

# Preface

The scientific literature offers many excellent textbooks on the elasto-plastic behavior of structural members. However, many students who are entering this research area find it difficult to master this topic based on classical textbooks. To facilitate the understanding of the theory, concepts, and involved algorithms, this textbook offers for students and researchers who are new in this topic a simpler introduction to elasto-plastic material behavior and its computational treatment.

In this intention, the focus of this textbook is on two simple one-dimensional structural members, i.e., the rod and the beam. A classical approach is presented where first the governing differential equations are derived based on the basic equations of continuum mechanics. The corresponding analytical solutions serve many times to judge the accuracy of derived approximate solutions. The focus in regard to approximate solutions is on the finite element method. This method can be regarded nowadays as the standard tool to solve engineering problems in the industrial context. The treatment of one-dimensional members allows to keep the mathematical notation and treatment relatively simple. Nevertheless, the derivations are presented in such a way that a transfer to higher dimensions can be achieved. Thus, the considered one-dimensional elements are from an educational point of view a good demonstrator to introduce basic concepts of continuum mechanics and numerical strategies for approximate solutions. Pure elasticity is treated first and then the important case of nonlinear material behavior in the elasto-plastic range is considered.

Furthermore, rod and beam elements have structural analogies in higher dimensions. The procedures applied to the rod element can be transferred to three-dimensional solid elements. Knowing the theory of one-dimensional beam elements allows the understanding of two-dimensional plate elements. Thus, a solid understanding of the basic equations and procedures for one-dimensional elements allows an easier access to elements of the two- and three-dimensional space.

Chapter 1 illustrates a few technical applications, which can be modeled in a simplified approach based on one-dimensional elements. Chapter 2 introduces the basic equations of plasticity theory. The yield condition, the flow rule, and the hardening rule are introduced. After presenting the equations for the

one-dimensional stress and strain state, the equations are generalized for a two-component case. Looking at the two-component case, the notation stays relatively simple and the major differences between the one-dimensional case and a higher dimensionality can easily be shown. Chapter 3 covers the analytical description of rod or bar members. Based on the three basic equations, i.e., the kinematics relationship, the constitutive law, and the equilibrium equation, the partial differential equation which describes the problem is derived. Analytical solutions in the elastic and elasto-plastic range for different loading and boundary conditions are derived and discussed. Chapter 4 follows the approach of the previous chapter and introduces beam elements. Three different types of theories are treated, i.e., the EULER–BERNOULLI theory, the TIMOSHENKO theory and higher-order beam theories. Chapter 5 introduces the finite element theory for elastic problems based on rod, beam, and generalized beam elements. First, each element type is considered separately, then in pure one-dimensional structures and finally the case of plane frame structures which allows to arrange and combine different elements. Chapter 6 covers elasto-plastic finite element simulations, an important type of nonlinear problems. The solution strategy is derived for pure one-dimensional problems. However, the derivations are presented in such a way that a transfer to higher dimensions is easily possible. Chapter 7 is devoted to an alternative approach to derive approximate solutions. Based on one-dimensional problems, the finite difference method is introduced for elastic and elasto-plastic problems. Chapter 8 summarizes the basic equations for a three-dimensional continuum. The derived partial differential equations serve to derive finite element procedure for solid elements. The chapter concludes with an introduction to elasto-plastic finite element simulations.

In order to deepen the derived equations and theories, each technical chapter collects at its end supplementary problems. In total over 120 of such additional problems are provided, and a short solution for each problem is included in this book. It should be noted that these short solutions contain major steps for the solution of the problem and not only, for example, a numerical value for the final result. This should ensure that students are able to successfully master these problems. I hope that students find this book a useful complement to many classical textbooks. I look forward to receive their comments and suggestions.

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