economy which induces a smaller loss (Poole 1970, Spahn 2009).

With this simple model, Poole demonstrates that, while the reaction functions (2.2.19) and (2.2.20) remain the same, whether uncertainty is prevailing or not, the value of the loss functions, Equation (2.2.30) and (2.2.31), vary as well. However, once the instrument is being fixed, *certainty equivalence* holds. But, although *certainty equivalence* prevails and the reaction function is not altered by the source and size of the shock, it could be utile to decide which instrument to choose in order to minimize the overall loss, given the relative importance of the random shock and the parameters of the model.

The choice of instrument, between the short-term interest rate and some monetary aggregate, is in fact a matter of discussion. As Bindseil (2004) mentions there is nearly no way of controlling monetary aggregates, even in a very narrow band of, say, base money, than using the interest rate. Accordingly, the monetary base is often rather seen as an intermediate target than a direct policy instrument. Controlling the monetary aggregate would thus always imply the control of the nominal short-term interest rate. Accounting for that, there is no operational choice in conducting monetary policy via rates or quantities. Thus, the question changes from whether reacting with some monetary aggregate or the interest rate, to the question of how strong and how fast a reaction should be conducted (Bindseil 2004).

2.3. Parametric Uncertainty: The Brainard Example

The most prominent form of uncertainty is labeled parametric or multiplicative uncertainty. It is strongly connected to the seminal work of Brainard (1967), who analyzed

the effectiveness of monetary policy in the presence of parametric uncertainty. His result, which has later been labeled *Brainard conservatism* (Blinder 1998), suggests that under parametric uncertainty the central bank must react less intense in response to a shock compared to the situation of certainty or certainty equivalence. Nevertheless, this result has been a matter of discussion during the years and it has been shown that the opposite reaction – to be more aggressive than under certainty – can also be supported (Craine 1979, Söderström 1999). An important feature of parametric uncertainty is the vaporization of *certainty equivalence*. While under additive uncertainty the policy rule is independent from the uncertainty added to the model, this no longer holds for uncertainty concerning the parameters of the model.

The original problem of Brainard (1967) can be illustrated as follows. It is assumed that the central bank is uncertain about the impact of her action concerning her target variable y_f (e.g., output or inflation). The target variable shall be controlled by an instrument variable, r . However, the impact from the control variable to the target variable is determined by some response or transmission coefficient, a . As before the model economy is hit by an exogenous shock, u . This relation is given as

$$y = ar + u. \quad (2.3.1)$$

For convenience, the nomenclature differs from Brainard (1967) and opposed to the original problem I presume the transmission variable and the shock being independent from each other for the rest of this work. In Equation (2.3.1) two distinct sources of uncertainty exist. First of all, the influence of the shock variable on output, which leads to the problem of a proper reaction of the policy instrument. This form of uncertainty has already been examined in the previous section. It has shown that due to certainty equivalence this poses no challenge to the monetary authority. The second source of uncertainty arises from the reaction of the monetary authority itself, and pertains its uncertain influence on the target variable.

It is assumed that the exogenous disturbance is represented by an i.i.d. error term with zero mean and constant, positive variance. The mean of the transmission parameter, however, is non-zero and said to be known to the central bank. The variance is constant and positive as well. The uncertainty surrounding the target variable must be a combination

of the uncertainty – expressed by the variance – of the transmission parameter and the exogenous shock vector. It is given by

$$\text{Var}(y) = \text{Var}(ar + u) \quad (2.3.2)$$

$$\begin{aligned} &= \text{Var}(ar) + \text{Var}(u) \\ \sigma_y^2 &= \sigma_a^2 r^2 + \sigma_u^2. \end{aligned} \quad (2.3.3)$$

Compared to the former case of additive uncertainty the variance of the target variable depends not only on the variance of the additive shock vector, but also on the variance of the transmission parameter. The influence of the latter, however, depends crucially on the size of the reaction parameter, which is under the control of the monetary authority. This already indicates that the monetary authority must take into account the uncertainty of the transmission parameter when calculating her optimal policy response. Certainty equivalence ceases to hold.

For further investigation it is assumed that the monetary authority wants to minimize a loss function, which penalizes positive as well as negative deviations from the target variable and is thus of the form

$$L = (y - y_f)^2. \quad (2.3.4)$$

The expected loss is attained by plugging Equation (2.3.1) into (2.3.4) to get

$$\begin{aligned} E[L] &= E[ar + u - y_f]^2 \\ &= E[a^2] r^2 + E[u^2] + y_f^2 - 2E[u] y_f - 2E[a] r y_f + 2E[a] r E[u] \\ &= E[a^2] r^2 + E[u^2] + y_f^2 - 2E[a] r y_f. \end{aligned} \quad (2.3.5)$$

The expectation operator only applies to the transmission parameter a and the shock vector u . Furthermore, as has been noted before for the mean and the variance of those variables it holds that $E[u] = 0$, $E[u^2] = \sigma_u^2$, and $E(a) = \hat{a}$, $E[a - \hat{a}]^2 = \sigma_a^2$. Making use of the computational formula for the variance one can rewrite $E[a^2] = \sigma_a^2 + (E[\hat{a}])^2$.

Taking into account Equation (2.3.3) the expected loss can be simplified to

$$\begin{aligned} E[L] &= (\sigma_a^2 + E[\hat{a}^2]) r^2 + \sigma_u^2 + y_f^2 - 2E[a] r y_f \\ &= \sigma_a^2 r^2 + \sigma_u^2 + y_f^2 - 2E[a] r y_f + E[\hat{a}^2] r^2 \\ &= \sigma_y^2 + y_f^2 - 2E[a] r y_f + E[\hat{a}^2] r^2. \end{aligned} \quad (2.3.6)$$

Hence, the loss is defined as

$$\begin{aligned} L &= \sigma_y^2 + (\hat{a}r - y_f)^2 \\ &= \sigma_a^2 r^2 + \sigma_u^2 + (\hat{a}r - y_f)^2, \end{aligned} \quad (2.3.7)$$

which is the sum of the variance of the target variable, plus the expected deviation from its target level. In other words, the expected value of the random target variable equals its variance plus the squared bias. Equation (2.3.7) shows how the policy instruments affects the variance as well as the deviation of the target variable. The minimization of both parts with only one instrument can only be done by taking into account some trade-off between those two parts (Martin 1999).

Differentiating Equation (2.3.7) delivers the optimal rule for the policy instrument under parametric uncertainty

$$r^* = \frac{\hat{a}y_f}{\sigma_a^2 + \hat{a}^2}. \quad (2.3.8)$$

With the emergence of uncertainty a smaller reaction of the interest rate compared to the certainty equivalence case is demanded. This more reluctant policy stance is called *conservatism*. Reformulating Equation (2.3.8) makes this conservatism even more clear. Replacing $\sigma_a/\hat{a}$ with ζ and define g as $1/(1 + \zeta^2)$ delivers

$$r^* = g \frac{1}{\hat{a}} y_f. \quad (2.3.9)$$

Equation (2.3.9) shows how the optimal setting of the policy parameter is affected by the size of uncertainty concerning the transmission, represented by σ_a^2 . The policy reaction is only a fraction of the certainty case, due to the fact that g is less than one. This becomes most obvious if Equation (2.3.9) is compared to the certainty (equivalence) reaction function

$$r^* = \frac{1}{a_1} y_f, \quad (2.3.10)$$

where the g is missing.

Growing uncertainty, indicated by a growing variance, calls for a less active policy stance, hence, a smaller reaction via r^* is demanded. For the extreme case of unlimited uncertainty, $\sigma_a^2 \rightarrow \infty$, r should be set to zero. For the case of no uncertainty concerning the response coefficient, i.e., $\sigma_a^2 = 0$, r equals the certainty equivalence case of Equation (2.3.10) as g approaches one (Brainard 1967, Martin 1999, Martin and Salmon 1999).

A more conservative policy in the presence of multiplicative uncertainty comes as no surprise. The expected error which is intended to be minimized consists of two sources. The first one, the deviation of the target level from its realized level, is minimized by the standard certainty equivalence reaction. The second source, the variance of the target variable, is minimized by a zero action policy. Thus, the optimal reaction must be somewhat in between these two policy rules. The actual reaction, however, depends on whether the policy is set rather with respect to the target breach or the variance concerns (Onatski 2000).

Furthermore, Martin (1999) and Martin and Salmon (1999) identify two additional consequences of uncertainty, which they label *gradualism* and *caution*. By gradualism, they define the smoothing of the response to a shock. Due to the fact that only a part of the shock is offset, each period of the target variable becomes autocorrelated and since the interest rate is set in response to this lasting deviation, it becomes autocorrelated as well.⁹ With a lower variance of the transmission parameter the degree of autocorrelation thus declines as well and gradualism disappears in the limit. It is absent if only additive uncertainty is prevailing (Martin and Salmon 1999).

Caution measures the magnitude of the total response, i.e., the cumulative interest rate response. Under certainty this is clearly $1/a$. The shock is completely offset in one period as it is costless to do so. Under parametric uncertainty, this response is smaller, due to the natural decay as time goes by. However, this does not hold for output and inflation. So in sum, under parametric uncertainty, the interest rate deviation from the desired – neutral – level is smaller, more phased in and the cumulative response is less than under certainty (Martin and Salmon 1999).

The example of Brainard (1967) shows how *certainty equivalence* ceases in the presence of parametric uncertainty. The policy rule is now affected by the size of the variance of the unknown parameter. Offsetting deviations of the target variable by large interest rate steps induces more variance. Changing the interest rate is not costless anymore, and thus, it is no longer optimal to completely offset a shock in one period as it is the case under certainty. On the other hand, the size of the disturbance vector is not taken into account in setting the policy instrument. This confirms the previous finding that additive disturbances, while they might affect the loss function, they do not affect the policy rule (Brainard 1967, Martin 1999).

The so far mentioned findings of this section gave birth to the *Brainard conservatism* and have been restated various times, underpinning the early findings of Brainard (1967). During the years there have been counter examples, which show that also a more aggressive

⁹ Martin and Salmon (1999) further distinguish between *nominal* and *real* interest rate responses/paths, although the conclusion is not significantly altered.

policy could be appropriate, especially when there is uncertainty about the *dynamics* of the economy rather than the *impact* of some policy action (see, e.g., Söderström 2002).

Beside technical reasons for a more or less aggressive policy reaction in the presence of uncertainty, compared to an environment of certainty equivalence, Caplin and Leahy (1996) highlight that a too cautious policy may indeed prove to be ineffectual to stimulate economic activity.

In the recessionary set up of Caplin and Leahy (1996) the policy maker faces the problem that she does not know, whether the intended interest rate cut is sufficient to stimulate the economy. Two possibilities emerge. In the first case, a too cautious policy reaction will pursue the ongoing recession and makes further interest rates cuts necessary. In the second — more aggressive — case, inflationary pressure will emerge. The task is thus to find an optimal policy in the sense that it should be as aggressive as needed but also as cautious as possible, considering that the central bank as well as the agents learn from the others action and reaction.

As an outcome of this game monetary policy authorities indeed should react more aggressive than actually needed for the intended outcome. It results from the fact that agents will know that any failure in stimulating the economy will be followed by further rate cuts, which gives them an incentive to wait and postpone action in the prospect of even better future conditions. Additionally, any policy failure will reduce the probability of a successful future policy as it reveals the prevailing bad condition of the economy.¹⁰

If, however, a more aggressive rate cut is considered, a situation which promotes economic activities can be achieved due to the fact that the agents of the economy expected this cut to be successfully and thus only of a temporary nature. This in turn creates a climate of urgency to invest and to benefit from the extraordinary financing conditions.

Never-the-less, despite these counter findings the *Brainard conservatism* represents still the most prominent guideline in practical policy making (Blinder 1998, Goodhart 1999).

Comparison between Additive and Parametric Uncertainty

To show the difference between additive, i.e., certainty equivalence, and parametric uncertainty, the following example is considered. No matter what kind of uncertainty prevails, the social loss is defined as

$$E[L] = (y - y_f)^2 \quad (2.3.11)$$

$$= \sigma_y^2 + (E[y] - y_f)^2 \quad (2.3.12)$$

¹⁰ This result can also be applied to other situations of uncertainty, such as fiscal or tax policy issues or even private selling and bargaining activities (Caplin and Leahy 1996).

with

$$\sigma_y^2 = \text{Var}(ar + u), \quad (2.3.13)$$

$$E[y] = E[ar + u]. \quad (2.3.14)$$

However, σ_y^2 and $E[y]$ differ, depending on which kind of uncertainty is assumed.¹¹ Under additive uncertainty the expected loss and the resulting optimal rate of interest are given by

$$E[L] = \sigma_u^2 + (ar + \hat{u} - y_f)^2, \quad (2.3.15)$$

$$r^* = \frac{y_f - \hat{u}}{a}. \quad (2.3.16)$$

Under parametric uncertainty the loss as well as the optimal interest rate are given by

$$E[L] = r^2 \sigma_a^2 + \sigma_u^2 + (\hat{a}r + \hat{u} - y_f)^2, \quad (2.3.17)$$

$$r^* = \frac{\hat{a}(y_f - \hat{u})}{\sigma_a^2 + \hat{a}^2}. \quad (2.3.18)$$

For the example the following parameter values are assumed, $y_f = 0$, $\hat{u} = 6$, $\sigma_u = 1$, $\hat{a} = -2$. Under additive uncertainty the variance of the transmission parameter is set zero, $\sigma_a = 0$. Yet, under parametric uncertainty it holds that $\sigma_a = 2$.

Thus, according to Equation (2.3.12), the social loss is defined with two components, the variance of the output and the expected deviation from the target output level. Figure 2.2 plots in the upper part the expected output deviation (see, Equation (2.3.14)) on the vertical axes, and the variance of the output (Equation (2.3.13)) on the horizontal axes.¹²

In both situations the starting point is given by $E[y] = 6$ and $\sigma_y = 1$, where no interest rate action is taken at all. Subsequently, the interest rate is raised in order to contain the loss caused by the additive shock. Taking into account Equation (2.3.13) and (2.3.14), it becomes obvious that under additive uncertainty the variance of output stays the same no matter the size of the interest rate. Thus changing the interest rate only affects the expected deviation from the target output. Hence, if additive shock terms are the only source of uncertainty, a rising interest rate is equivalent to moving down the dotted line. It shows that only the variance of the error term affects the value of the target variable. There is no trade off situation.

On the other hand, under parametric uncertainty a change in the interest rate affects the expected deviation of the target variable as well as the variance. A rise in the interest

¹¹ It is assumed that $\sigma_{au}^2 = 0$

¹² For plotting reasons actually the square root of the variance, the standard deviation, is taken.

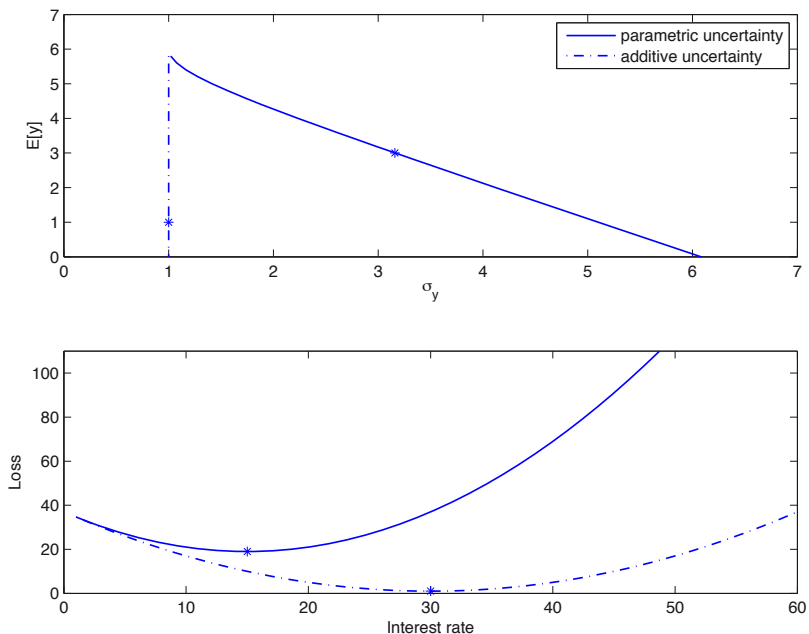


Figure 2.2.: Additive vs. parametric uncertainty

rate is equivalent to moving down the solid line. In Figure 2.2 a rising interest rate lowers the expected deviation of the target variable, but at the same time increases the social cost by a rising variance due to the parametric uncertainty.

The lower part of Figure 2.2 depicts the loss associated with both uncertainty situations. Again, the solid line represents the situation under parametric uncertainty, while the dotted line represents the situation under additive uncertainty only. The loss is calculated by Equation (2.3.15) and (2.3.17), respectively. Accordingly, the minimal loss under additive uncertainty is achieved by an interest rate of 3%, whilst under parametric uncertainty the minimal loss is achieved with an interest rate of only 1.5%. Both optimal situations are depicted by the asterisk. The interest rates levels which are associated with the optimal trade-off situations are also marked in the upper part of Figure 2.2 (Brainard 1967, Kähler and Pinkwart 2009).

This example offers insights on various aspects, which have already been suggested. Firstly, the so-called Brainard conservatism concept is confirmed. The optimal interest rate is lower under parametric uncertainty, compared to a situation with only additive

disturbances. Secondly, the upper part of Figure 2.2 shows that under parametric uncertainty, when it is utile for the central bank to react less aggressive, the central bank accepts a higher deviation from the the target level in order to keep the variance low. Thirdly, under parametric uncertainty the loss is always higher compared to a situation under additive uncertainty as long as the interest rate reaction is larger than zero.

2.4. Data Uncertainty: The Use of the Optimal Estimate

2.4.1. Data Revisions: Real-Time and Final Estimates

Most economic key indicators are released as a preliminary forecast, being revisited several times in the subsequent periods. Especially output data – no matter if actual output, or potential, or trend output – is susceptible to measurement errors. This comes as no surprise, due to the difficulties in the measurement and calculation of production and income. Koske and Pain (2008) find that not so much the wrong estimation of potential output causes data revisions of the output gap, but rather the estimation of actual data. Likewise Marcellino and Musso (2011) list potential sources of uncertainty for the estimation of the output gap. They especially highlight errors due to model uncertainty, parameter uncertainty, and parameter instability. Model uncertainty causes measurement errors due to the variety of alternative models resulting in significantly different estimates. Parametric uncertainty as well as instability causes wrong estimates due to the change of parameters over time. This is of special concern when different vintages are used to calculate, e.g., recursively the development of the output gap. However, this can not be an excuse as monetary policy must act, even though the working data might be wrong or no more than approximations to the true data, which are only available with a considerable delay.

Figure 2.3 plots data from OECD (2008) concerning the assessment of the output gap during the time 1991 until 2007.¹³ The upper panel plots German data, the lower panel US data. For both regions it can be seen that there is a significant difference between the first estimate (real-time data), the revision two years later, and the final data, which is defined as vintage at least three years after the first inquiry. The difference between the second year revision and the final data is mostly negligible. However, the difference between the real-time data and second estimate is considerable. For the US information on data revisions concerning inflation and output prior to 1991 can be found in Orphanides

¹³ The output gap is defined according to OECD (2013) as the difference between actual and potential GDP as a per cent of potential GDP.

(2003b). For the time period 1966 until 1994 he shows that the US output gap is constantly underestimated during the first estimation, from which he concludes that policy failures during the late sixties and seventies are *inter alia* owed to a wrong assessment of the output gap associated with the productivity slowdown (Orphanides 2003b).

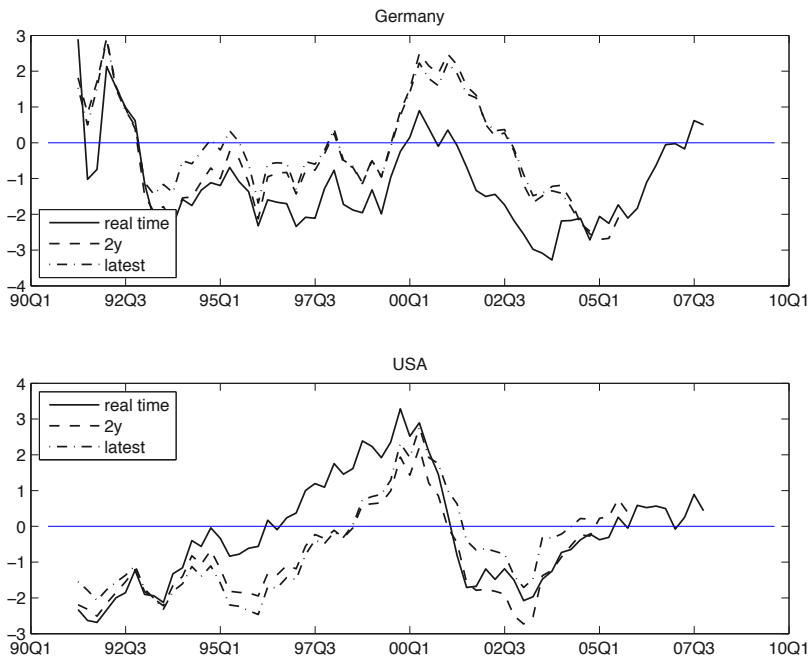


Figure 2.3.: Output gap estimates, Germany (top) and USA (bottom)

According to the OECD (2008) data set an underestimation of the output gap is also true for Germany, with respect to the observed period. The picture changes, however, for the US. From 1994 onwards the output gap is significantly overestimated before it switches back again around the year 2001. Figure 2.4 especially highlights the size and direction of data revisions in both regions. It shows the more or less persistent under- and overestimation of the output gap in Germany and the USA.

Goodfriend (2002) attest the Federal Reserve a successfully tightening of her monetary policy during the early nineties, which has followed an expansive stance in the mid-nineties. However, it could also be that the Fed was just very lucky. While she was reacting on real-time data, which shows a high output gap, in fact, she was only reacting

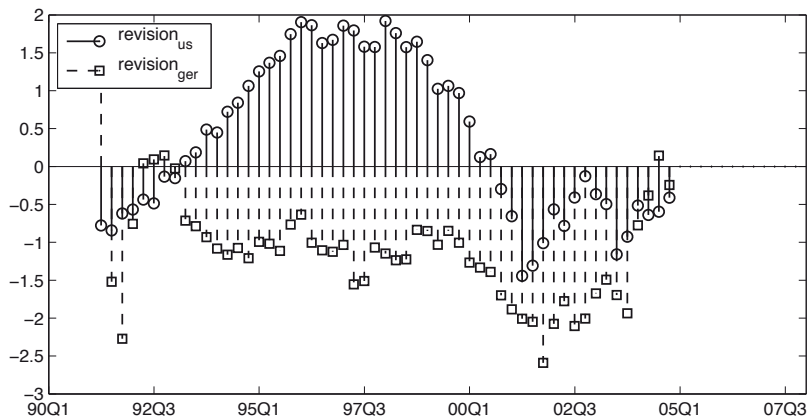


Figure 2.4.: Output gap revisions, Germany (top) and USA (bottom)

to a very low output gap according to the revised data. Hence, her policy was supported by a wrong estimation of the output gap. Against this background, the expansive policy of the Fed at the beginning of the millennium would be appropriate in the light of an underestimation of the output gap. Yet, these examples are highly speculative in many ways. For example, the Federal Reserve has a multidimensional target, and as such a high discretionary leeway. The output gap is just one of many factors that might influence the decision of the open market committee, but it again underlines the importance of data uncertainty and particularly its possible consequences.

The good news is, Tosetto (2008) shows that the correlation between preliminary estimates and further revision, indeed, exhibits a high value of at least 0.75, and Koske and Pain (2008) conclude that even preliminary estimates give a good picture of the business cycle. Only turning points pose a big challenge. During such times data revisions are more likely and also of greater extent, compared to other times (Orphanides and van Norden 1999, 2002). Figure 2.3 and 2.4 support this finding. For the years around 2001 the revision of the German data seems to be extra large. Yet, for the US data, the hypothesis can not be supported. The largest deviations of the sample period are in the mid-nineties. Although, other reasons might be responsible for this period.

For some periods it even seems like an improvement of the estimates, see, e.g., the period starting at the year 2004 in the US, has been realized. Yet, Chiu and Wieladek (2012) find for the time span 1991 until the end of 2005 for 15 OECD countries no evidence for an improvement of estimates, rather the contrary is true. Given the fact of a better

inclusion of contemporaneous information, and thus, less noisy forecasts the authors call difficulties in disentangling trend and cycle to account for the bad outcome (Chiu and Wieladek 2012).

2.4.2. Imperfect Observation vs. Signal Extraction

Dealing with a model which accounts for data uncertainty, the formulation of the problem becomes crucial. If the system is defined as a linear relationship of the form $y = a_1 r + u$ and the loss function penalizes positive and negative deviations from the target output y_f , the policy rule is given by

$$r^* = \frac{1}{a_1} y_f. \quad (2.4.1)$$

This is the standard certainty solution, which has been derived before. For this setting, even though there is uncertainty, certainty equivalence prevails, i.e., the policy rule is impervious of an additive shock.

If, however, data uncertainty enters the set-up, some pre-considerations need to be done. In a first scenario the monetary authority observes a level of output, and it is assumed that

$$y_t = y_{t|t} + \eta_t. \quad (2.4.2)$$

Thus, the true value of output, y_t , equals the observed value, $y_{t|t}$, plus a zero mean random variable, η_t . Hence, the true value of output is measured with noise. From Equation (2.4.2) it can be inferred that $E_t(y_t) = y_{t|t}$. So the real-time or observed value equals the expected value of output, which is equal to the best estimate one can make.

Using the method described above and substituting output by its best estimate one will quickly see that there is nearly no difference to the example of additive uncertainty or in other words, again certainty equivalence holds. Given the loss function which penalizes deviations from potential output, but now working with estimates instead of true values, would deliver

$$L = E \left[(a_1 r + u + \eta - y_f)^2 \right]. \quad (2.4.3)$$

Due to the zero mean of the disturbance vector and the (best) expectation of output, which equals the observed or real-time variable, the optimal interest rate is again defined as

$$r^* = \frac{1}{a_1} y_f, \quad (2.4.4)$$

which confirms the certainty equivalence proposition of the previous section. From this setting it becomes obvious why some authors call data uncertainty a form of additive uncertainty. If data uncertainty is merely understood as a somewhat sloppy observation of the current values the simple replacement of the actual variable by its best estimate, in fact, conveys the problem to a much easier one (see, for example, Swanson 2000).

Yet, this treatment of data uncertainty falls short of some important aspects. To say that data uncertainty implies certainty equivalence would be to hasty – at least with no further explanation. In the above calculation it has been implicitly assumed that the estimate of output is also the *best* estimate, hence, it is not biased.

The situation changes if instead of Equation (2.4.2) it is assumed that

$$y_{t|t} = y_t + \eta_t, \quad (2.4.5)$$

where y_t is the true value underlying the signal, hence, is not directly observable but can only be inferred by other data. The noise is again represented by η_t .

Although the difference seems to be marginal and maybe negligible it changes the whole character of the situation. Whilst before $y_{t|t} = E_t(y_t)$, this no longer holds for Equation (2.4.5). In fact, $y_{t|t} \neq E_t(y_t)$ but $E_t(y_t) = [\sigma_y^2 / (\sigma_y^2 + \sigma_\eta^2)] y_{t|t}$. An increase in the variance of η_t now poses an effect on the best estimate. The reason for this change originates from the different way the problem is tackled now (Swanson 2004).

In Equation (2.4.2) the estimate was already declared as being *best*. In this case data uncertainty collapses merely into additive uncertainty, i.e., certainty equivalence. Data uncertainty does not impose any restriction or constraint to the policy maker. In fact, to her there is no problem at all as she behaves ‘as if’ everything is known with certainty. Working with observed variables which fulfill the condition of being the best estimate vanish somewhat the problem of uncertainty.

However, if instead the formulation of Equation (2.4.5) is used, the problem becomes one of *signal extraction*. Now, the policy maker is faced with the problem of inferring the true value of a variable which can not be observed but must be deduced from a signal which stands in an assumed relationship to the the variable of concern (Orphanides 2003a, Swanson 2004).

If this problem of potentially noisy data is ignored and the interest rate policy is conducted in the way of Equation (2.4.1), i.e., utilizing $y_{t|t}$ instead of the *best* estimate, which leads to a distorted policy reaction function. Working with the observed signal would deliver over- or under-reactions depending on the noise, leading to a non-optimal policy response, and thus, to a non-efficient situation. This becomes obvious if a simple rule in which the interest rate is set in accordance only with the observed state variable gap of the form

$$i_t = \gamma_x x_{t|t} \quad (2.4.6)$$

is assumed.

If Equation (2.4.5) holds, that is, the noisy observation is used instead of the best estimate the interest rate is set according to

$$i_t = \gamma_x x_{t|t} \quad (2.4.7)$$

$$= \gamma_x x_t + \gamma_x \eta_t. \quad (2.4.8)$$

The underestimation of the output gap, for example, due to an overestimation of the potential output, could thus lead to an expansionary policy, which is in fact not adequate. Clearly this naive policy is absolutely adequate if there is no noise. In this case the problem collapses into a simple optimization exercise (Orphanides 2003a, Walsh 2003b).

In the presence of noisy data the central bank reacts inadvertently to the noise. The greater the noise is the more volatile interest rate changes become. Taking further noisy variables into account could exacerbate the problem even more. If for example the interest rate is additionally set as well in accordance with the inflation gap

$$i_t = \gamma_x x_{t|t} + \gamma_\pi (\pi_{t|t} - \pi^*) \quad (2.4.9)$$

$$= \gamma_x x_t + \gamma_\pi (\pi_t - \pi^*) + \gamma_x \eta_x + \gamma_\pi \eta_\pi, \quad (2.4.10)$$

the error extends to $\gamma_x \eta_x + \gamma_\pi \eta_\pi$, consisting of the noise related to the output gap and the noise related to the inflation gap.

Orphanides (2003b) shows how the the misconception of the US production capacity may have led to an inflationary environment during the 1970s by restricting the ex post analysis to the information set up of those days. For a period from the mid eighties until the mid nineties Orphanides (2001) highlights that, despite output and inflation revisions, which may have contributed to the difference between the actual federal fund rate and a hypothetical Taylor rate based on final data, historical interpretations may also have fostered this effect. Thus, deviations of the actual rate compared to a Taylor rate with

final data may have been also undergone evaluations which were interpreted differently these days than it would be today.

Figure 2.5 plots the actual federal fund rate together with a hypothetical Taylor-rule type interest rate.¹⁴ The latter has been calculated using two different vintages of output gap estimates. What matters is not the comparison of the estimated rate and the actual interest rate, but the difference between using real-time data (first estimate) and revised data (final estimate). The wrong appraisal of the output gap (see the lower part of Figure 2.3) transmits itself into a possibly wrong interest rate setting. Thus, quite illustrative for the period 1994 until 2000, it can be seen that an over-estimation of the output gap is accompanied by a higher Taylor rate compared to the rate which would be realized if the final values for the output gap were employed.¹⁵

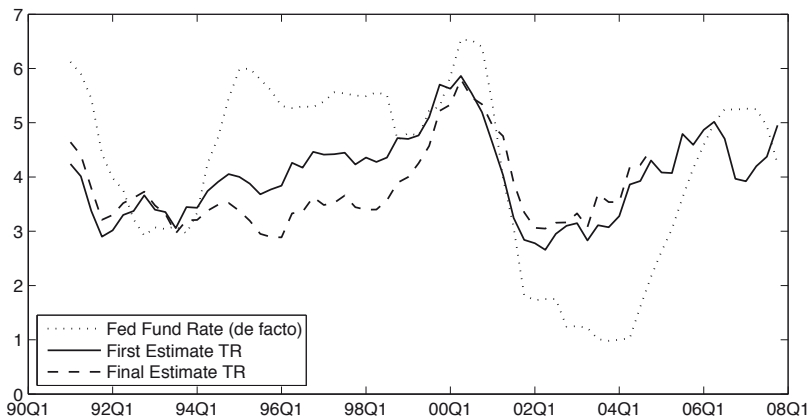


Figure 2.5.: Taylor rates with different vintages of output gap estimates

The more noisy variables are taken into account, the larger the possible error which could be made by the monetary authority. If, however, less variables can serve a less volatile interest rate, due to a smaller additional reaction term, a generally good advice would suggest the central bank merely ‘less activism’ in her policy, especially when the noise is plenty.

¹⁴ The assumed Taylor rule has the form $i_t = 4 + 0.5(\pi_t - 2) + 1.5y_t$ with π_t the actual inflation rate and y_t the OECD output gap estimate (Woodford 2001). It is acknowledged that there are several ways to calculate a Taylor interest rate using, e.g., different values for target variables or utilizing average values for a smoother character. However, the present rule is sufficient for this analysis.

¹⁵ Of course it is again noticed that several factors constitute the decision of the Fed and the output gap is just one of many, which can explain as well the difference between the hypothetical Taylor rate and the actual Fed rate like Orphanides (2003b) mentions.

With respect to a stabilizing policy, Orphanides (2003b) brings up two alternative rules to overcome the information problem of level values. In the case of high uncertainty over a state variable estimate, it could be utile to exclude this variable from the policy rule at all. In case of a noisy output gap, it could thus be superior to head for a strict inflation targeting, and by that ignoring GDP measurement errors at all. The effect of an exclusion of one variable can be seen by comparing Equation (2.4.7) with (2.4.9) where the first equation excludes the inflation noise by excluding inflation from the target catalog. One possible outcome of this procedure could be a smoothing of the interest rate and in fact, Rudebusch (2002) argues that, interest rate smoothing can be ascribed to incorrectly excluded variables from the reaction function.

Another possible way, could be the replacement of the output level by its growth rate due to the fact that the change of a gap is less sensitive to measurement problems. Orphanides (2003b) labels those alternatives as ‘prudent’ reaction functions. Under perfect information, however, those rules are not superior. They unfold only if noisy data is taken into account.

2.5. A Further Note on Certainty Equivalence

2.5.1. Certainty Equivalence in Risk Theory

If a system exhibits some special properties, it is said to be certainty equivalent. These necessary conditions are a quadratic loss function in combination with a linear constraint. However, due to the fact that researchers who come from risk theory and those who come from monetary policy theory might have a different sentiment about it, I will discuss this concept briefly at this juncture, and show the interrelations it has to the so far discussed forms of uncertainty.

Since the utility of consumption is a non-linear function it can not be assumed that $E[u(C)] = u[E(C)]$, that is, the expected utility of a given consumption level does not equal the utility of an expected consumption level. A rising curvature of the utility function is accompanied by a rise of this discrepancy. In risk theory this curvature represents the risk aversion of the agent. In general, an agent is risk averse if $u'' < 0$, risk neutral if $u'' = 0$, and risk loving if $u'' > 0$.

Figure 2.6 captures the idea. On the vertical axis utility, and on the horizontal axis consumption is depicted. Both consumption levels, C_1 and C_2 , deliver a utility level via the utility function of $u(C_1)$ and $u(C_2)$, respectively. If uncertainty about the attainable level of consumption prevails, the expected level of consumption would be merely the arithmetic average of C_1 and C_2 . If both consumption levels are equiprobable, hence,

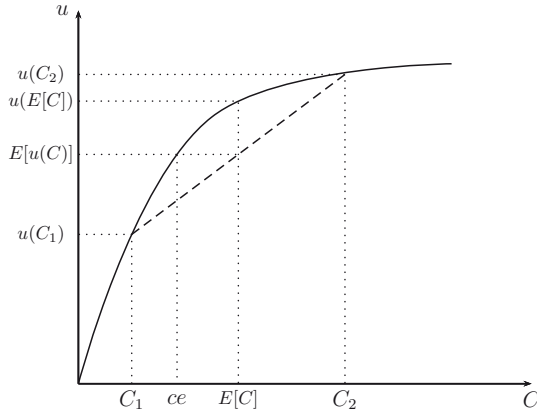


Figure 2.6.: Non-linear utility function

$p = 0.5$, the expected level of consumption lies right in the middle of C_1 and C_2 , designated by $E[C] = p \times C_1 + p \times C_2$.

The attained utility from this expected consumption level is achieved by connecting the two possible extreme outcomes of consumption on the utility function with a straight line, and plumbing it to the vertical axis. It delivers the expected utility $E[u(C)] = p \times u(C_1) + p \times u(C_2)$. As can be seen in Figure 2.6, the expected utility level deduced from the connecting line lies below the utility that would be achieved if the hypothetical consumption level would be actually realized, $E[u(C)] < u(E[C])$. The opposite holds for a risk loving agent, i.e., $E[u(C)] > u(E[C])$. This implies that a risk averse agent would rather take the expected level of consumption than to play.

To make this point clear consider the following numeric example. The utility function is given by $u(C) = \sqrt{C}$. Let $C_1 = 1$ and $C_2 = 9$ and the probability of each outcome be 0.5. Hence, $E[u(C)] = E[0.5\sqrt{1} + 0.5\sqrt{9}] = 2$. Yet, $u(E[C]) = \sqrt{5} = 2.24$, which states that this agent is clearly risk averse. The difference between these two utility levels is that one represents a certain outcome, whilst the other represents merely an expected, and thus, uncertain outcome. The question arises how much consumption with certainty would be needed to be exactly as well off as if the risky opportunity would be chosen, not knowing whether C_1 or C_2 is obtained.

The agent would be indifferent if both utility levels equal each other, that is, the utility received in the certainty case and the utility received in the uncertainty case. However, in this case the level of consumption would differ. The difference between the certain consumption level, marked as ce , and the expected level, $E(C)$, which give both the same

utility, is known as the risk-premium. It can be stated that if $ce < E[C]$ the agent is risk averse, hence, she is willing to accept a less but certain level of consumption in favor of an expected higher but uncertain level of consumption. Clearly the opposite holds for a risk loving agent, i.e., $ce > E[C]$.

The more curvature the utility function exhibits the larger is the discrepancy between the certain payoff and its expected counterpart. The curvature represents the risk awareness of the consumer. As she becomes more and more risk averse she is more and more willing to accept a smaller amount of certain consumption, compared to the expected, but uncertain amount of consumption. The less curvature the utility function exhibits, the smaller the risk premium gets. In the extreme case of no curvature, i.e., the utility function is a straight linear line, the risk premium approaches zero and it always holds that $E[u(C)] = u(E[C])$.

It can easily be understood that a high risk awareness is more sensitive to uncertainty than a low one, and that risk neutrality is not affected by uncertainty at all. Figure 2.7 displays two different utility functions, U_1 and U_2 . Further, different amounts of uncertainty, expressed by a different variance of the consumption level, are given. It can be seen that a higher amount of uncertainty calls for a higher risk premium to compensate the agent if U_1 prevails. The higher the variance, the greater the discrepancy between the certainty equivalence value and $E[C]$.

If, however, the agent becomes less risk averse, the utility function U_1 shapes more and more towards U_2 . If the utility function is actually linear, i.e., the agent is risk-neutral, the degree of uncertainty about the consumption level has no influence on the expected utility of consumption at all. Considering the same choice as before, i.e., between a certain payoff and a risky one, ce and $E[C]$ of Figure 2.6 are always congruent if U_2 prevails. A rising degree of uncertainty, which is equivalent to a wider range between the two possible payoffs, has no influence any more (Varian 2000).

2.5.2. Parametric Uncertainty and Certainty Equivalence

Following the previous argumentation, yet, applied on monetary policy under uncertainty yields the following. If the transition equation of the economy is linear, and the loss function has a quadratic form (positive and negative deviations are penalized equally), the degree of uncertainty in the system does not influence the reaction function except that only expected values of the system are taken into account. Nevertheless the utility is certainly affected.

To see why certainty equivalence ceases to hold in the case of multiplicative uncertainty, opposed to additive uncertainty, Figure 2.9 covers the case of additive uncertainty and

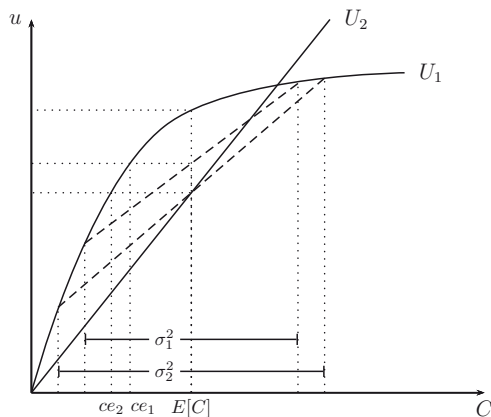


Figure 2.7.: Utility function and uncertainty

Figure 2.10 the case of multiplicative uncertainty. In both figures the policy instrument r is given on the horizontal axis, and the target variable y on the vertical axis. The relation between the policy variable and its target variable shall be given by the solid, linear, and negative sloped line and is of the form $y = c - ar$ with c being a constant and a some sort of reaction coefficient. The intuition is, given some value for c a rising interest rate depresses the overall economic output, i.e., a lower y . Hence, there is a negative connection between the policy instrument and the target variable.

In a certain environment each change in the policy variable would induce a predictable change in the target variable. Or in a revers order, to find a desired level of output, the necessary interest rate level can be easily deduced via the linear relationship. This is shown in Figure 2.8. It is assumed that the level of output is below its desired level, marked by the subscript f . In order to achieve this desired level the interest rate must be lowered on $r^* < r$.

If additive uncertainty enters the set up the expected value of the target variable and the realized value may differ. The simple linear relationship of before is amended by an exogenous, zero mean shock vector and changes into $y = c - ar + \epsilon$. In Figure 2.9 the expected level of output, given some interest rate level r , is marked by the solid line. The actual realized value of output could be within a band around its expected value which is suggested by the dashed lines. Hence, setting the policy instrument at r_1 would on average result in y_1^E , but could also be somewhere within the range y_{11} and y_{12} . This range is also shown by the vertical solid lines.

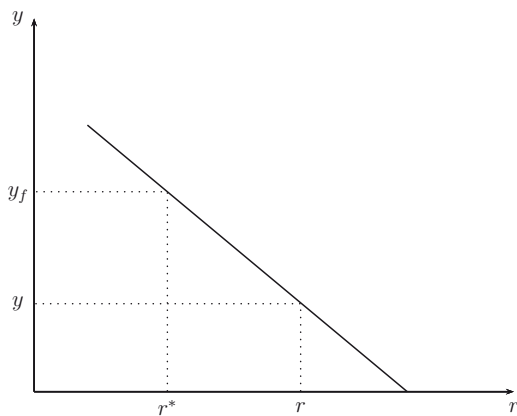


Figure 2.8.: Certainty in a linear framework

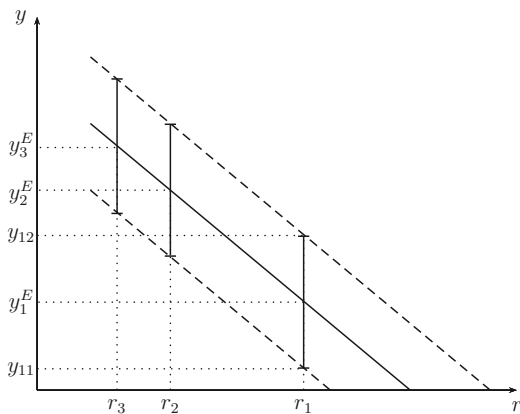


Figure 2.9.: Additive uncertainty in a linear framework

If again the output is lower than desired, the policy maker would implement the same policy as under certainty to reach the desired level of output. On average she achieves with the interest rate r_2 the output y_2^E , but again she could also end up in the range marked by the middle horizontal solid line. Figure 2.9 shows very nicely how the variance of the error term is transmitted one-to-one to the target variable, which itself has now become a random variable with $\sigma_y^2 = \sigma_\epsilon^2$.

What is striking is the fact that the uncertainty can not be reduced actively by the policy maker. Setting the interest rate from r_1 to r_3 would not be different from a change of r_1 to r_2 in terms of potential output deviations. The variance of output remains the same no matter the change of the control variable. The variance of the target variable is thus independent of the policy parameter setting. The policy maker acts in a situation of certainty equivalence. Although the setting of the policy parameter is not different under certainty than under additive uncertainty, clearly the policy maker is worse-off under uncertainty due to the potential output deviations from its desired level (Sack and Wieland 2000).

With multiplicative uncertainty the picture changes. Due to the fact that the transmission from the policy instrument to the target variable, given by the parameter a , is uncertain, the variance of the target variable is now $\sigma_y^2 = \sigma_a^2 r^2 + \sigma_\varepsilon^2$. Thus, the variance of output depends from the uncertainty of the slope, but is as well increasing with the size of the policy reaction parameter. The comparison in Figure 2.10 shows, the size of an interest rate change matters for the potential output deviation marked again by the solid horizontal lines. In case of a wrong estimation concerning the transmission parameter, setting the interest rate from r_1 to r_3 creates a potentially larger output deviation than raising the interest rate only from r_1 to r_2 . It becomes evident that certainty equivalence ceases to hold. The variance of the output can be actively reduced by the setting of the policy instrument. From an analytical point of view the expected value of the value function depends on the variance-covariance matrix of the state vector. If all parameters are non-stochastic, this matrix must coincide with the variance matrix of the disturbance vector and is thus independent of the instrument variable. If on the other hand the variance-covariance matrix depends also on the state or instrument, this matrix can be influenced by the central bank (Sack and Wieland 2000, Söderström 2002).

This setting shows the earlier discussed trade-off situation for the policy maker. On the one hand an aggressive policy reaction will move output closer to the desired target level on average, but on the other hand this will also create a higher uncertainty. Hence, a higher variance of the resulting level of output. A possible solution to this problem would be the partition of one big step into several small steps as shown in Figure 2.11. This confirms the previous finding, whereby under parametric uncertainty the optimal policy reaction does not yield a complete target achievement. The underlying idea of this behavior is a ‘learning’ policy maker. Given that some desired level of output is only achievable with tremendous uncertainty concerning the transmission, and thus, the resulting output levels, sequential steps could improve the situation. Instead of one big interest rate step, the policy maker only sets the rate from r_1 to r_2 . After that she observes the consequences of his action and takes these into account for his second assessment, setting the rate from

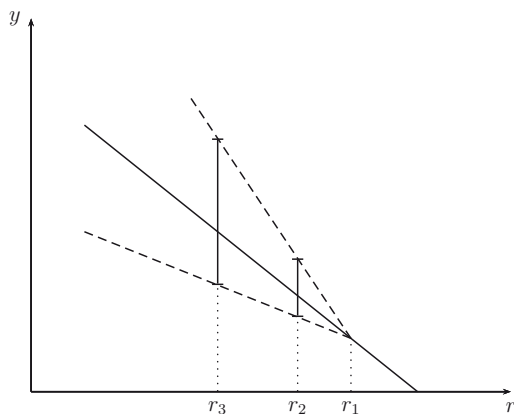


Figure 2.10.: Multiplicative uncertainty in a linear framework

r_2 to r_3 . By that she narrows the potential uncertainty of missing the desired level of output y_f (Sack and Wieland 2000).

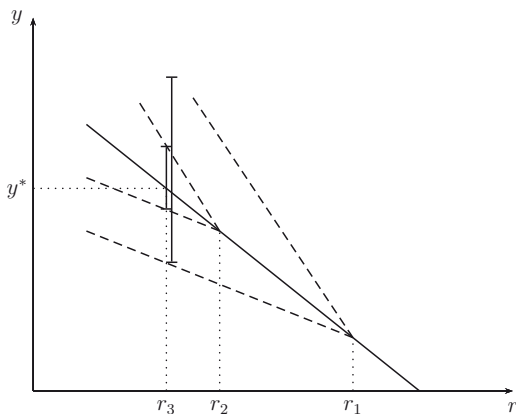


Figure 2.11.: Multiplicative uncertainty, several vs. one step

2.5.3. Data Uncertainty and Certainty Equivalence

Confronted with the problem of signal extraction and not merely with an additive error term, the monetary authority needs to work with the *best* estimate – or, in other words,

a not biased estimate – to react ‘optimal’. Yet, the question arises whether the forming of an optimal estimate can be done without any reference to the control problem. If so certainty equivalence should prevail and monetary policy can be conducted the standard way, replacing uncertain variables simply by their best estimates.

In fact, this separation is admissible. The *separation principle* states that in order to behave optimal, the monetary authority should first optimize ‘as if’ everything is known with certainty and should secondly substitute her optimal forecast for the unknown variable (Hansen and Sargent 2005). Figure 2.12 captures this idea.

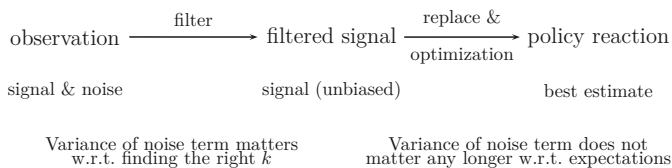


Figure 2.12.: Separation principle

It is assumed that the observation of a variable is contaminated by noise. This noise is filtered out by an appropriate filter such as the Kalman routine which uses all available measurement data to calculate in a predictor-corrector manner the best estimate. Using an initial guess or measurement, which is compared with a second observation, delivers a more accurate data point, which can be used again to be compared with a third measurement and so on. The goal of the process is to find the best estimate of the unobservable or noisy variable. Thereby best is defined as the minimum variance estimate. At the end there should be an estimate of an unobserved variable, which indeed has the lowest, that is best, variance.

The big advantage of this filter algorithm is that not every past data must be kept in storage. All past data are included in the latest best estimate. Hence, the problem can always be treated as a two period problem with ‘old’ and ‘new’ information. This improves the filter also from a technical point of view and contributes to its popularity (Maybeck 1979).

Not to go too much into detail, Equation (2.5.1) shows that the assessment of the variable $x_{t|t}$ depends on the past estimate $x_{t|t-1}$, and the new information gathered by observing z_t .

$$x_{t|t} = (1 - k_t)x_{t|t-1} + k_t z_t \quad (2.5.1)$$

So the best estimate of $x_{t|t}$ is constituted by the estimate made without the information of the new observation, $x_{t|t-1}$, and the new information, z_t . Thereby, more weight is given to the less uncertain part. The weight parameter k_t is obtained by differentiating Equation (2.5.1). Equipped with the new information of the observation z_t and the optimal weight k_t the posterior variance is calculated. This is in turn used to form an estimate and the according variance for the next period.

In general, the so-called *Kalman gain* which minimizes the variance of the estimate is given as

$$k_t = \frac{p_{t|t-1}}{(p_{t|t-1} + \sigma_v^2)}. \quad (2.5.2)$$

with $p_{t|t-1}$ denoting the prior variance and σ_v^2 the noise of the observation.

The general prior variance is

$$\begin{aligned} p_{t|t-1} &= \rho^2 p_{t-1|t-1} + \sigma_u^2 \\ &= \rho^2 \left(\frac{1}{p_{t-1|t-2}} + \frac{1}{\sigma_v^2} \right)^{-1} + \sigma_u^2, \end{aligned}$$

with ρ depicting the parameters, $p_{t|t}$ the posterior variance, and σ_u^2 some additive noise term of the process of interest. It can be seen from Equation (2.5.2) if the noise of the measurement becomes zero the Kalman gain equals one, that is, the observations are fully taken into account in Equation (2.5.1). If, however, the prior variance gets smaller compared to the noise variance, the Kalman gain gets smaller, and thus, the newly measurements become less important compared to the prediction based on the past observations. The estimate gets better and better the more observations are used. At the end of this process stands the best estimate of the unobserved variable. This estimate entitles the decision maker to form unbiased expectations regarding the unobservable variable.

The proper working of this filter is shown by Figure 2.13. For 200 periods it is assumed that the true process follows a state equation of the form $x_{t+1} = ax_t + u_{t+1}$. The observation equation is given by $z_t = cx_t + v_t$.¹⁶ The simulated true time series is observed with noise. Yet, despite the noise in the observation, the Kalman filter delivers a series, which is most of the time quite close to the true values.

For the calculations of the filtered variables the size of the variance of the disturbance is absolutely relevant. It defines the size of the Kalman gain vector k , and thus, the weight placed on the new information, seen in Equation (2.5.2) and (2.5.1). However, after the

¹⁶ The parameterization, albeit not crucial if in certain ranges, is $a = 0.8$, $c = 0.7$, $\sigma_u^2 = 0.001$, $\sigma_v^2 = 0.01$. The Kalman filter can be used to overcome data uncertainty, however, the most popular usage is the determination of model parameter in combination with a maximum likelihood estimation.

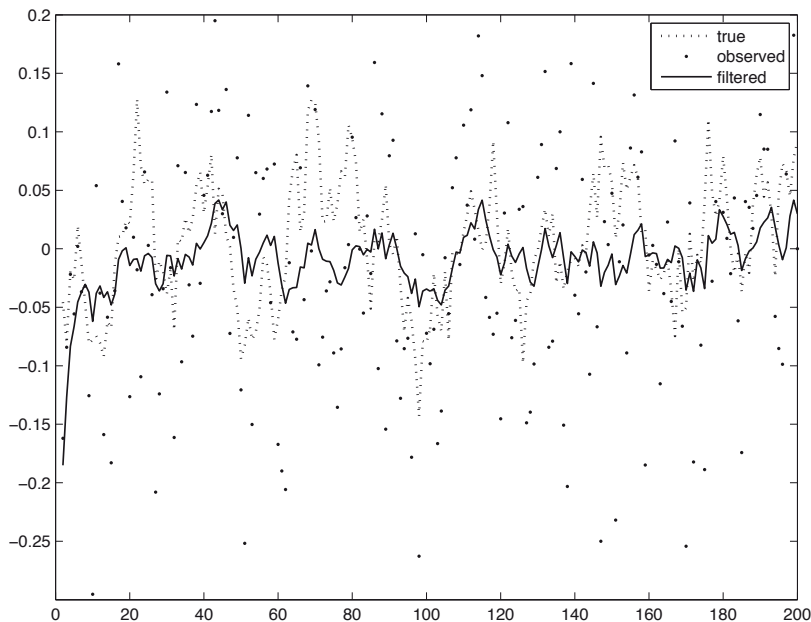


Figure 2.13.: Simulated times series with Kalman filter

observations have undergone the filter process the resulting best estimate entitles the decision maker to calculate the optimal policy reaction (Kalman 1960, Hamilton 1994).

Orphanides (2003a) clarifies with respect to data uncertainty that it is not true, whereas this ‘additive’ uncertainty does not matter at all. The uncertainty, measured by the variance of the error term, matters with respect to finding the optimal weight, which varies with the size of the error term. But the measurement error, or better its variance, does not matter with respect to the conditional expectations once the noise is filtered from the data.

The *separation principle* states the separation between optimizing and filtering. The order of these two steps is irrelevant. Whether first the optimization is conducted and second expectations are formed or the other way around does not matter. It is admissible due to the fact that an optimization is feasible even without knowing the true value of the exogenous variable by solving the decision makers Euler equation for the control variable. After that, in a second step, the conditional expectation of the exogenous variable can be deduced, given the distribution function of the shock and replacing the variable from the first step (Svensson and Woodford 2003b, Hansen and Sargent 2005).

Hence, it can be said that data uncertainty *per se* does not exhibit certainty equivalence. It depends whether the issue is tackled merely as an additive uncertainty problem or as a signal extraction problem, i.e., if the estimate of a given variable is biased or not. An unbiased estimate changes the problem into one of additive uncertainty, where the principle of certainty equivalence is fulfilled. If the problem is one of signal extraction and one has to infer a signal from noisy observations, the uncertainty or noise surrounding this signal is crucial and must be filtered out. In this sense, the filtering process is sensitive to uncertainty, however, once the noise has been removed, a standard optimization problem remains.

2.6. Interest Rate Smoothing due to Uncertainty

To give a short summary, this chapter has dealt with three different forms of uncertainty – namely additive, parametric, and data uncertainty – and already quarried some important insights. It has been shown that under additive uncertainty the policy rule is impervious to the exogenous shock vector. This finding, which only applies to very restrictive cases, is known as the *certainty equivalence* principle. On the other hand, Poole (1970) has shown that while the policy rule might be unaffected by additive uncertainty, the loss of the central bank in terms of deviations from a given target variable is not. Poole (1970) demonstrates that depending on the source of the shock, it could be more or less favorable to utilize the interest rate or the money supply as appropriate instrument of choice, due to the different impact they have on the loss function.

The most prominent example on monetary policy under uncertainty is by all means Brainard (1967). His seminal work demonstrates how the optimal rule for setting the interest rate is affected by the uncertainty surrounding the model parameter. He shows that under multiplicative uncertainty the concept of certainty equivalence ceases, thus the monetary authority must take account of the uncertainty compared to the case with additive uncertainty. The so-called *Brainard conservatism* suggests a rather moderate reaction to shocks in an uncertain environment. Although this concept has been criticized in many ways it still prevails in conducting monetary policy today.

Furthermore, the example of data uncertainty has revealed some important findings regarding the fact that most of the underlying economic data is only available with a significant lag and object of several data revisions. It has been shown that data uncertainty *per se* is not sufficient to tell whether or not certainty equivalence prevails. If the estimate of an uncertain variable is called to be *best*, the monetary authority can go on in setting her policy working with this estimate instead of the true but hidden value. Nevertheless,

in achieving this *best* estimate an appropriate filter algorithm – like the Kalman filter routine – is needed, which definitely takes account for the uncertainty.

These findings are mirrored in the path of the main refinancing rate of the most important central banks such as the Federal Reserve, the Bank of England or the European Central Bank, all of which plotted in Figure 2.14. They all offer a similar pattern. Most of the time interest rates (i) move for a long time into the same direction, that is reversals are kept on a minimum. (ii) Steps into one direction are rather parceled into small but many than big but few steps. (iii) Interest rates are often kept constant for several subsequent periods. This pattern of partial adjustment has often been labeled gradualism or interest rate smoothing. The literature offers several explanations for this property, which can be divided into two big classes. The first class can be described by the expressions ‘shaping expectations’. The second can be labeled ‘uncertainty’ (Goodhart 1999, Sack and Wieland 2000, Gerlach-Kristen 2004).

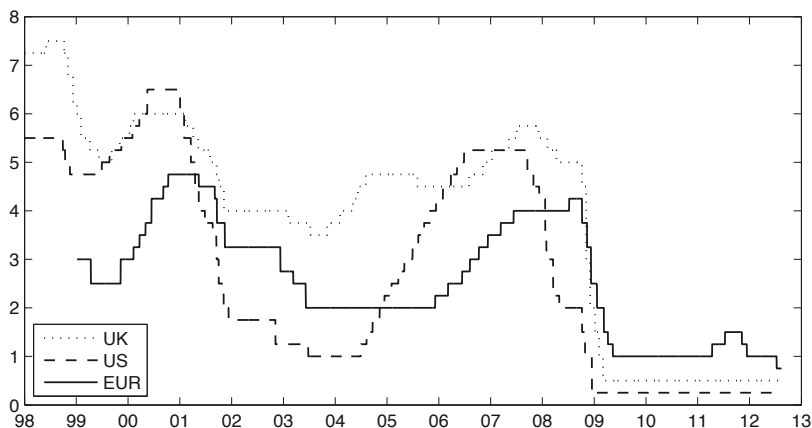


Figure 2.14.: Interest rates of major central banks

For the first class it can be stated that, interest rates move slow due to some sort of dealing or utilizing a forward-looking behavior of market participants. The agents infer future steps of the central bank by observing past and present decisions. Small steps into one direction (for a sufficient long period) would thus shape expectations of future steps and provide a clear guidance to the public. By that, controlling the short-term interest rate would trigger off long-term interest rates and enforcing the power of the short-term nominal interest rate. Combined with a sufficient communication this policy

would increase the effectiveness of monetary policy even though only small interest rate changes are conducted (Goodfriend 1991, Goodhart 1999).

Another explanation within this class is given by Goodfriend (1987). To him central banks smooth interest rates in order “to maintain ‘orderly money markets’ ” (Goodfriend 1987:339). This behavior would preserve the economy of large swings in asset prices followed by a higher risk of bankruptcies and banking crises. These kinds of factors, where the central bank actively manages the interest rate in order to shape expectations, are also called *intrinsic* or *endogenous*. A typical setting would be the partition of a desired interest rate change into several small steps, dragging the adjustment process over a longer period (Rudebusch 2006). Other endogenous reasons might be the building up of reputation and the consensus decision-making process on part of the central bank (Sack and Wieland 2000).

Despite these explanations, and first and foremost to a greater extent relevant for this work, uncertainties can provide a good reasoning for a gradual stance of monetary policy. This has been shown in the previous sections. If the central bank lacks of information, and thus, is unable to set its interest rate in an accurate and appropriate manner it creates a persistence of the policy rate. This kind of interest rate inertia is said to be *extrinsic* or *exogenous*. The policy rate reacts with respect to the validity of given parameters and is set with caution. As the information level changes policy actions may change as well (Rudebusch 2006).

For the case of data uncertainty most of the literature considers a less aggressive interest rate policy as appropriate. These findings hold under simple rules, such as the Taylor rule in form of reduced response coefficients, as well as under optimal policy rules (see, e.g., Cagliarini and Debelle 2000, Sack and Wieland 2000, Rudebusch 2006). This chapter has shown, if noise prevails in the data, a monetary authority would inadvertently react on this noise with the consequence of a higher interest rate volatility. A reaction function based on noisy data is likely to over- or underestimate the proper interest rate reaction, also inducing unnecessary interest movements. So the mere awareness of noise in the data should lead any central bank to be react more caution which could be the explanation for a smooth interest rate path (Cagliarini and Debelle 2000, Smets 2002, Orphanides 2003a).

Under parametric uncertainty, as the Brainard example has demonstrated, uncertainty enters the reaction function of the central bank, recommending a more cautious policy than under certainty due to the fact that certainty equivalence does not apply. This was demonstrated before in Section 2.5.1. Not knowing the exact transmission of interest movements should call for a less aggressive stance in monetary policy in order to avoid exaggerations due to unpredicted consequences. Although, this result, as shown for example in Craine (1979), Cagliarini and Debelle (2000), or Söderström (2002), does not

hold in general. The question, whether one should be more or less aggressive cumulates in the case of model uncertainty, which some authors interpret as parameter uncertainty as well. However, in this case nearly no general answer about how the optimal interest rate rule should be set can be given, due to the fact that the right answer depends on so many factors (Rudebusch 2006).

Concluding, interest rate inertia maybe due to an active stance of monetary policy aiming for some sort of expectation shaping. Nevertheless, without assuming an active, or endogenous interest rate smoothing, uncertainty may be the main cause for smoothing interest rates. This again highlights the very early mentioned two challenges of monetary policy, which have been on the one hand the appropriate handling of uncertainty and on the other the active shaping and control of expectations.

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