Preface

This volume reports on recent developments in the theory of vertex operator algebras (VOAs) and their applications to mathematics and physics. Historically the mathematical theory of VOAs originated from the famous monstrous moonshine conjectures of J.H. Conway and S.P. Norton, which predicted a deep relationship between the characters of the largest simple finite sporadic group, the Monster, and the theory of modular forms inspired by the observations of J. MacKay and J. Thompson.

Although perhaps implicitly present earlier in conformal field theory, the precise mathematical notion of vertex algebras first emerged from the work of I. Frenkel, J. Lepowsky and A. Meurman and their purely algebraic construction of a vertex algebra with a natural action of the Monster group, laying the foundations for Borcherds' later proof of the moonshine conjectures. Indeed, by isolating the underlying mechanism from analytical aspects, physical field theories and the explicit examples thus shaped the axiomatic definition of vertex algebras as purely algebraic objects, opening a rich new field. By studying them for their own sake, R. Borcherds not only gave the theory its precise form, but also succeeded in proving the moonshine conjectures with the aid of these new concepts. So, it is quite interesting that unlike other algebraic structures like fields, rings, algebras or Lie algebras, the concept of vertex algebras appeared comparatively late in the mathematical literature. However, looking back, even today the underlying mechanism played by vertex operator algebras connecting representations of certain simple groups and modular forms remains to be mysterious. Altogether, Borcherds' results on the monstrous moonshine conjecture relating the representation theory of the monster group with modular forms using the construction of the vertex algebra $V^{\natural}$, whose graded dimensions are the Fourier coefficients of $j(q) - 744$, are a major landmark in the theory. They have led to many subsequent investigations of the structure of VOAs, some of which are addressed in this volume. Another theoretical milestone was Y. Zhu's finding that for rational VOAs (essentially those vertex operator algebras whose category of admissible modules is semisimple), every irreducible representation of a rational VOA gives rise to an elliptic modular form. Hence rational VOAs and certain generalizations have since been studied intensively.

Independently from Zhu's work it should be mentioned that also more general types of modular forms, defined by their Fourier expansion as counting functions, naturally arise in the theory of vertex algebras via the Kac denominator formula of generalized Kac-Moody Lie algebras derived from certain distinguished VOAs and derived generalized Lie algebras. In fact, thereby not only elliptic modular forms seem to appear naturally in the theory, but also modular forms of several variables. Of course there are other interesting circumstances under which modular forms of higher genus naturally occur, some of which will be addressed in this volume.

In a remarkable development, A. Beilinson succeeded in further generalizing the concept of vertex algebras by stressing the importance of the underlying geometric space. Thanks to the combined achievements of Beilinson and V. Drinfeld we now know that vertex algebras are a special case of chiral algebras, where these chiral algebras are certain sheaves on algebraic varieties. It seems, at least if the underlying geometry comes from a Riemann surface, that this new aspect indicates deep connections between conformal field theories, class field theory and various other branches of mathematics.

Quite recently, the study of representations of vertex algebras has produced surprising new developments for simple groups G other than the Monster group. Here, especially the Mathieu groups play a prominent part, with interesting applications to the theory of black holes, which has since become a very active field.

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We wish to extend our sincere thanks to all contributors to this volume and all conference participants, with special thanks to Geoffrey Mason and Miranda Cheng for their excellent preparatory courses that were held prior to the conference. Geoffrey's course entitled Vertex Operator Algebras, Modular Forms and Moonshine is included as an appendix to this volume.

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